# The Trader's Dilemma: Trading Strategies and Endogenous Pricing in an Illiquid Market

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## Abstract

We investigate a large trader's trading strategies in a decentralized market, in which all traders are subject to type switching. This trader has pressure to liquidate her position by the end of the horizon so as to avoid extra holding costs. She faces a trade-off: if she trades quickly, she moves the price too much; if she trades slowly, she may not be able to find counterparties in the market in later periods. We derive subgame perfect equilibria under three different spot market structures. These structures are chosen to show various degrees of competitive bargaining. We show that, in each equilibrium, the large trader chooses the optimal trading strategy to take into account both the price impact effect and liquidity uncertainty. Thus, asset prices are generated endogenously through a dynamic bargaining and trading process and reflect the impact of the large trader's trades. Small traders, who possess little market power, cannot be ignored because their reactions to the large trader's trading strategy jointly determines market liquidity. We show that limiting case of competitive pricing occurs when there are enough small traders, or there are many trading periods. Illiquidity is a result of the thin market for buyers, and their limited capacity to buy the asset sold by the large trader.

## 1. Introduction

There has been increasing interest in the impact on asset prices of large traders' trading strategies. The fact that a large trader's actions can be significant enough to move prices is a significant concern for large institutional investors. This price impact of trading has been verified by many empirical studies and exists in almost all kinds of markets.[1] Financial distress can occur when investors find themselves in desperate need to close out long positions and market liquidity dries up. A recent well-known occurrence was the LTCM crisis in 1998. Studies of this crisis have shown that, in addition to poor risk management, it is suspected that LTCM became a victim of predatory trading. Studying the trading behaviour of market makers during the crisis using a unique dataset of audit trail transactions, Cai(2003) infers that market makers exploited their information on customers' order-flows (LTCM needed to cover its short position in the treasury bond future market) and front ran their customers' trades.

The 1998 turmoil would not have happened had the market been perfectly liquid. This aspect of asset market illiquidity arises from imperfect competition [Basak(1997), Kihlstrom(2000), Pritsker(2004)]. Other explanations for illiquidity include exogenous transaction cost - either deterministic [(Amihud and Mendelson(1986), Constantinides(1986), Vayanos(1998), Vayanos and Vila(1999), Huang(2003), Duffie, Garleanu and Pedersen(2004a, b)], or stochastic [Acharya and Pedersen(2004)] – as well as asymmetric information [Kyle(1985, 1989), Vayanos(1999, 2001)]. These various

[1] For example, Holthausen, Leftwich and Mayers (1990) examine price effects associated with block trades by investigating the largest 50 trades for 109 firms traded on the NYSE in 1983 and find that most of the price effects are permanent and related to block size. They report a price impact of around 1 percent. Keim and Madhavan (1996) report an even larger price impact (8 percent) in an up-stairs market. Harris and Piwowar (2004) study transaction costs and trading volumes in the U.S. municipal bond market and find that municipal bond trades are substantially more expensive than similar sized equity trades due to the lack of price transparency.

facets have long been studied in the market microstructure literature but could not completely explain the 1998 illiquidity.

Market liquidity anomalies have aroused a lot of interest, but the models have not provided convincing explanations. For example, Longstaff (2001) defined an illiquid market as one in which traders were unable to initiate or unwind a position, and studied a trader's optimal portfolio selection problem. The illiquid market could be regarded as an exogenous trading constraint faced by market participants. However, his model does not provide an explanation for how such an extreme situation comes into being, or why market participants retreated from trading under such circumstances.

Other issues arise from studying the performance of "large" traders, such as hedge and mutual funds. If large traders have superior analytical or technological skills and information, how could they not consistently "beat the market"? Contrary to popular belief, analyses by Braas and Bralver(2003) on the trading profits of more than 40 large trading rooms throughout the world conclude that speculative positioning cannot be the major source of trading revenues. More often than not, this practice loses money rather than makes money. These authors imply that the profits are obtained through strategic trading on, for example, inter-dealer markets or retail markets, where they have dominant market power.

We may then infer that a large trader, whose trades impact prices, can either benefit or suffer from her own market power, conditioned on the state of market conditions. To better understand the impact of large traders' activities on market illiquidity, we need to introduce a large trader into a model with a "thin market" and examine how her behaviour reacts to market conditions.

In this paper, we study a large trader's strategy in a scenario where distressed sales occur and show how her trading strategy impacts the intertemporal equilibrium price process. This is accomplished by adapting the basic structure of the model by Duffie, Garleanu and Pedersen (2004a; henceforth DGP), in which agents switch randomly between high and low expectations, and thus are motivated to trade a single claim to future consumptions. Whereas DGP study limiting behaviour with large numbers of traders in a stationary stochastic environment, our model assumes a limited number of traders and focuses on short-run strategic trading, and its impact on the price process. We assume symmetric information so that there are no incentives to signal or act in a manner to exploit informational monopoly power. Because we wish to model "thin" markets, we model asset sales in each period by a decentralised bargaining process where traders bargain over the transaction price. (Examples of such markets are over-the-counter markets for derivative securities or corporate bonds.) Traders have heterogeneous initial endowments (i.e., one large trader with two shares vs. two small traders who have a maximum capacity to own one share each) and differential bargaining power. They also differ by their intrinsic types (high-type versus low-type) in the sense that they have different asset valuations. In addition to different valuations on assets, a low-type asset owner, who places a low valuation on the asset she holds, also incurs a holding cost. We assume that intrinsic types are subject to random change over time: this generates uncertainty over future types, and in turn, induces randomness over the future number of small liquidity providers. In this sense, there is uncertainty over future market liquidity. There are even scenarios where the large trader cannot find anyone to trade with profitably in future periods. (This framework allows us to rationalise Longstaff's (2001) idea of the market "drying up".) We are able to study a dilemma often faced by large traders: trade fast and you move the market too much against you; wait to trade and the market moves around you. Large trader choose trading strategies that take into account the inherent trade-offs between these two effects.

Our formulation is sufficiently flexible that we can study traders' behaviour under different multilateral bargaining game structures. These bargaining games take place at each date and are contingent on the intrinsic types of the traders and their asset holdings. In particular, we explore three bargaining games. The first bargaining game models the large trader as holding a privileged position in trading with small traders who cannot communicate. The second game models a situation where all traders are on an equal footing in bargaining, but negotiation is a one-shot game at each trading date; and the third game assumes all traders can renegotiate repeatedly to mimic a semi-competitive situation. Using each of the component bargaining games in turn, we analyse the dynamic game to deduce the trading strategies and price process. As a general result, we show that the large trader chooses optimal selling strategies, trading off the initial price impact of the large trader's monopolistic market power against the uncertainty of market liquidity in the future periods. The extent of the price impact also varies with the constituent bargaining game structure. For example, in the first type of bargaining game - where the large trader faces two small traders who cannot trade between themselves - the large trader's first period trade incurs a large price impact. That is, she obtains a lower price if she sells two shares in one period as opposed to spreading the sale over two periods. With the other bargaining games, where the small traders are less constrained in their bargaining (mimicking a more competitive outcome) the price impact becomes less evident. The large trader's monopoly power weakens when the market becomes more transparent (in terms of the bargaining process) and competitive. She may have to choose to spread trades over two periods because the cost to induce small traders to buy in the first period is just too high. For this reason, the large trader may benefit from the improvement in market transparency to the extent that small traders have higher expected payoffs from better trading opportunities. When bargaining with small traders, the large trader gains more by giving up some immediate monopoly advantage, especially when her relative bargaining power over small traders is very high.

Our model differs from the competitive search model of DGP (extended by Vayanos and Wang(2003) and Weill(2003)) in several ways. They assume a continuum of identical small traders and focus on steady-state equilibria, while we have only a limited number of traders and solve the model by characterizing the subgame perfect equilibrium of a dynamic game. The reduction in the number of counterparties, plus the probability of type switching, gives rise to uncertainty over future trading opportunities. Traders are thus faced with the "illiquidity uncertainty" of not being able to find a counterparty in some future period. Both aspects of "illiquidity" - the limited number of counterparties and uncertainty over availability of counterparties in the future – underlie the "illiquidity uncertainty". We show that when the number of traders or trading periods becomes larger, the market becomes more liquid in the sense that a trader can trade at any speed or at any time she chooses.

In addition, we introduce a large trader into the model, which generates a number of different results. Firstly, the shift of a large trader's type has a greater impact on the security's demand or supply than does type switching by a small trader. Secondly, a large trader is able to choose trading strategies so as to maximize liquidation value, which in turn influences future market liquidity. Therefore, when choosing trading strategies, the large trader takes both the price impact and liquidity uncertainty into consideration, which endogenizes illiquidity cost and price impact.

In reality, large traders sometimes hold dominant market power relative to their dealers or other traders, and extract more value from bargaining. Braas and Bralver's (2003) analysis of the trading profits of large intermediaries demonstrates that trading profits from market making and from customer business are a function of the relative power of the two trading parties. Green, Hollifield and Schurhoff(2004), estimating a structural bargaining model using transaction data of the U.S. municipal bond market, attribute their finding of decreasing profits on trade sizes to the dealers' relative market power.

Our results contribute to the market microstructure literature in several ways. We show that even without asymmetric information [Kyle(1985)] or the need to share risk [Vayanos(1999, 2001)] large traders trade strategically when the market is illiquid. Moreover, our study shows that the price impact could be magnified by market illiquidity; monopoly power has less of an effect in a more liquid market.

Our model and methodology also contribute to a recent strand of literature trying to incorporate liquidity risk into asset pricing by endogenizing illiquidity costs into asset prices. For example, Pritsker (2004) studies a general equilibrium model in which the competitive fringe takes prices as given, whereas large investors face prices as a function of their own orderflows. Illiquidity in that model stems from imperfect competition. He is able to derive a multi-factor asset pricing formula, capturing the imperfect risk sharing with temporary factors[2] in addition to the market risk factor. Acharya and Pedersen (2004), on the other hand, assume a stochastic illiquidity cost and develop a liquidity adjusted CAPM model. Since the stochastic transaction cost is exogenous, the net-of-transaction-cost returns should satisfy the CAPM in a frictionless economy. Using this insight they are able to derive asset prices in an overlapping generation model. They show that in the liquidity-adjusted CAPM, the expected return of an asset has a four-factor structure with a non-zero constant term representing the expected illiquidity cost. Vayanos(2004) complements Acharya and Pedersen (2004) by introducing a link between liquidity and volatility. Instead of a time-varying transaction cost as assumed by Acharya

[2] These risk factors are temporary in that it is the deviations from Pareto optimal asset holdings by large investors that affect asset prices and these deviations will eventually disappear when the investors' risky asset holdings converge to the competitive levels as time goes to infinity.

and Pedersen, he assumes a constant transaction cost, but a time-varying horizon which depends on the volatility of market returns. By modeling investors as fund managers subject to performance-based withdrawals, he shows that assets in equilibrium can be priced by a conditional two-factor CAPM adjusted for the transaction cost: the two factors are the market risk and volatility.

We do not assume a deterministic or stochastic illiquidity cost. Rather, we model illiquidity as arising from both imperfect competition and liquidity uncertainty, so that trading and price impacts are endogenized in the process of bargaining and trading. In addition, the existence of a large trader and a few small traders alters the bargaining situation from bilateral to multilateral bargaining. This introduces more complexity into the model by requiring us to analyse a large number of contingent trading strategies. On the plus side, it allows us to better examine the dynamics of trading strategies and how market competition influences prices.

Lastly, our model provides a theoretical base to Longstaff's (2001) interpretation of an illiquid market. Our results show that there is some probability that there may be no counterparty on the other side of the market, either because of the sudden co-switching of traders (from high-type to low-type, or vice versa) or because traders are not willing to trade due to the high uncertainty of liquidity. In either case, markets "disappear" temporarily.

Our work is also related to the literature on market manipulation. For example, Jarrow(1992) investigates market manipulation trading strategies by large traders when their trades move prices. He studies the conditions on the price process under which large traders generate profits at no risk. Subramanian and Jarrow(2001) study the liquidity cost when a trader's trades have a price impact and there are execution lags in trading. There are a number of differences between their model and ours. In their model, the price process and price impact function are assumed, while in our model prices are produced endogenously and price impacts exist as a result of imperfect competition and liquidity uncertainty. Secondly, they study the large trader's behaviour in a partial equilibrium model while we study the large trader's behaviour in a dynamic game. Finally, there is no liquidity uncertainty in their model.

The rest of our paper is organized as follows. Section 2 describes the basic model. The security market resembles an over-the-counter market in which traders contact potential trading counterparties and bargain over prices. Section 3 analyses the model and describes the optimal trading strategy for the large trader under different bargaining game structures. Section 4 explores the model in a situation where the low-type non-owners exit the market so that the large trader may not be able to find any trading counterparties in the marketplace. This variation introduces an extreme situation of illiquidity. Section 5 briefly discusses the case of a monopolistic buyer: we show that we can apply our earlier findings to obtain symmetric results for the case of a large buyer. Section 6 extends our model to $n$ small traders and $t$ trading periods: we show how an infinite number of small traders and trading periods affect market liquidity and pricing, making them more competitive and reducing the price impact. Conclusions and further implications are discussed in Section 7. Calculations and proofs can be found in Appendices.

## 2. The Basic Model

This is a three-date model. People trade at $t_1$ and $t_2$. No trade takes place at the last date, $t_3$. Investors can either invest in a perfectly liquid, risk-free money market account with a return of *r,* or an illiquid security in an over-the-counter market, paying a dividend $\bar{D} > r$ at date $t_3$. Borrowing or short selling is not allowed.

There are three risk-neutral traders in the market. Each trader can be characterized by a triple set $\{\upsilon, \gamma, m\}$. Let $\upsilon \in \{h, l\}$ denote an agent's valuation for the illiquid asset that can be either high (*h*) or low (*l*). We regard $\upsilon$ as an agent's intrinsic type. An agent with $\upsilon = h$ values the asset at $\bar{D} > r$, while an agent with $\upsilon = l$ only values the asset at $\bar{D} - \varepsilon \ll r$. $\gamma \in \{0,1,2\}$ is an agent's position in the illiquid asset. Also, an agent may own $m$ dollars in his money market account, $m \in [0, M]$. At $t_1$, one big trader (*B*) is endowed with two shares of the illiquid security but no money, i.e., $\gamma_{t_1}^{B} = 2$ and $m_{t_1}^{B} = 0$; and two small traders (*S*) are each endowed with $m_{t_1}^{S} = M$ dollars, where $\bar{D} < M < 2\bar{D}$.[3] That is, a small trader can only afford one share of the asset at any bilaterally agreed price.

In addition, investors' intrinsic types are subject to change. The transition function is defined as $\rho\left(\upsilon^{t+1} \middle| \upsilon^{t}\right)$. Let $\rho\left(l^{t+1} \middle| h^{t}\right) = \rho_d$ be the probability of switching rate from the "high-type" to the "low-type", and $\rho\left(h^{t+1} \middle| l^{t}\right) = \rho_u$ the opposite switching rate from the "low-type" to the "high-type". $\rho_{u/d} \in (0,1)$. The type switching probabilities are public information. The investor intrinsic types and changes capture the effects of several situations, such as a liquidity shock (i.e., a need for cash), a risk management requirement

[3] Note that the exact number of shares of the security held by an agent is not particular significant. What the numbers try to capture here, is that a large trader is one who owns significantly more shares than a small trader.

(e.g., to meet VaR restrictions or hedging needs), or low utility for an asset (e.g., a low expectation of future dividend flow.)

We thus denote an investor's type as one from the set {*high-type owner, high-type non-owner, low-type owner, low-type non-owner*}, or $I = \{ho, hn, lo, ln\}$. When the intrinsic type of an investor switches from *high* to *low,* the investor's valuation of the asset becomes lower, and she wants to liquidate the asset. Similarly when an investor's type switches from *low* to *high*, she may want to buy the illiquid asset and consume the dividend. Thus, the asset will be transferred between investors with different expected payoffs (e.g., from a low-type owner to a high-type non-owner.)

The market, however, is decentralized in the sense that buyers and sellers are separated. When an agent has a need to trade, she automatically contacts other agents in the market and bargains with them over the price. The timing of events proceeds as follows (see Figure 1). The game starts with "nature" choosing the types of three traders - they are randomly drawn from the type set {*ho, hn, lo, ln*}. At the beginning of each date, each trader recognizes her own type and position, and decides whether or not to trade in this period. An agent who decides to trade contacts some other agents. When two agents start a negotiation, they immediately reveal their types and bargain over the transaction prices. Engaging in bargaining, however, does not guarantee a deal. An agent, who may bargain with more than one other agent, will trade at the most advantageous price. If the two parties engaged in bargaining reach an agreement, a transaction occurs. If negotiations breakdown, they have to wait until the next trading date to resume trading. Between transaction dates, intrinsic types are subject to change. At the start of the next date in the sequence, agents learn their new types, and trade if necessary.

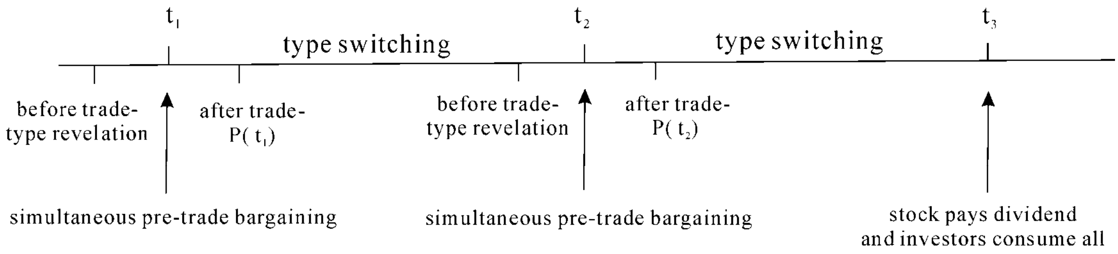


Figure 1. Timing of type switching and trade

At this point, we specify the bargaining/trading process at each date in more detail. It is modeled as a two-stage mechanism. In the first stage, traders simultaneously engage in pre-trade bilateral negotiations. They reveal their types, $\upsilon$, when they start negotiation. We model the pre-trade bilateral bargaining price by the Nash bargaining solution to keep the model simple and tractable. Thus the price is given by

$$P(t) = q_s \Delta V_b(t) + q_b \Delta V_s(t) \tag{1}$$

where $q_s$ and $q_b$ are the bargaining power of the seller and the buyer, and $\Delta V_b$ and $\Delta V_s$ refer to the reservation values of the buyer and the seller respectively.

In the second stage, traders choose their actions: how many shares to trade and with whom to trade. Actions that a trader $i$ chooses to play at time $t$ is defined as $a_t^i \in \{-\gamma_t^i, \ldots, -1, 0, 1, \ldots, \gamma_t^i\}$. For example, the big trader $B$ can choose to sell two shares, sell one share or not to trade at $t_1$, i.e., $a_{t_1}^B \in \{-2, -1, 0\}$. Let $h_t = (a_1, \ldots, a_{t-1})$ be the realized choices of actions at all periods before $t$. Trader $i$'s payoff is thus defined as his expected value function over the rest of the trading horizon. That is,

$$u^i = EV_t^i\left(a_t^i(h_t)\right)$$

Traders may bargain with more than one counterparty in the first stage, but they will only trade at the most advantageous price in the second stage.

However, this trading rule does not rule out the possibility that strategic negotiations may go on for many rounds among the three traders. To study the effect of different trading mechanisms on traders' actions, we look at three mechanisms that are chosen to demonstrate various degrees of competitive bargaining. We first assume that small traders are "geographically separate", such that they are unable to contact each other. In other words, they can only be reached by the large trader. This can be thought of as an example of a monopolist in a market where small traders have very limit contacts. We then remove this assumption so that the three traders are allowed to contact each other but restrict pre-trade negotiations to one-round. That is, a trader cannot re-open negotiations with another trader if they fail to reach an agreement during a trading date. Rather they have to wait till the next trading date to make contact and negotiate with each other all over again. This mimics a fast-paced and high pressured situation where a trader has to make quick decisions. Lastly, we allow traders to negotiate iteratively and infinitely at each trading date. They can strategically delay or decline negotiations if desired. With this structure, the bargaining solution depends heavily on the market supply and demand at that time.

Since we are interested in a large trader's strategy in the situation of forced liquidation, we only focus on the following case:[4] at $t_1$, the large trader $B$ is of low-type and thus wants to sell two shares of the illiquid asset within two periods. The other two small traders are both high-type agents, each with an endowment of $M$ dollars to buy one share. In order to study the particular case of a distressed sale by the large trader, we assume that once the large owner becomes a low-type (either initially, or through switching), she cannot switch to high-type unless she first unwinds her long position.[5] In other words, if

[4] Note that any combination of the three traders' types constitutes a game. Some combinations are conducive to trades, whereas others are not and are thus trivial. To avoid repetition, we will not study every single case in detail.

[5] This can occur in real-world situations such as: a) a large trader facing margin calls from her broker; b) a fund manager facing sudden withdrawals by fund holders; or c) a risk manager facing a binding constraint (e.g., VaR constraint). In all these cases the trader has to liquidate at least part of her position to meet the cash need. She cannot wait for the situation to improve by itself.

the large trader, being a low type owner at the beginning, sells only one share (or none) in the first period, she will still be a low type owner in the second period, and cannot switch to being a high type. However, if she sells off two shares at $t_1$ (becoming a low type non-owner after trade), she is then freely subject to type switching and hence symmetric to small traders. With this assumption, the distressed large trader who suffers a sudden liquidity shock cannot take her chances by doing nothing and hoping the situation will improve itself. Small traders, however, are not subject to this type-switching restriction.

Lastly, we assume small traders' relative bargaining power with respect to the big trader is $q$, $0 \leq q < 1/2$. Two small traders are identical at $t_1$ in terms of their relative bargaining power to each other (i.e., 1/2) and their initial endowments. The notion of bargaining power partly captures the idea of the "market power", in the sense that it gives an agent control over bargains. In this model, "market power" is also reflected in how many shares a trader owns.

The model described thus far clearly demonstrates where "illiquidity" comes from. Since there are only a small number of traders in the market, it is "illiquid" in that there are only a limited number of counterparties and thus, limited trading opportunities. This can be defined as "exogenous" illiquidity. Moreover, traders fear that their types may change in the future. They are thus only willing to trade at a discounted price in early periods to avoid being stuck in such a situation where they cannot find any counterparty at all. This reduction in trades can be regarded as "endogenous" illiquidity. We define "illiquidity uncertainty" in this model as the uncertainty that a trader cannot find enough counterparties to trade with.

It is important to note that, throughout this study, we assume transparency of information in that no trader can hide her identity when she enters negotiations. This distinguishes our model from many market microstructure models, such as Kyle(1985) and others, in which information asymmetry is the major incentive for some market participants to trade strategically.

## 3. Distressed Sales and Asset Prices

In this section we study the distressed large trader's behavior under three different bargaining/trading structures. We consider in detail the case where at $t_1$ the large trader is the only low-type owner with two shares to sell while the two small traders are both high-type non-owners, who each would like to buy one share.[6] Figure 2 describes the dynamics of the population structure, which evolves according to trades and type switching.

Before we define the subgame perfect equilibrium of this dynamic game, we would like to explain the structure, shown in Figures 2 and 3, in more detail. In these two figures, a cluster of ovals represents a trader configuration at some specific time. Since the large trader is the only *lo* trader who has two shares of the security, she can choose to trade one share, two shares, or not trade at all. This is shown in Figure 2 as three branches leading to three trader distributions after trading at $t_1$. Before the next trading date arrives, traders' types are subject to change. This is represented in the figure as dotted lines leading to possible trader distributions at $t_2$, following by the associated probabilities. Upon arriving at $t_2$, agents find themselves in one cluster of ovals (a particular configuration of types), which develops into a subgame numbered from (i) to (xiii). For instance, the trader configuration highlighted in the rectangle evolves into the subgame demonstrated in Figure 3. Even with two periods, the structure of the game can become quite complicated.

At the beginning of each period, the large trader chooses the optimal strategy to maximize her expected payoff at the last date. Since she has two shares to sell and she can sell only one share to any buyer she encounters, her major concerns are "when" to

[6] We examine a similar case of the large trader being the only buyer (*hn*) at $t_1$ in a later section.

sell and "how many shares" to sell at each period. By selling quickly (i.e., selling two shares in the first period), she may not get a very good price, but then the liquidation pressure is gone and the payoff is guaranteed. Employing the strategy of smoothing the sales across two periods may be conducive to a higher transaction price - by exploiting the large trader's monopoly power - but the uncertainty of not being able to trade in a later period increases (due to the probability of type switching by *hn* traders). Thus, the tradeoff faced by the large trader is between the price impact of trading and the possibility of market deterioration.

We must now consider the outcomes of this game, and in particular, define an equilibrium outcome.

***Definition***: An outcome profile consists of a trading strategy profile and the associated transaction prices $\left(\psi(t), P(t)\right)$. An *equilibrium outcome profile* $\left(\psi(t)^*, P(t)^*\right)$ is an outcome profile such that for a particular trader configuration at each time, given split-the-difference negotiations, the large trader cannot improve her expected payoff by adopting any other strategy profile $\left(\psi(t)', P(t)'\right)$, and no small trader can improve his expected payoff in pair-wise negotiations with the large trader.

We solve the model by backward induction. This approach can be summarized as follows. In this two-period game, traders have two opportunities to trade: times $t_1$ and $t_2$. Each trader seeks to maximize her value function at $t_3$. She solves the dynamic programming problem

$$\max_{a_{t_1}, a_{t_2}} \quad EV_{t_3}$$

by choosing trading strategies $a_{t_1}$ , $a_{t_2}$ at each date, where $a_{t_1}$ , $a_{t_2} \in \{-2, -1, 0\}$ . In equilibrium, the trading activity chosen by a trader must be the best response to the other traders' trading actions.

The second period: We first determine each trader's payoff at the last date, $X_{t_3}^i$. Then, for the subgame at $t_2$ we compute the large trader's value function, $V_{t_2}^B\left(\Gamma\left(\bullet, t_2\right)\right)$

$$V_{t_2}^B\left(\Gamma\left(\bullet, t_2\right)\right) = \max_{a_{t_2}} E\left[X_{t_3}^B \middle| a_{t_2}\right],$$

by comparing her expected payoffs across different trading strategies, $a_{t_2}$, in the game $\Gamma\left(\bullet, t_2\right)$, where the dot describes the trader distribution of this game.

The first period (see figure 2): In the first period, we determine the large trader's optimal trading strategy by comparing her payoffs of adopting different actions. $B$'s value function resulting from taking an action $a_{t_1}$ in the game $\Gamma\left(\bullet, t_1\right)$ is given by

$$V_{t_1}^B\left(a_{t_1}, \Gamma\left(\bullet, t_1\right)\right) = E\left[V_{t_2}^B\left(\Gamma\left(\bullet, t_2\right) \middle| a_{t_1}\right)\right],$$

which is her expected utility over all possible outcomes of subgames resulting from the action $a_{t_1}$. The optimal trading strategy of the large trader is the strategy which maximizes her expected utility at $t_1$. $\left(a_{t_1}^*, a_{t_2}^*\right)$ constitutes the large trader's optimal trading strategy profile.

Next, we solve the model under three different designs for the market structure. We first study the game in which small traders are "geographically separate". We then allow small traders to contact each other, but restrict traders from re-opening negotiations at a trading date once pair-wise negotiations are closed between any two. Finally, we relax all the above restrictions such that traders are free to contact anyone and may start and stop renegotiations with any other trader any number of times.

### 3.1 Geographically Separated Small Traders

#### 3.1.1 Subgames in the Second Period

The subgames starting at $t_2$ are numbered from (i) to (xiii) in Figure 2. To demonstrate how a trader makes a trading decision, it is useful to first compute the expected payoffs to owning one share for traders of all four types. For a high type trader, whether large or small, the expected payoff to owning one share at $t_2$ is

$$X_{t_2}^{h} = \frac{\bar{D}-\varepsilon}{r}\left(\rho_d \Delta t\right) + \frac{\bar{D}}{r}\left(1-\rho_d \Delta t\right) = \frac{\bar{D}-\rho_d \Delta t \varepsilon}{r}. \tag{2}$$

However for low type traders, the large trader's expected payoff is different from that of a small trader:

$$X_{t_2}^{S_{lo}} = X^{S_{ln}}\left(t_2\right) = \frac{\bar{D}-\varepsilon}{r}\left(1-\rho_u \Delta t\right) + \frac{\bar{D}}{r}\left(\rho_u \Delta t\right) = \frac{\bar{D}-\left(1-\rho_u \Delta t\right)\varepsilon}{r}, \tag{3}$$

$$X_{t_2}^{B_{lo}} = \frac{\bar{D}-\varepsilon}{r}. \tag{4}$$

We assume $1-\rho_u \Delta t - \rho_d \Delta t > 0$ to insure that a high type trader is willing to buy from a low type trader (i.e., $X_{t_2}^{h} > X_{t_2}^{l}$). Also note that $X_{t_2}^{S_{ln}}$ is greater than $X_{t_2}^{B_{lo}}$ because the large, low-type owner cannot switch to being a high type in the second period whereas a small low-type trader may. Thus, trades will take place between a $S_{ln}$ and the $B_{lo}$, but not between two small traders.

The large trader can always sell to small non-owners as long as she can find one. However, the price at which she sells to a small high type non-owner is different from that at which she sells to a small low type non-owner. The bargaining-derived price $P_{t_2}^{B_{lo}-S_{hn}}$ between $B_{lo}$ and $S_{hn}$ is determined by

$$q\left[P_{t_2}^{B_{lo}-S_{hn}} - X_{t_2}^{B_{lo}}\right] = \left(1-q\right)\left[X_{t_2}^{S_{hn}} - P_{t_2}^{B_{lo}-S_{hn}}\right] \tag{5}$$

$$P_{t_2}^{B_{lo}-S_{hn}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right] + \left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\rho_d \Delta t \varepsilon\right)\right] \tag{6}$$

Similarly, the price outcome of bargaining between $B_{lo}$ and $S_{ln}$ is given by

$$P_{t_2}^{B_{lo}-S_{ln}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right]+\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon\right)\right] \tag{7}$$

Thus in subgame (i), the large seller simultaneously contacts both small high type non-owners and sells one share to each trader at $P_{t_2}^{B_{lo}-S_{hn}}$. In subgame (ii), *B* sells one share each to $S_{hn}$ and $S_{ln}$ at $P_{t_2}^{B_{lo}-S_{hn}}$ and $P_{t_2}^{B_{lo}-S_{ln}}$ respectively. In subgame (iii), where both of the small high type non-owners switch to become low type between $t_1$ and $t_2$, *B* sells two shares to two $S_{ln}$'s at $P_{t_2}^{B_{lo}-S_{ln}}$. Therefore, the value function of the large trader adopting the action $a_{t_1}^B = 0$, is her expected payoff to this action at $t_1$.

$$\begin{aligned}
V_{t_1}^{B_{lo}}\left(a_{t_1}^B = 0, \Gamma\left(B_{lo}, 2S_{hn}, t_1\right)\right) = \frac{1}{r}\Bigg[ & \left(1-\rho_d\Delta t\right)^2 \frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r} \\
& +2\rho_d\Delta t\left(1-\rho_d\Delta t\right)\frac{2\bar{D}-\varepsilon\left[2q+\left(1-q\right)\left(1-\rho_u\Delta t+\rho_d\Delta t\right)\right]}{r} \\
& +\left(\rho_d\Delta t\right)^2 \frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\left(1-\rho_u\Delta t\right)\right]}{r}\Bigg] \\
= & \frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\left(\rho_d\Delta t\right)^2-\left(1-q\right)\rho_d\rho_u\Delta t^2\right]
\end{aligned} \tag{8}$$

Sugames (iv) to (vii) describe all possible trader configurations at $t_2$ if the large trader sells one share at $t_1$, i.e., $a_{t_1}^B = -1$. If there is no type switching between two trading dates, i.e., subgame (iv), *B* contacts $S_{hn}$ and sells her one remaining share at $t_2$ for the price $P_{t_2}^{B_{lo}-S_{hn}}$. Thus, the large trader's value function for this subgame is the proceeds from selling two shares at two dates, specifically,

$$V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{hn}, S_{ho}, t_2\right)\right) = rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B = -1\right)+P_{t_2}^{B_{lo}-S_{hn}} \tag{9}$$

If the large trader finds herself in subgame (v) or (vii), where she only finds one small low-type non-owner, she sells the share to $S_{ln}$ at $P_{t_2}^{B_{lo}-S_{ln}}$. Her value function is the same in both of these subgames, namely:

$$V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},S_{ln},S_{ho},t_2\right)\right)=V_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo},S_{ln},S_{lo},t_2\right)\right)=rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)+P_{t_2}^{B_{lo}-S_{ln}} \tag{10}$$

In subgame (vi), differing game structures come into effect. Now there is only one buyer, $S_{hn}$, facing two heterogeneous low type owners, $S_{lo}$ and $B_{lo}$. Since the expected payoffs to holding one share at $t_2$ are different for $S_{lo}$ and $B_{lo}$, the price resulting from bilateral bargaining between $S_{hn}$ and $B_{lo}$ and between $S_{hn}$ and $S_{lo}$ must be different as well. If $S_{hn}$ can contact both $S_{lo}$ and $B_{lo}$, whom she buys from depends entirely on the negotiated price. The assumption that two small traders are "geographically separate", however, simplifies the analysis here by eliminating the possibility of trade between $S_{hn}$ and $S_{lo}$. The upshot is that under this game structure, $S_{hn}$ buys from the large trader $B_{lo}$ at the bilateral bargaining price, $P_{t_2}^{B_{lo}-S_{hn}}$.

We then can compute the large trader's value function of selling one share at $t_1$.

$$V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right)=P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)+\frac{\bar{D}}{r^2} \\ -\frac{\varepsilon}{r^2}\left[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\left(\rho_d\Delta t\right)^2-\left(1-q\right)\rho_d\rho_u\Delta t^2\right] \tag{11}$$

If the large trader sells two shares in the first period, she becomes a low-type non-owner after trading, and is then subject to potential type switching. Trades will occur in the second period only if the large trader switches up to high type, and at the same time one or both high type small traders switch down to low type: these situations are reflected in subgames (xii) and (xiii). The large trader now becomes a large buyer so that these two cases are symmetric to subgame (i) and (ii). She will buy back as many share as possible in these two subgames at the bilateral bargaining price $P_{t_2}^{B_{hn}-S_{lo}}$.

$$P_{t_2}^{B_{hn}-S_{lo}}=\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon\right)\right]+q\left[\frac{1}{r}\left(\bar{D}-\rho_d\Delta t\varepsilon\right)\right] \tag{12}$$

The large trader's value functions for subgames (xii) and (xiii) are

$$V_{t_2}^{B_{hn}}\left(\Gamma\left(B_{hn},S_{ho},S_{lo},t_2\right)\right)=\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+2rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-P_{t_2}^{B_{hn}-S_{lo}}$$

$$=2rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)+\frac{\varepsilon}{r}(1-q)(1-\rho_u\Delta t-\rho_d\Delta t) \qquad (13)$$

$$V_{t_2}^{B_{hn}}\left(\Gamma\left(B_{hn},2S_{lo},t_2\right)\right)=2\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+2rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-2P_{t_2}^{B_{hn}-S_{lo}}$$

$$=2rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)+\frac{2\varepsilon}{r}(1-q)(1-\rho_u\Delta t-\rho_d\Delta t) \qquad (14)$$

There is no further trade in other subgames. The large trader's value functions are the same and equal to $2rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)$, which is the proceeds of selling two shares at $t_1$.

The overall large trader's value function of selling two shares at $t_1$ is

$$\begin{aligned}V_{t_1}^{B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)&=2P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)\\&\quad+\frac{2\varepsilon}{r^2}(1-q)\rho_d\rho_u\Delta t^2(1-\rho_u\Delta t-\rho_d\Delta t)\end{aligned} \qquad (15)$$

### 3.1.2 Subgame in the First Period

To determine the large trader's optimal strategy in the first period, we compare his value functions for all strategies. Before we can make these comparisons, we need to determine the transaction price at $t_1$. The large seller may contact and negotiate with one or two small buyers simultaneously. The question is, how does she choose one trading strategy over another?

Consider the outcome if the large trader sells only one share. The Nash bargaining outcome requires that the large seller and a small buyer split the joint surplus of trading according to their bargaining power, thereby satisfying

$$\begin{aligned}&q\left[V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)-V_{t_1}^{B_{lo}}\left(a_{t_1}^B=0,\Gamma(t_1)\right)\right]\\&\qquad=(1-q)\left[V_{t_1}^{S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-1\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)\right]\end{aligned}$$

(16)

where $V_{t_1}^{S_{ho}}\left(a_{t_1}^{S}=1,\Gamma(t_1)\middle|a_{t_1}^{B}=-1\right)$ is the expected payoff for the small trader who buys one share given that the large trader sells one share at $t_1$.

$$V_{t_1}^{S_{ho}}\left(a_{t_1}^{S}=1,\Gamma(t_1)\middle|a_{t_1}^{B}=-1\right)=\frac{1}{r}\left[(1-\rho_d\Delta t)\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+\rho_d\Delta t\frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r}\right]$$
$$=\frac{1}{r^2}\left[\bar{D}-(2-\rho_d\Delta t-\rho_u\Delta t)\rho_d\Delta t\varepsilon\right]$$

(17)

Substituting $V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma(t_1)\right)$, $V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=0,\Gamma(t_1)\right)$ and $V_{t_1}^{S_{ho}}\left(a_{t_1}^{S}=1,\Gamma(t_1)\middle|a_{t_1}^{B}=-1\right)$ into (17), we derive $P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)$ as

$$P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2-\left(1-q^2\right)\rho_d\rho_u\Delta t^2\right]$$

(18)

The price is equal to the present value of the dividend paid at $t_3$ minus a discount, which is a function of the relative bargaining power and the rate of type switching.

Substituting the price back into $V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma(t_1)\right)$, we have the value function for the large trader selling one share

$$V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma(t_1)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[q+q^2+2(1-q)(2+q)\rho_d\Delta t$$
$$-(1-q)(2+q)(\rho_d\Delta t)^2-(1-q)(2+q)\rho_d\rho_u\Delta t^2\Big] \qquad (19)$$

Similarly, if selling two shares is the optimal strategy in the first period, then the following equation should be satisfied.

$$q\left[V_{t_1}^{B_{ln}}\left(a_{t_1}^{B}=-2,\Gamma(t_1)\right)-V_{t_1}^{B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma(t_1)\right)\right]$$
$$=(1-q)\left[V_{t_1}^{S_{ho}}\left(a_{t_1}^{S}=1,\Gamma(t_1)\middle|a_{t_1}^{B}=-2\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-2\right)\right]$$

(20)

This equation states that the large trader keeps selling until the portion of the profit from selling the second share given up to the second buyer is equal to the portion of the gain that can be claimed from this buyer. The second buyer's expected payoff for buying one share at $t_1$ is

$$V_{t_1}^{S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-2\right)=\frac{1}{r}\left\{(1-\rho_d\Delta t)\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+\rho_d\Delta t(1-\rho_u\Delta t)\frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r}\right.$$
$$\left.+\rho_d\Delta t\rho_u\Delta t\frac{\bar{D}-\left[(1-q)(1-\rho_u\Delta t)+q\rho_d\Delta t\right]\varepsilon}{r}\right\}$$

(21)

The price at which the large trader sells one share to each $S_{hn}$ is then given by

$$P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2(1+q)}\left[q^2(1+q)+2(1-q)(1+q)^2\rho_d\Delta t-(1-q)(1+q)^2\rho_d^2\Delta t^2\right.$$
$$\left.-(1-q)\left(1+q+2q^2+q^3\right)\rho_d\rho_u\Delta t^2-q(1-q)\rho_d^2\rho_u\Delta t^3-q(1-q)\rho_d\rho_u^2\Delta t^3\right]$$

(22)

And the value function of selling two shares for the large trader is

$$V_{t_1}^{B_{ln}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left[q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2\right.$$
$$\left.-\frac{(1-q)\left[q(1+q)^2+2+q\right]}{1+q}\rho_d\rho_u\Delta t^2+\frac{1-q}{1+q}\rho_d^2\rho_u\Delta t^3+\frac{1-q}{1+q}\rho_d\rho_u^2\Delta t^3\right]$$

(23)

Of particular interest is equation (20), which can be rewritten by substituting $V_{t_1}^{B_{ln}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right)$ and $V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)$ from (15) and (11) to get

$$q\left\{\left[P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-h\left(\bar{D},\varepsilon\right)\right]+\left[P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)\right]\right\}$$
$$=(1-q)\left[V_{t_1}^{S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-2\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)\right]$$

(24)

where,

$$h\left(\bar{D},\varepsilon\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[q+2\left(1-q\right)\rho_d\Delta t-\left(1-q\right)\rho_d^2\Delta t^2$$
$$+\left(1-q\right)\rho_d\rho_u\Delta t-2\left(1-q\right)\rho_d^2\rho_u\Delta t^3-2\left(1-q\right)\rho_d\rho_u^2\Delta t^3\Big]$$

The left-hand side of (24) shows that by selling an additional share, the large seller not only gains $P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-h\left(\bar{D},\varepsilon\right)$ at the margin, but also incurs the *price impact* captured by $P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)-P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)$. This secondary price effect is crucial in our analysis: when the large trader sells an additional share she has to take into consideration the effect of her own trading on the price. The marginal costs of selling one share and selling two shares are different due to the additional uncertainty of being able to trade with a high-type non-owner in a later period.

The large trader chooses the optimal strategy by comparing the payoffs of the three trading strategies. Simple algebra gives us the following proposition describing her optimal strategy.

**Proposition 1 (geographically separate small traders)**.

(i) *When* $0\leq q<1/2$ *,* $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in\left(0,1\right)$ *, there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell two shares in the first period, that is,* $a_{t_1}^B=-2$.

(ii) *The large trader's trading incurs a price impact due to her trading, i.e., she receives different prices from selling one share and selling two shares. More specifically,* $P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)>P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)$ *when* $1-\rho_d\Delta t-\rho_u\Delta t>q\left(1+q\right)$.

### 3.2 One Round Multilateral Negotiation

In this sub-section, we relax the previous assumption that small traders are unable to approach each other. Instead, we allow the three traders to freely contact one another. At each trading date, however, they have to make quick decisions since negotiations between any two traders are one-shot affairs. That is, a trader cannot re-open negotiations with another trader once she departs without reaching an agreement.

Analysis of the game with this structure follows the same logic as the "separate small trader" case. Only subgame (vi) at $t_2$ displays differences between these two structures. Thus we analyze subgame (vi) here and leave the remaining derivations to the appendix.

Subgame (vi) describes the situation in which one small buyer (*hn*-type) faces two sellers (*lo*-type), one small and one large, each of whom has one share to sell. The small buyer will contact both sellers and enter into bargaining with each of them. The bilateral bargaining price between $S_{hn}$ and $S_{lo}$ is

$$P_{t_2}^{MN,S_{lo}-S_{hn}} = \frac{1}{2}\left[\frac{1}{r}\left(\bar{D}-(1-\rho_u\Delta t)\varepsilon\right)\right]+\frac{1}{2}\left[\frac{1}{r}\left(\bar{D}-\rho_d\Delta t\varepsilon\right)\right] \tag{25}$$

and the bargaining price between $S_{hn}$ and $B_{lo}$ is

$$P_{t_2}^{MN,B_{lo}-S_{hn}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right]+(1-q)\left[\frac{1}{r}\left(\bar{D}-\rho_d\Delta t\varepsilon\right)\right] \tag{26}$$

where "*MN*" denotes the structure "one-round ***M***ultilateral ***N***egotiation". Since a trader can only enter negotiations with another trader once at each trading date, she cannot strategically delay or decline in the hope of provoking competition between the other traders. Rather, the buyer, $S_{hn}$, will simply compare two prices and buy at the lower one.

Therefore, the small buyer buys from the large seller ($B_{lo}$) if $P_{t_2}^{MN,B_{lo}-S_{hn}} \le P_{t_2}^{MN,S_{lo}-S_{hn}}$ and buys from the small seller ($S_{lo}$) otherwise.[7] Comparing these two prices $P_{t_2}^{MN,B_{lo}-S_{hn}}$ and $P_{t_2}^{MN,S_{lo}-S_{hn}}$, we find that $P_{t_2}^{MN,B_{lo}-S_{hn}} \le P_{t_2}^{MN,S_{lo}-S_{hn}}$ if and only if $2q \ge \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$. The large seller's value function in this game is thus

[7] We simply assume here that when $P_{t_2}^{S_{hn}-B_{lo}} = P_{t_2}^{S_{hn}-S_{lo}}$, the small buyer buys from the large seller.

$$V_{t_2}^{MN,B_{lo}}\left(\Gamma\left(B_{lo},S_{lo},S_{hn},t_2\right)\right)=\begin{cases} rP_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)+P_{t_2}^{MN,B_{lo}-S_{hn}} & \text{when} \quad 2q\geq\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \\ rP_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)+\dfrac{\bar{D}-\varepsilon}{r} & \text{when} \quad 2q<\dfrac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t} \end{cases} \tag{27}$$

where $V_{t_2}^{MN,B_{lo}}$ denotes the value function of $B_{lo}$ at $t_2$ under the game structure "one-round *M*ultilateral *N*egotiation".

The condition, $2q\geq\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$ (or $2q<\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$), results from the small buyer comparing his expected payoffs from trading with the small seller and the large seller. It also reveals the key trade-off when a trader chooses his trading counterparty: uncertainty over future type switching vs. his bargaining power. Thus, we denote the right hand side of the inequality as "*CTS*" (Chances of Type Switching). The condition is then simplified as $2q\geq CTS$ (or $2q<CTS$).

Continuing by backward induction, we derive the optimal outcome of the game as stated in the following proposition.

**Proposition 2 (one round multilateral negotiation)**.

(i) *When* $2q\geq CTS$ *, the equilibrium outcome profile is the same as that with "geographically separate" small traders, i.e.,* $a_{t_1}^{B}=-2$.

(ii) *When* $2q<CTS$ , $q\in\left[0,1/2\right)$ , $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in\left(0,1\right)$*, there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell one share or two shares in the first period, i.e.,* $a_{t_1}^{B}=\left\{-2,-1\right\}$*, depending on the relationship between q,* $\rho_d\Delta t$ *and* $\rho_u\Delta t$ .

Therefore, the equilibrium outcome of the game is the same as that under the structure of "geographically separate small traders" when $2q\geq CTS$ . When $2q<CTS$ , the large

trader's optimal strategy in the first period can either be $a_{t_1}^B = -1$ or $a_{t_1}^B = -2$ in the first period, depending on the relationship between her relative bargaining power $q$, and type switching probabilities $\rho_d \Delta t$ and $\rho_u \Delta t$. We provide examples to illustrate the sensitivity of the large trader's equilibrium strategy to these parameters in Appendix C.

### 3.3 Iterative Limiting Case

We next consider a setting where the three traders are free to contact each other and engage in an arbitrary number of pair-wise negotiations prior to trading. In each bargaining round, any trader can commit to the number of shares to trade and re-open negotiations over the transaction price. Again, we only analyze subgames whose outcomes are affected by this game structure.

When traders are allowed to iteratively bargain with others, they can strategically delay or decline to reach an agreement in any bargaining round without worrying about the consequences of breakdown. Hence, the outcome rests more with the role a trader plays in the market.

Let's first look at the subgame (i) in which the large trader (*lo*) faces two small traders (both *hn's*)[8]. The large trader has three strategies to choose from in the first period. The "no trade" strategy is ruled out immediately because: a) this is the last period; and b) she always gains from trading. Since she is the only seller, she could exploit her monopoly power by committing to sell only one share, thus creating competition between the two small traders which would drive the price up to their expected payoffs of owning one share, i.e., $\left(\bar{D} - \rho_d \Delta t \varepsilon\right) / r$. Alternatively, the large trader could commit to sell two shares at the bilateral bargaining price

[8] Subgame (xiii) is the case where $B_{lo}$ is the monopoly buyer. It is symmetric to subgame (i), so we do not analyze it in detail here.

$$P_{t_2}^{IN,B_{lo}-S_{hn}} = q\left[\frac{1}{r}\left(\bar{D}-\varepsilon\right)\right]+\left(1-q\right)\left[\frac{1}{r}\left(\bar{D}-\rho_d\Delta t\varepsilon\right)\right]$$

given in equation (6). Here, the superscript "*IN*" indicates the structure "***I***terative ***N***egotiation." Her decision in this game depends on which strategy has a higher payoff.

$$V_{t_2}^{IN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=\frac{\bar{D}-\varepsilon}{r}+\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}$$

$$V_{t_2}^{IN,B_{lo}}\left(a_{t_1}^B=-2,\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=2P_{t_2}^{IN,B_{lo}-S_{hn}}=\frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r}$$

Comparing the two value functions, it is easy to see that she will sell two shares as long as $q<1/2$, which is satisfied by our assumption that the large trader's relative bargaining power is greater than that of the small traders. Therefore, the large trader's value function for this subgame is

$$V_{t_2}^{IN,B_{lo}}\left(\Gamma\left(B_{lo},2S_{hn},t_2\right)\right)=\frac{2\bar{D}-2\varepsilon\left[q+\left(1-q\right)\rho_d\Delta t\right]}{r}.$$

Another subgame we must consider is (vi), in which the small high type non-owner is the only buyer, and two sellers, $B_{lo}$ and $S_{lo}$, differ in their expected payoffs of holding one share at $t_2$. If there is only one seller on the market, $B_{lo}$ or $S_{lo}$, the bilateral bargaining price between $S_{hn}$ and $B_{lo}$ (or $S_{lo}$) is $P_{t_2}^{IN,B_{lo}-S_{hn}}$ (or $P_{t_2}^{IN,S_{lo}-S_{hn}}$) given by equation (26) (or 25). Of course, the monopoly buyer $S_{hn}$ would like to buy at the lower price. But now she can do more than that. She keeps negotiating repeatedly with two sellers until the price is driven down to the level at which one seller has no gain from trading and drops out the competition. Who will drop out first?

We already know that the expected payoff of "no trade" to $S_{lo}$, i.e., $\left[\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon\right]/r$, is higher than that to $B_{lo}$, $\left(\bar{D}-\varepsilon\right)/r$, which are their "disagreement payoffs" in bargains with $S_{hn}$. In the competition with $S_{lo}$, $B_{lo}$ would still benefit from trading at the price $\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r$, while $S_{lo}$ is indifferent between selling and holding her share at this level. If the bilateral bargaining price $P_{t_2}^{IN,B_{lo}-S_{hn}}$ is lower than $\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r$, the small buyer $S_{ho}$ buys from the large seller at $P_{t_2}^{IN,B_{lo}-S_{hn}}$ immediately, because the small seller $S_{lo}$ will not compete with $B_{lo}$. However, if $P_{t_2}^{IN,B_{lo}-S_{hn}}$ is greater than $\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r$, the competition between $S_{lo}$ and $B_{lo}$ will bring the price down to $\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r$. By asking a price just a little bit lower, the large trader wins the competition (we assume that at $\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon/r\right]$, $S_{lo}$ would rather hold her share than sell it). Therefore, it is always the big seller, $B_{lo}$, who sells to $S_{hn}$, but at a different price such as

$$\left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r \text{ when } P_{t_2}^{IN,B_{lo}-S_{hn}} \geq \left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r \Rightarrow q \leq \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t};$$

$$\text{or } P_{t_2}^{IN,B_{lo}-S_{hn}} \text{ when } P_{t_2}^{IN,B_{lo}-S_{hn}} < \left[\bar{D}-(1-\rho_u\Delta t)\varepsilon\right]/r \Rightarrow q > \frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}.$$

Again, we replace $\frac{1-\rho_d\Delta t-\rho_u\Delta t}{1-\rho_d\Delta t}$ by $CTS$. The large trader's value function in this subgame then becomes

$$V_{t_2}^{IN,B_{lo}}\left(\Gamma(B_{lo},S_{lo},S_{hn},t_2)\right)=\begin{cases} rP_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+\dfrac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r} & \text{when} \quad q\leq CTS \\ rP_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+P_{t_2}^{IN,B_{lo}-S_{hn}} & \text{when} \quad q>CTS \end{cases} \tag{28}$$

The small buyer's expected payoff in this subgame then becomes

$$V_{t_2}^{IN,S_{hn}}\left(\Gamma\left(B_{lo},S_{lo},S_{hn},t_2\right)\right)=\begin{cases}\dfrac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\dfrac{\bar{D}-\left(1-\rho_u\Delta t\right)\varepsilon}{r} & \text{when } q\le CTS\\ \dfrac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\dfrac{\bar{D}-\left[q+\left(1-q\right)\rho_d\Delta t\right]\varepsilon}{r} & \text{when } q> CTS\end{cases}\tag{29}$$

Outcomes of other subgames at $t_2$ are the same under this structure as with the previous structures. Having obtained the large trader's value functions for all three strategies, we go back to the first period to decide her optimal strategy at $t_1$.

At the beginning of the first period, the large trader faces the same decision as in subgame (i) at $t_2$, in which two small buyers intend to buy one share each. If she commits to sell one share only, the competition between two small buyers will drive the price up to such a level that a small high type non-owner is indifferent between acquiring a share now and waiting till the next trading date. That is

$$V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)=V_{t_1}^{IN,S_{hn}}\left(a_{t_1}^S=0,\Gamma\left(t_1\right)\right)\tag{30}$$

where $V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\right)$ and $V_{t_1}^{IN,S_{hn}}\left(a_{t_1}^S=0,\Gamma\left(t_1\right)\right)$ are the value functions for a small trader "buying one share" and "not buying", respectively, in the first period when $B_{lo}$ offers to sell one share.

If the large trader commits to sell two shares, she contacts both of the small traders and bargains with them. We need the following lemma to determine a small trader's strategies and value functions. Small traders must either trade or not trade with the large trader at the same time. This is because if the large trader reaches an agreement with one small trader, this small trader must obtain a higher utility by trading than by not trading. Since both small traders are identical, the other small trader would be better off by re-opening negotiations with the large trader and mimicking the first small trader. By symmetry, the situation where one trades and one does not, cannot arise in an equilibrium.

Small traders, simultaneously contacted by the large trader, either both trade at the same time or neither of them trades.

**Lemma 1**: *When the large trader commits to sell two shares, and both small traders are able to re-negotiate with the large trader over their transaction prices, the small traders will choose the same action: either both trading or both not trading.*

Note that small traders take the same action not because they collaborate, but rather because this is the equilibrium outcome from which neither of them wants to deviate. This implies that they bargain over the price until both small traders are indifferent between buying and waiting. The large trader's value function, conditional on both small traders trading, is $V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(t_1\right)\right)$, and her value function, conditional on neither small traders trading, is $V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(t_1\right)\right)$. For a small trader, the gain from trading is $V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)$. The large trader bargains with both small buyers, and the price is given by

$$q\left[V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(t_1\right)\right)\right] \\ =2\left(1-q\right)\left[V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\middle|a_{t_1}^B=-2\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)\right] \tag{31}$$

From (30) and (31) we get the prices at which the large trader sells one share and sells two shares.

The large trader chooses the optimal strategy at $t_1$ by comparing her expected value functions for three actions.

**Proposition 3 (iterative limiting case):**

(i) *When* $q\le CTS$, $q\in\left[0,1/2\right)$, $\rho_{u/d}\in\left(0,1\right)$ *and* $\rho_d\Delta t+\rho_u\Delta t<1$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell either one share or two shares in the first period, i.e.,* $a_{t_1}^B=\left\{-2,-1\right\}$, *depending on the parameter values.*

(ii) *When* $q > CTS$, $q \in [0,1/2)$, $\rho_{u/d} \in (0,1)$ *and* $\rho_d \Delta t + \rho_u \Delta t < 1$, *there exists a unique subgame perfect equilibrium in this game, in which the large trader sells two shares in the first period, i.e.,* $a_{t_1}^B = -2$.

Similar to the games with "geographically separated small traders" and "one round negotiation", "no trade" is never the optimal strategy in this game when all traders are able to iteratively bargain with each other. When $q \le CTS$, the large trader's action in the first period may be to sell one share or two shares, depending on the values of the model parameters $q, \rho_d$, $\rho_u$ and $\Delta t$. When $q > CTS$, the payoffs to actions "sell one share" and "no trade" are the same. Thus, both actions are dominated by the action of "sell two shares". Examples in Appendix C show how the large trader's strategy is affected by parameter values in this game.

### 3.4 Effects of Different Market Structures on Trading Decision and Prices

The three game forms discussed above represent different market structures. For example, the model with "geographically separated" small traders can be thought of as an over-the-counter market where only the "dealer" ($B_{lo}$) can locate other traders. In such a market, the large trader faces no competition and hence has absolute control over when to contact small traders. Analysis shows that she always finds it optimal to trade quickly in the first period so as to fully exploit her monopoly power.

When the market becomes more transparent, in the sense that all traders are able to contact and negotiate with each other, the large trader's status as a monopolist is challenged in some situations, e.g., subgame (vi) at $t_2$, in which her trading with a high type small buyer is no longer guaranteed. As a result, selling quickly in the first period is not always optimal. In contrast, she may either spread the sales over two periods or dump two shares in the first period, depending on whether the price impact effect or the liquidity uncertainty effect is dominant.

The market becomes even more competitive when renegotiations become possible. This can be thought of as a setting in which both small traders participate in a batch auction, which takes place at $t_1$ and $t_2$. In these auctions, the distressed trader acts as a dealer trading for her own account, and receives orders from small traders. With this structure, the large trader behaves strategically, but so can the small traders. Again, the large trader may have to spread her sales over two periods so as to maintain her monopoly position.

The following table summarizes and compares the large trader's optimal strategies at $t_1$ under different market structures.

| Game structures | $CTS < q$ | $q \le CTS \le 2q$ | $CTS > 2q$ |
|---|---|---|---|
| Geographically separate small traders (GS) | $a_{t_1}^B = -2$ | $a_{t_1}^B = -2$ | $a_{t_1}^B = -2$ |
| One-round multilateral negotiation (MN) | $a_{t_1}^B = -2$ | $a_{t_1}^B = -2$ | $a_{t_1}^B = \{-2, -1\}$ |
| Iterative negotiation (IN) | $a_{t_1}^B = -2$ | $a_{t_1}^B = \{-2, -1\}$ | $a_{t_1}^B = \{-2, -1\}$ |

Table 1. The Large Trader's Optimal Strategies $t_1$ and Game Structures

According to Propositions 1-3, the large trader's optimal strategy depends on the relationship between her relative bargaining power and the type switching probabilities. Given the level of liquidity uncertainty (i.e., the values of $\rho_d \Delta t$ and $\rho_u \Delta t$ ), when $q > CTS$ the large trader's optimal strategy is the same under the three different market structures, namely, to sell two shares in the first period. When $q$ decreases to a level such that $q \le CTS \le 2q$ (or equivalently, $CTS/2 \le q < CTS$ ), the large trader's optimal strategy is to sell two shares under the "separate small traders" and the "one round multilateral negotiation", but to sell one or two shares under the structure of "iterative negotiation" structures. As $q$ decreases further such that $CTS > 2q$ (or $q < CTS/2$), "sell two shares" is the optimal strategy only under the "separate small traders" structure, while "sell one or two shares" becomes the optimal strategy under the other structures. Hence, for the later two market structures, "one round multilateral negotiation" and "iterative negotiation", given the values of $\rho_d \Delta t$ and $\rho_u \Delta t$, the large trader tends to trade more slowly as her relative bargaining power, 1- $q$, increases. She also tends to trade more slowly as the market becomes more competitive, other things being equal, so as to mitigate the price impact of her trades.

However, the large trader's expected value is not monotonically decreasing in market competitive. For the same values of model parameters $q$, $\rho_d \Delta t$ and $\rho_u \Delta t$, the large trader may better off in a more competitive market. This is especially true when her relative bargaining power, (1-$q$), is extremely high. For example, compare the large trader's value functions under different market structures with the following set of parameter values: $\rho_d = \rho_u = 0.2$, $\Delta t = 1$ and $q$ = 0.1. The large trader's expected value is highest when she sells two shares at $t_1$ in the market with "one round multilateral negotiation". However, when $q$ is increased to 0.4, other parameters being equal, the large trader is better off in the most competitive market of the three, i.e., the market with "iterative negotiations". Increasing $q$ further to 0.45, we find that the large trader prefers to trade in the market with "geographically separate small traders", the least competitive market among the three. Improved trading opportunities in a more transparent market increase the small traders' expected payoffs, which in turn increases the large trader's expected value directly from bilateral bargaining. This benefit may be offset by the loss, in a more competitive market, of her monopoly payoff. Thus, the large trader may have some incentive to improve trading opportunities for small trader under certain levels of liquidity uncertainty and bargaining power.

Note that thus far in the discussion, the large trader can always liquidate her position within two periods. Even in the worst-case scenario in which she can only find low type non-owners to trade with, she still can unwind her position at a sufficiently low price. We next consider the question of how the large trader's strategy is affected if trading with low type non-owners is forbidden, such as when these non-owners' fund are held for collateral or for the purpose of risk management. In the next section, we study the model with a further assumption that low type non-owners must exit the market and will not be able to trade until they experience a switch to the high type. We repeat the above analyses under the three game structures and study the large trader's trading behavior in this more stringent market.

## 4. Distressed Sales and Temporary Market Disappearance

In this section, we study the large trader's strategy and the resulting price function when she may not be able to find trading counterparties in the second period. In contrast to the analyses in section 3, we assume that low type non-owners must exit the market and cannot come back until their type switches from low to high. With this assumption, the distressed large seller ($B_{lo}$) cannot trade with a low type non-owner as in the previous section. Only low type owners and high type non-owners participate in trading. Hence there will be no trade in certain subgames at $t_2$: in particular, $B_{lo}$ cannot find any high type non-owners in subgames (iii), (v) and (vii).

We briefly analyze the subgames that are affected by this assumption. In subgame (ii), $B_{lo}$ finds only one $S_{hn}$ (the other small trader undergoes a type switching and exits the market) so that she can only sell one share to this small trader at the bilateral bargaining price. In subgames (iii), (v), (vii), there is no high type non-owner in the market so that the large distressed trader, $B_{lo}$, cannot trade at all. However, if the large trader sells two shares in the first period, she is not affected by this assumption. Having liquidated the entire position, she becomes a low type non-owner and exits the market.

We state the outcomes of the game for different market structures in the following propositions.

**Proposition 4 (geographically-separate-small-traders case with low type non-owners exiting the market):**

(i) *There exists a unique subgame perfect equilibrium of this game. When* $0 \le q < 1/2$*,* $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in(0,1)$*, the large trader sells two shares in the first period, i.e.,* $a_{t_1}^B=-2$.

(ii) *The large trader's trading incurs a price impact, i.e.* $\tilde{P}_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)<\tilde{P}_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)$*, for all values of q,* $\rho_d\Delta t$ *and* $\rho_u\Delta t$ *.*

This result provides evidence that a large trader will accelerates trading when faced with the risk of not being able to trade later, even though she has to trade at a more disadvantageous price than when she spreads the trades.

**Proposition 5 (one-round-multilateral-negotiation case with low type non-owners exiting the market):**

(i) *When* $2q \geq CTS$ *, the equilibrium outcome profile is the same as that with "geographically separated" small traders: that is,* $a_{t_1}^B=-2$*.*

(ii) *When* $2q < CTS$ *,* $q\in[0,1/2)$ *,* $1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in(0,1)$*, there exists a unique subgame perfect equilibrium in this game, in which the large trader chooses to sell either one share or two shares in the first period (*$a_{t_1}^B=\{-2,-1\}$*) depending on the relationship between q,* $\rho_d\Delta t$ *and* $\rho_u\Delta t$ *.*

When small traders can contact each other, the large trader may lose out to a small trader in subgame (vi) because the small trader is willing to sell at a lower price. This possible outcome affects the large trader's decision in the first period such that she may choose to spread the sale over two periods, or sell quickly in the first period, depending on the relationship between her relative bargaining power and the type switching rates. Afraid of being unable to trade in the second period (such as in subgames (iii), (v), (vii)), the large trader never leaves all the trades to the second period.

**Proposition 6 (iterative-limiting case with low type non-owners exiting the market):**

*There exists a unique subgame perfect equilibrium of this game. When* $0\leq q<1/2, 1-\rho_u\Delta t-\rho_d\Delta t>0$ *and* $\rho_{u/d}\in(0,1)$*, the large trader sells two shares in the first period, i.e.,* $a_{t_1}^B=-2$*.*

The outcome of the iterative limiting case when low type non-owners must exit the market is quite different from that with low type non-owners remaining in the market. In equilibrium, the large trader responds to this harsher market by dumping her entire position quickly in the first period. Intuitively, when the market becomes more transparent, the large trader makes less profit because her monopoly power is weakened. Therefore, she tends to trade more slowly to avoid too much competition with small traders. This is certainly the case when she can always trade with low-type non-owners in the second period. However, when low type non-owners exit the market, the possibility that the large distressed trader may not be able to trade at all in the second period becomes a concern. Weighing her chances to trade in the second period against the proportion of gain that will be given up to small traders for trading in the first period, the large trader finds it optimal to sell two shares in the first period in all circumstances.

In summation, when low type non-owners are unable to trade, the large trader loses some trading opportunities in the second period, which thereby decreases her expected payoff from trading in the second period, and consequently accelerates the speed of liquidation.

## 5. A Symmetric Case: A Large Buyer vs. Two Small Sellers

It is natural to ask whether these arguments apply in a symmetric case, that is, where a large buyer (*hn*-type) seeks to buy two shares in two periods and two small traders (*lo*-type) hold one share each and hence are eager to sell before $t_3$. Similar to the scenario of a distressed sale, the large buyer must buy two shares before the end of the second period to avoid a penalty of $\delta$ per share, where $\delta > \bar{D}$. This assumption can be interpreted as a situation where, for example, a large trader faces margin calls and is forced to cover a short position; or a fund manager has to close her short position in the case of an adverse market movement. In such cases, the large trader will trade aggressively to avoid the penalty. Much like the large seller liquidating a "long" position, here the large buyer may be forced to liquidate her "short" position.

The large buyer faces the same strategic dilemma: buying aggressively, she may push up the price; waiting to buy at a better price, she could miss the last opportunity to purchase. On the other side of the market, small sellers are balancing the payoff of selling a share today versus the expected payoff of keeping it until the next period. Given the same bargaining and trading procedures, we can find the subgame perfect equilibrium for the "large buyer" game which is symmetric to the "large seller" case. As in the former game, we expect that prices in the first period are functions of the relative bargaining power, type switching rates, $\rho_u$ and $\rho_d$, the holding cost $\varepsilon$ for the low-type owner and the penalty $\delta$ for the large high-type non-owner. The large trader's strategy should depend on the current market liquidity and the expected future market liquidity. Therefore, conditional on the current market situation, there should exist some condition under which the large trader would rather bargain harder now than wait, and vice versa.

## 6. Extensions and Further Interpretations of the Model

Unfortunately, there is no unanimous definition for or measurement of a liquid (or illiquid) market. That said, Kyle(1985) does provide a thorough characterization of "market liquidity", which is widely accepted by academics and practitioners. He describes three aspects of market liquidity: tightness, depth and resiliency.

In this paper we also try to provide some insight into the definition "market liquidity", a term which has two levels of meaning in our model. It refers to both current and future liquidity levels, i.e., the liquidity providers available in the market. To model these two aspects of market liquidity, we assume a limited number of small traders and type switching rates which introduces uncertainty to the future liquidity level. We have shown that such a market, from the large trader's perspective, is neither infinitely tight (i.e., she cannot turn over a position without cost in two periods even if she can perfectly discriminate across small traders), nor deep enough to avoid a price impact.

This, however, is not the case if there are a large number of small traders in the market, or many trading periods. To show this, we now extend our model to *n* small traders, each being either a high-type non-owner or a low-type non-owner, with probability $p_h$ and $1-p_h$, respectively. In any period, the probability of there being at least two high-type non-owners is $1-\left[\left(1-p_h\right)^n + np_h\left(1-p_h\right)^{n-1}\right]$, which is asymptotically equal to one for very large *n*. This implies that the large trader is almost certain to find somebody in the market to trade with, even when type switching probabilities are significantly greater than zero. Therefore, the market is perfectly liquid in the sense that she can sell whatever number of shares, whenever she wants.

Next we consider a market with a limited number of small traders and a fixed horizon, but $t$ trading periods. Suppose that the large trader only knows that at time $\tau_1$ the probability of a small trader being a high-type is *p*. Then, after $\tau$ periods, the probability of a small trader being a high-type is

$$p-\left[p\rho_d\Delta t-(1-p)\rho_u\Delta t\right]\left(\frac{1-(1-\rho_d\Delta t-\rho_u\Delta t)^{\tau}}{\rho_d\Delta t+\rho_u\Delta t}\right)$$

which is $\rho_u/(\rho_u+\rho_d)$ in the limit as $\tau$ goes to infinity.[9] It is easy to see that when two switching rates are equivalent, the probability that the large trader will find at least one small trader to trade with is approximately ½. The probability will be less than ½ when the downward switching rate $\rho_d$ is greater than the upward switching rate $\rho_u$, and be greater than ½ when $\rho_d$ is less than $\rho_u$. Whichever rate dominates, the probability of the availability of at least one high-type small trader is a constant in the limit, and is

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[9] We provide a brief derivation here. At time $\tau_1$, the probability that a small trader being a high-type is *p*, i.e., $p_h(\tau_1)=p$, $p_l(\tau_1)=1-p$.

After one period,

$$\begin{aligned}p_h(\tau_1+\Delta t)&=p(1-\rho_d\Delta t)+(1-p)\rho_u\Delta t\\&=p-\left[p\rho_d\Delta t-(1-p)\rho_u\Delta t\right]\\&=p-\gamma\end{aligned}$$

and

$$p_l(\tau_1+\Delta t)=1-p+\gamma$$

where $\gamma=p\rho_d\Delta t-(1-p)\rho_u\Delta t$.

After two periods,

$$\begin{aligned}p_h(\tau_1+2\Delta t)&=p_h(\tau_1+\Delta t)(1-\rho_d\Delta t)+p_l(\tau_1+\Delta t)\rho_u\Delta t\\&=p-\gamma(1+1-\rho_d\Delta t-\rho_u\Delta t)\\&=p-\gamma(1+x)\\p_l(\tau_1+2\Delta t)&=1-p+\gamma(1+x)\end{aligned}$$

where $x=1-\rho_d\Delta t-\rho_u\Delta t$.

Following the same method, we have

$$p_h(\tau_1+3\Delta t)=p-\gamma(1+x+x^2).$$

We can show by induction that after $\tau$ periods,

$$\begin{aligned}p_h(\tau_1+\tau\Delta t)&=p-\gamma(1+x+x^2+\cdots+x^{\tau-1})\\&=p-\gamma\left(\frac{1-x^{\tau}}{1-x}\right)\end{aligned}$$

Because $x\in(0,1)$, $\lim_{\tau\to\infty}p_h(\tau_1+\tau\Delta t)=p-\dfrac{p\rho_d\Delta t-(1-p)\rho_u\Delta t}{\rho_d\Delta t+\rho_u\Delta t}=\dfrac{\rho_u}{\rho_d+\rho_u}$.

significantly greater than zero.  Therefore, if the large trader is allowed to trade frequently enough, she can always liquidate her position without disturbing the price or worrying about illiquidity.

If the number of trading period is finite, then the probability that there is no high-type small trader is non-zero in some period.  This could even last for several periods, which, to the large trader, would seem as if the market had disappeared.

The above two extensions show that a limited number of traders, and limited trading opportunities are both crucial to market illiquidity.  Our model also provides a theoretical basis for the definition of illiquidity in Longstaff(2001), in which a trader is unable to trade because the market has disappeared.  Our model shows that such an occurrence is indeed possible.

## 7. Conclusions and Future Research

The main purpose of the simplified three-date model was to illustrate the impact of trading strategies on prices under different spot market structures. We demonstrated that with three traders (one large trader and two small traders), the transaction price of the security is determined by the future dividend flow, the traders' type-switching rates, and their bargaining power.

By studying the large trader's strategy, we show how asset prices are jointly affected by the market conditions for trading and by the large trader's own trading strategy. The risk neutrality of all market participants ensures that the liquidity effect is purely a consideration of future market liquidity. We show that, firstly, the large trader's activity does have a price impact in a "thin" market; this impact varies with different game structures. Secondly, we derive a unique subgame perfect equilibrium for the game under each structure. The large trader's equilibrium strategy varies with her relative market power, type switching probabilities, and market structures. She chooses an optimal strategy in essence by considering her chance of trading in the second period and how much she has to give up to small traders for trading in the first period.

Furthermore, with different game structures we are able to study how market structure affects both the traders' strategies and asset prices. We show that the subgame perfect equilibrium strategy for the large trader can be quite subtle and sensitive to market structure. Essentially, in a less competitive market, such as a market with separate small traders, the large trader can quickly dump her shares without worrying too much about price depression. Her monopoly power weakens as the market becomes more competitive (e.g., the iterative limiting case). She may have to choose to spread the trades over two periods under some conditions because the cost to induce small traders to buy in the first period is just too high. Finally, we explore limiting pricing results by extending the number of small traders and the number of trading periods and show that competitive intuition applies in the model.

This simple multi-period model can be extended in several ways. (1) Extend the game to multiple large traders and study how the existence of other large traders would affect individual large trader's trading strategies. It is interesting to explore the actions of other large traders when one of the large traders is in financial distress. In particular we would like to know whether we can observe front-running as part of a strategic response to a large trader's distressed selling.[10] This extension is reported in Liang(2005). (2) We could generalise the model by letting bargaining power be a function of the shares held, and study how this impacts the distressed large trader's trading strategy and asset prices. (3) We could add into the large trader's portfolio a derivative and study how hedging strategies change due to imperfect competition and liquidity risks on the underlying asset market.

[10] Predatory trading and front-running are also studied in Brunnermeier and Pedersen (2004), Attari, Mello and Ruckes (2002) and Pritsker (2004).

# Appendix

## A: Games with *ln*-traders staying on the market

### Proposition 2: (One round multilateral negotiation)

This game structure does not affect the large trader's value functions when she does not sell or sells two shares in the first period. That is

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(B_{lo},2S_{hn},t_1\right)\right)=V_{t_1}^{B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(B_{lo},2S_{hn},t_1\right)\right)$$

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)=V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-2,\Gamma\left(B_{ln},2S_{ho},t_1\right)\right)$$

given by (8) and (15) respectively. The superscript *MN* in the value function indicates the game structure of "*M*ultilateral *N*egotiation".

The large trader's value function of trading one share in the first period is thus different from that under the assumption of "separate small traders".

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right)=\begin{cases}V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-1,\Gamma\left(B_{lo},S_{hn},S_{ho},t_1\right)\right) & \text{when} \quad 2q\ge CTS\\ EV_{t_2}^{MN,B_{lo}}\left(\Gamma\left(\bullet,t_2\right)\middle|a_{t_1}^B=-1\right) & \text{when} \quad 2q<CTS\end{cases}$$

$$=\begin{cases}P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2\left(1-q\right)\rho_d\Delta t\\ \quad -\left(1-q\right)\rho_d^2\Delta t^2-\left(1-q\right)\rho_d\rho_u\Delta t^2\Big] \quad \text{when} \quad 2q\ge CTS\\ P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+3\left(1-q\right)\rho_d\Delta t-3\left(1-q\right)\rho_d^2\Delta t^2\\ \quad +\left(1-q\right)\rho_d^3\Delta t^3-\left(1-q\right)\rho_d\rho_u\Delta t^2\Big] \quad \text{when} \quad 2q<CTS\end{cases}$$

(A.1)

In the first period, if the large trader determines to sell one share, then the following equation is satisfied.

$$q\left[V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma\left(t_1\right)\right)-V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(t_1\right)\right)\right]$$
$$=\left(1-q\right)\left[V_{t_1}^{MN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\middle|a_{t_1}^B=-1\right)-P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)\right]$$

(A.2)

where

$$V_{t_1}^{MN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-1\right)=\begin{cases}V_{t_1}^{S_{lo}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-1\right) & \text{when} \quad 2q\ge CTS\\ \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[\dfrac{3}{2}\rho_d\Delta t-\rho_d^2\Delta t^2+\dfrac{1}{2}\rho_d^3\Delta t^3\right. & \\ \qquad\left.-\dfrac{1}{2}\rho_d\rho_u\Delta t^2-\dfrac{1}{2}\rho_d^2\rho_u\Delta t^3\right] & \text{when} \quad 2q<CTS\end{cases}$$

(A.3)

This is because, when $2q\ge CTS$, this small trader cannot sell in subgame (vi), in which her type switches to low type at $t_2$; but when $2q<CTS$, she is able to sell one share in this subgame. Substituting $V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)$, $V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=0,\Gamma(t_1)\right)$ and $V_{t_1}^{MN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-1\right)$ back to (A.2), we get the price as

$$P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)=\begin{cases}P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right) & \text{when} \quad 2q\ge CTS\\ \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[q^2+(1-q)\left(q+\dfrac{3}{2}\right)\rho_d\Delta t-(1-q)^2\rho_d^2\Delta t^2+(1-q)\left(\dfrac{1}{2}-q\right)\rho_d^3\Delta t^3\right. & \\ \left.-(1-q)\left(\dfrac{1}{2}+q\right)\rho_d\rho_u\Delta t^2-\dfrac{1}{2}(1-q)\rho_d^2\rho_u\Delta t^3\right] & \text{when} \quad 2q<CTS\end{cases}$$

(A.4)

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)=\begin{cases}V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right) & \text{when} \quad 2q\ge CTS\\ \dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[q+q^2+(1-q)\left(q+\dfrac{9}{2}\right)\rho_d\Delta t+(q-4)(1-q)\rho_d^2\Delta t^2\right. & \\ \left.+(1-q)\left(\dfrac{3}{2}-q\right)\rho_d^3\Delta t^3-(1-q)\left(\dfrac{3}{2}+q\right)\rho_d\rho_u\Delta t^2-\dfrac{1}{2}(1-q)\rho_d^2\rho_u\Delta t^3\right] & \\ & \text{when} \quad 2q<CTS\end{cases}$$

(A.5)

Similarly, when the large trader decides to sell two shares at $t_1$, then

$$q\left[V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right)-V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)\right]$$
$$=(1-q)\left[V_{t_1}^{MN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-2\right)-P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)\right]$$

(A.6)

where $V_{t_1}^{MN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-2\right)=V_{t_1}^{S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\middle|a_{t_1}^B=-2\right)$, given by (22).

$$P_{t_1}^{MN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)=\begin{cases}P_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right) & \text{when}\quad 2q\geq CTS\\ \frac{\bar{D}}{r^2}-\frac{\varepsilon}{(1+q)r^2}\Big[q^2(1+q)+(1-q)\left(q^2+\frac{9}{2}q+2\right)\rho_d\Delta t+(1-q)\left(q^2-4q-1\right)\rho_d^2\Delta t^2 & \\ +q(1-q)\left(\frac{3}{2}-q\right)\rho_d^3\Delta t^3-(1-q)\left(q^2+\frac{1}{2}q+1\right)\rho_d\rho_u\Delta t^2 & \\ -\frac{3}{2}q(1-q)\rho_d^2\rho_u\Delta t^3-q(1-q)\rho_d\rho_u^2\Delta t^3\Big] & \text{when}\quad 2q<CTS\end{cases}\tag{A.7}$$

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right)=\begin{cases}V_{t_1}^{B_{lo}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right) & \text{when}\quad 2q\geq CTS\\ \frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{(1+q)r^2}\Big[q^2(1+q)+(1-q)\left(q^2+\frac{9}{2}q+2\right)\rho_d\Delta t & \\ +(1-q)\left(q^2-4q-1\right)\rho_d^2\Delta t^2+q(1-q)\left(\frac{3}{2}-q\right)\rho_d^3\Delta t^3 & \\ -(1-q)\left(q^2+\frac{3}{2}q+2\right)\rho_d\rho_u\Delta t^2-\frac{3}{2}q(1-q)\rho_d^2\rho_u\Delta t^2-q(1-q)\rho_d\rho_u^2\Delta t^3\Big] & \\ & \text{when}\quad 2q<CTS\end{cases}\tag{A.8}$$

Comparing value functions of three choices at $t_1$, the large trader chooses her optimal strategy as a function of $q, \rho_d$ , $\rho_u$ and $\Delta t$ . For example,

$$\begin{aligned}&V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=0,\Gamma(t_1)\right)-V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B=-2,\Gamma(t_1)\right)\\&=\frac{2(1-q)q}{(1+q)r^2}\varepsilon\Big[-(1+q)+\left(q+\frac{5}{2}\right)\rho_d\Delta t+(q-3)\rho_d^2\Delta t^2+\left(\frac{3}{2}-q\right)\rho_d^3\Delta t^3\\&-\left(q+\frac{1}{2}+\frac{1}{q}\right)\rho_d\rho_u\Delta t^2-\frac{3}{2}\rho_d^2\rho_u\Delta t^2-\rho_d\rho_u^2\Delta t^3\Big]\end{aligned}$$

Let $K(q,\rho_d,\rho_u,\Delta t)=-(1+q)+\left(q+\frac{5}{2}\right)\rho_d\Delta t+(q-3)\rho_d^2\Delta t^2+\left(\frac{3}{2}-q\right)\rho_d^3\Delta t^3$

$$\begin{aligned}\frac{\partial K}{\partial q}&=-1+\rho_d\Delta t+\left(\rho_d\Delta t\right)^2+\left(\rho_d\Delta t\right)^3\\&=\left(1-\rho_d\Delta t\right)\left[\left(\rho_d\Delta t\right)^2-1\right]\leq 0\end{aligned}$$

Since $q\in[0,1/2)$, $q$=0 maximizes $K$, i.e.,

$$K(q=0) = -1 + \frac{5}{2}\rho_d \Delta t - 3\rho_d^2 \Delta t^2 + \frac{3}{2}\rho_d^3 \Delta t^3 .$$

$$\frac{\partial K(q=0)}{\partial(\rho_d \Delta t)} = \frac{5}{2} - 6\rho_d \Delta t + \frac{9}{2}\rho_d^2 \Delta t^2 > 0$$

for $\rho_d \Delta t \in [0,1]$. Therefore, $K$ is maximized at $q = 0$ and $\rho_d \Delta t = 1$ and hence

$$K \le \overline{K}(q=0, \rho_d \Delta t = 1) = 0 .$$

It is then easy to see that

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right) - V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) < 0 .$$

Therefore, "no trade" is a strictly dominated strategy. The large trader would employ this strategy only when she has all the power in bargains with small traders, i.e., $q = 0$.

□

**Iterative limiting case:**

We first calculate the large traders' value functions for trading one share, trading two shares or no trade at $t_1$.

$$\begin{aligned} &V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(B_{lo}, 2S_{hn}, t_1)\right) = V_{t_1}^{B_{lo}}\left(a_{t_1}^B = 0, \Gamma(B_{lo}, 2S_{hn}, t_1)\right) \\ &= \frac{2\overline{D}}{r^2} - \frac{2\varepsilon}{r^2}\left[q + 2(1-q)\rho_d \Delta t - (1-q)\rho_d^2 \Delta t^2 - (1-q)\rho_d \rho_u \Delta t^2\right] \end{aligned} \tag{A.9}$$

$$\begin{aligned} &V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(B_{ln}, 2S_{ho}, t_1)\right) = V_{t_1}^{B_{ln}}\left(a_{t_1}^B = -2, \Gamma(B_{ln}, 2S_{ho}, t_1)\right) \\ &= 2P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B = -2\right) + \frac{2\varepsilon}{r^2}(1-q)\left(\rho_d \rho_u \Delta t^2 - \rho_d^2 \rho_u \Delta t^3 - \rho_d \rho_u^2 \Delta t^3\right) \end{aligned}$$

(A.10)

$$V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B=-1,\Gamma(t_1)\right)$$
$$=\begin{cases} P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q+2(1-q)\rho_d\Delta t-2(1-q)\rho_d^2\Delta t^2\right. \\ \quad \left.+(1-q)\rho_d^3\Delta t^3-\rho_d\rho_u\Delta t^2+\rho_d^2\rho_u\Delta t^3\right] \qquad \text{when} \quad q\le CTS \\ P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q+2(1-q)\rho_d\Delta t-(1-q)\rho_d^2\Delta t^2\right. \\ \quad \left.-(1-q)\rho_d\rho_u\Delta t^2\right] \qquad \text{when} \quad q>CTS \end{cases} \tag{A.11}$$

In the first period, the large trader will sell one share at a price such that a small trader is indifferent between acquiring a share now and waiting till the next trading date. That is

$$V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-1\right)=V_{t_1}^{IN,S_{hn}}\left(a_{t_1}^S=0,\Gamma(t_1)\right) \tag{A.12}$$

If a $S_{hn}$ buys one share at $t_1$, her expected payoff would be

$$V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma(t_1)\right)=\frac{1}{r}\left[(1-\rho_d\Delta t)\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}+\rho_d\Delta t\frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r}\right] \tag{A.13}$$

If she does not buy, her expected payoff of being a high type non-owner at $t_1$ would be

$$\begin{aligned} V_{t_1}^{IN,S_{hn}}\left(a_{t_1}^S=0,\Gamma(t_1)\right)=\frac{1}{r}\Bigg[&(1-\rho_d\Delta t)^2\left(\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\frac{\bar{D}-\varepsilon\left[q+(1-q)\rho_d\Delta t\right]}{r}\right) \\ &+\rho_d\Delta t\left(\frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r}-\frac{\bar{D}-\varepsilon\left[q+(1-q)(1-\rho_u\Delta t)\right]}{r}\right)+\rho_d\Delta t(1-\rho_d\Delta t) \\ &\left(\frac{\bar{D}-\rho_d\Delta t\varepsilon}{r}-\begin{cases}\frac{\bar{D}-(1-\rho_u\Delta t)\varepsilon}{r} & \text{when } q\le CTS \\ \frac{\bar{D}-\varepsilon\left[q+(1-q)\rho_d\Delta t\right]}{r} & \text{when } q>CTS\end{cases}\right)\Bigg] \end{aligned} \tag{A.14}$$

According to (A.12), the price at which the large trader to sell one share is

$$P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^{B}=-1\right)=\begin{cases}\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+3(1-q)\rho_d\Delta t-3(1-q)\rho_d^2\Delta t^2+(1-q)\rho_d^3\Delta t^3 \\ \quad +(q-2)\rho_d\rho_u\Delta t^2+\rho_d^2\rho_u\Delta t^3\Big] & \text{when } q\le CTS \\ \dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[q+2(1-q)\rho_d\Delta t-(1-q)\rho_d^2\Delta t^2 \\ \quad +(q-1)\rho_d\rho_u\Delta t^2\Big] & \text{when } q>CTS\end{cases}$$

(A.15)

Thereby,

$$V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^{B}=-1,\Gamma(t_1)\right)=\begin{cases}\dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[2q+5(1-q)\rho_d\Delta t-5(1-q)\rho_d^2\Delta t^2+2(1-q)\rho_d^3\Delta t^3 \\ \quad +(q-3)\rho_d\rho_u\Delta t^2+2\rho_d^2\rho_u\Delta t^3\Big] & \text{when } q\le CTS \\ \dfrac{2\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\Big[2q+4(1-q)\rho_d\Delta t-2(1-q)\rho_d^2\Delta t^2 \\ \quad +2(q-1)\rho_d\rho_u\Delta t^2\Big] & \text{when } q>CTS\end{cases}$$

(A.16)

If the large trader chooses to trade two shares in the first period, she contacts both small buyers simultaneously and bargains with each of them until both of them are indifferent between buying and not buying. We need the following lemma to determine the bargaining outcome.

**Lemma 1**: *When the large trader commits to sell two shares and both small traders are able to re-negotiate with the large trader over their transaction prices, small traders would either trade or not trade with the large trader at the same time.*

Proof: This is so because if the large trader only reaches an agreement with one small trader, this small trader must obtain a higher utility by trading than waiting. Since both small traders are identical, the other small trader would be better off by re-opening a negotiation with the large trader and mimicking the first small trader. By symmetry, such situation as, one trades while one does not, cannot arise in an equilibrium. Small traders, simultaneously contacted by the large trader, either both of them trade or none of them trade with the large trader at the same time. □

Since the large trader will trade with either both small traders or none, the solution to this bargaining game is that they split the joint surplus (from three parties) such as in a bilateral bargaining situation with one seller and one buyer.

$$q\left[V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B=0,\Gamma\left(t_1\right)\right)\right]$$
$$=2\left(1-q\right)\left[V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\middle|a_{t_1}^B=-2\right)-P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)\right] \tag{A.17}$$

where

$$V_{t_1}^{IN,S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\middle|a_{t_1}^B=-2\right)=V_{t_1}^{S_{ho}}\left(a_{t_1}^S=1,\Gamma\left(t_1\right)\middle|a_{t_1}^B=-2\right)$$

which is given by (21).

$$P_{t_1}^{IN,B_{lo}-S_{hn}}\left(a_{t_1}^B=-2\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left[q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2-\left(1-q^2\right)\rho_d\rho_u\Delta t^2\right] \tag{A.18}$$

$$V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(t_1\right)\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\Big[q^2+2\left(1-q^2\right)\rho_d\Delta t-\left(1-q^2\right)\rho_d^2\Delta t^2$$
$$-\left(1-q\right)\left(2+q\right)\rho_d\rho_u\Delta t^2+\left(1-q\right)\rho_d^2\rho_u\Delta t^3+\left(1-q\right)\rho_d\rho_u^2\Delta t^3\Big] \tag{A.19}$$

We can easily show that when $q\in\left[0,1/2\right)$, $\rho_{u/d}\in\left(0,1\right)$ and $\rho_d\Delta t+\rho_u\Delta t<1$, the strategy of "no trade" is strictly dominated by the strategy of "sell two shares".

$$V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=-2,\Gamma\left(t_1\right)\right)-V_{t_1}^{IN,B_{ln}}\left(a_{t_1}^B=0,\Gamma\left(t_1\right)\right)$$
$$=\frac{2\left(1-q\right)\varepsilon}{r^2}\left[q\left(1-\rho_d\Delta t\right)^2+\rho_d\rho_u\Delta t^2\left(1+q-\rho_d\Delta t-\rho_u\Delta t\right)\right]>0$$

Thus $B_{lo}$ will never choose to sell two shares in this game.

Whether the large trader would choose to sell one share or two shares in the first period depends on the relationship between $q$, $\rho_d$ and $\rho_u$. Examples in Appendix C show that either strategy can be optimal.

## B. Games when *ln*-traders must exit the market

With the assumption that low type non-owners exit the market, the large trader loses some trading opportunities in the second period, which makes trading in the first period more desirable. Subgames at $t_2$ that will be affected by this assumption include subgame (ii), (iii), (v) and (vii).

In subgame (ii), $B_{lo}$ can only trade with $S_{hn}$ but not $S_{ln}$. $B_{lo}$'s value function for this subgame is

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{hn}, S_{ln}, t_2\right)\middle| a_{t_1}^B = 0\right) = \frac{2\bar{D} - \varepsilon\left[1 + q + \left(1 - q\right)\rho_d \Delta t\right]}{r} \tag{B.1}$$

Even worse, in subgames (iii), (v) and (vii), $B_{lo}$ cannot find anyone to trade with.

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, 2S_{ln}, t_2\right)\middle| a_{t_1}^B = 0\right) = \frac{2\bar{D} - 2\varepsilon}{r} \tag{B.2}$$

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{ho}, S_{ln}, t_2\right)\middle| a_{t_1}^B = -1\right) = rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B = -1\right) + \frac{\bar{D} - \varepsilon}{r} \tag{B.3}$$

$$\tilde{V}_{t_2}^{B_{lo}}\left(\Gamma\left(B_{lo}, S_{lo}, S_{ln}, t_2\right)\middle| a_{t_1}^B = -1\right) = rP_{t_1}^{B_{lo}-S_{hn}}\left(a_{t_1}^B = -1\right) + \frac{\bar{D} - \varepsilon}{r} \tag{B.4}$$

These subgames are not affected by different game structures. Therefore, replacing traders' value functions of these subgames in the previous analyses with their value functions under this assumption, we get all the results.

## C. Numerical Examples

The large trader's optimal strategy in the first period is sensitive to parameters $q, \rho_d$ , $\rho_u$ and $\Delta t$ . We give examples in this appendix showing how the large trader's equilibrium strategy changes as parameters change under different specifications of game structures.

**(i) One round multilateral negotiation**

As shown in Proposition 2, when $2q \geq CTS$ , the large trader's equilibrium strategy in the first period is to sell two shares and the value functions are the same as those under the market structure "geographically separate small traders". On the contrary, when $2q < CTS$ , she may choose to sell either one share or two shares in the first period.

For example, let $\Delta t = 1$ , $\rho_d = 0.3$ , $\rho_u = 0.3$ , $q = 0.3 > CTS/2 \approx 0.2857$ .

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(1.188)$$

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(1.0662)$$

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}(0.8943)$$

Obviously,

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right)$$

and the large trader's optimal trading strategy in the first period is to sell two shares.

Let's again set $\Delta t = 1$ , $\rho_d = 0.3$ , $\rho_u = 0.3$ , but $q = 0.2 < CTS/2 \approx 0.2857$ . $V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right)$ is still the largest among the three value functions.

Let $\Delta t = 1$ , $\rho_d = 0.38$ , $\rho_u = 0.01$ and $q = 0.002$ , which is much less than $CTS/2 \approx 0.4919$ . In this case,

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right).$$

But when $q$ increases, e.g., $q = 0.05$, then

$$V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right) > V_{t_1}^{MN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right).$$

**(ii) Iterative limiting case**

Similarly, the large trader's optimal strategy in the first period under this game structure depends on the relationship between $q, \rho_d$, $\rho_u$ and $\Delta t$.

when $q \le CTS$, the large trader may choose to sell one share or two shares in the first period. For example, let $\Delta t = 1$, $\rho_d = 0.6$, $\rho_u = 0.05$ and $q = 0.05$.

$$V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right) > V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right) > V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right)$$

But when $\rho_u$ increase to 0.3, while $\Delta t$ and $q$ are kept unchanged, the large trader's optimal strategy is to sell two shares in the first period.

When $q > CTS$,

$$V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = 0, \Gamma(t_1)\right) = V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = -1, \Gamma(t_1)\right)$$

and both are less than $V_{t_1}^{IN,B_{lo}}\left(a_{t_1}^B = -2, \Gamma(t_1)\right)$. Thus the large trader's optimal strategy is to sell quickly, i.e., two shares, when $q > CTS$.

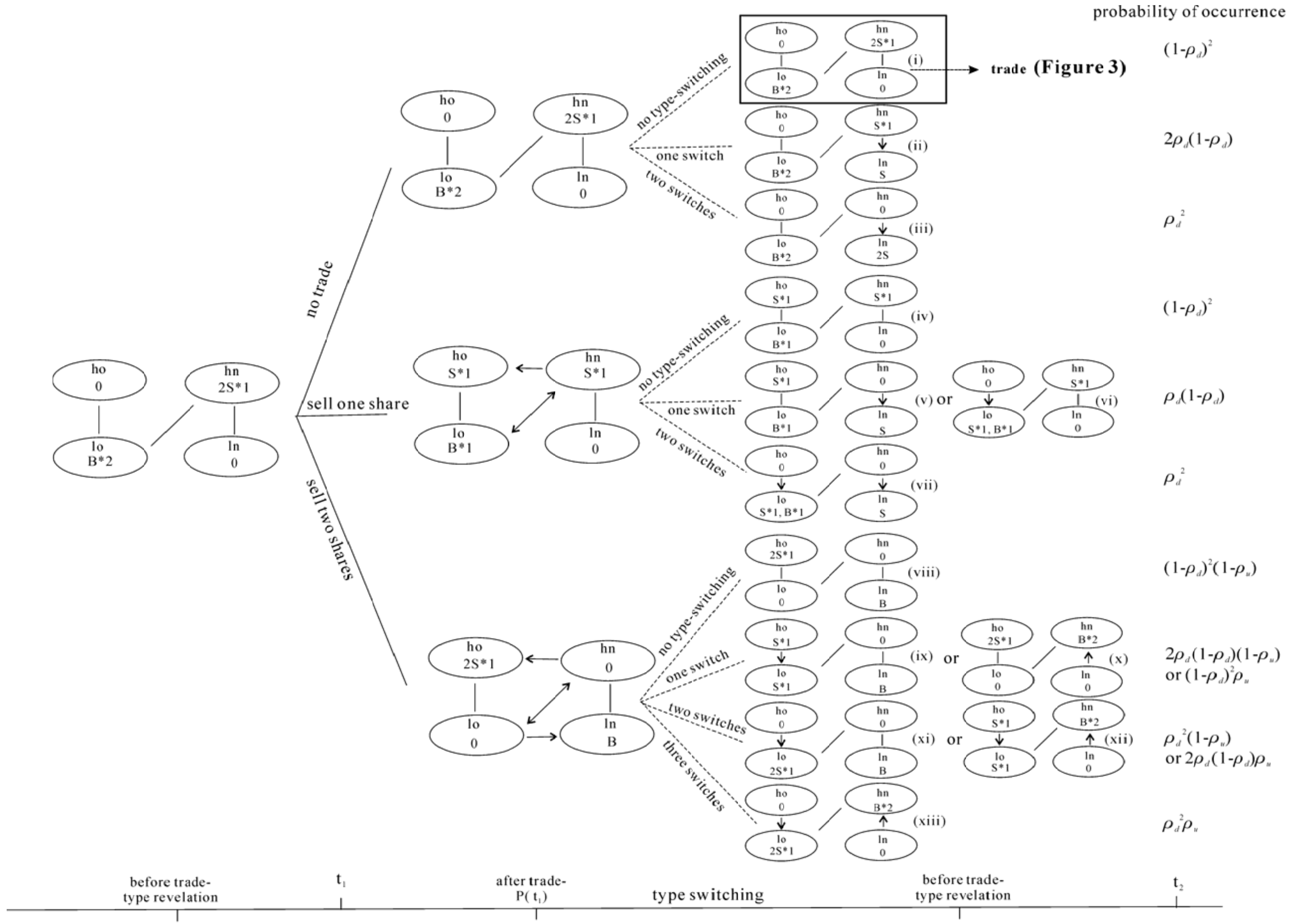


Figure 2. The dynamics of the trading and type evolution

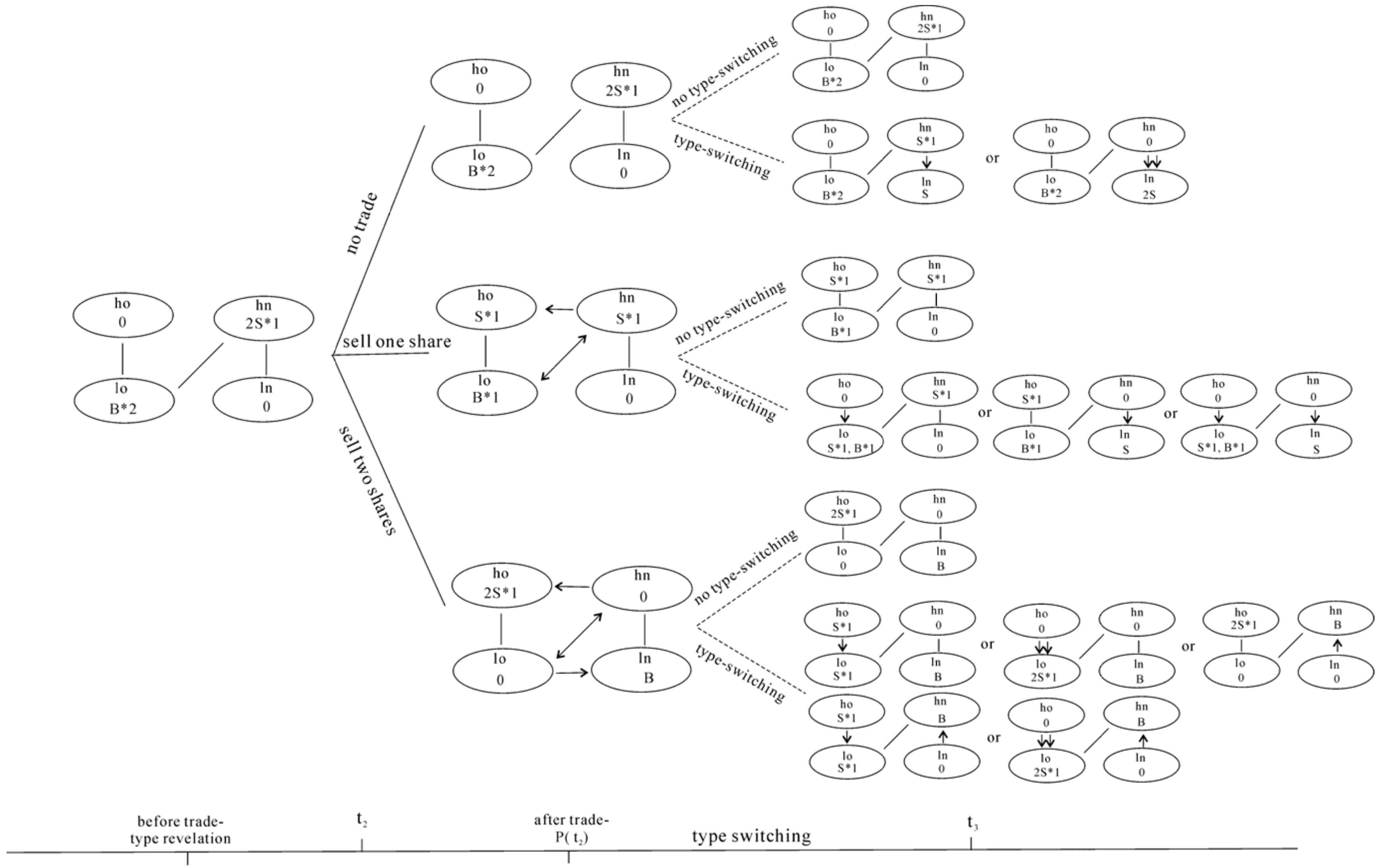


Figure 3. The evolution of a subgame begins at $t_2$

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