Abstract

We construct an equilibrium model of the labor market with on-the-job search where jobs are optimal dynamic contracts with endogenous termination. We first show that in the existing models of on-the-job search (e.g., Burdett and Mortensen, 1998; Burdett and Coles, 2003), if the contract is allowed to be contingent on the worker’s public outside offers, then the distribution of the contracts offered in equilibrium is degenerate, in which all workers are paid the same monopsony wage. That is, search, including on-the-job search, alone cannot support equilibrium contract dispersion. We then show that if the worker’s outside offers are private, not observable to the firm that currently employs him, then a non-degenerate distribution of wage-tenure contracts would arise in equilibrium. That is, search and private information combined could support equilibrium contract dispersion.
1 Introduction

Can search and wage posting support a non-degenerate equilibrium distribution of wages offered in a labor market with identical firms and homogenous workers? The search for a pure theory of wage dispersion starts with Diamond (1971) who, nevertheless, offers a negative answer to the above question. The only equilibrium he finds, in an environment that meets the qualifications of the above question, is one in which only the monopsony wage is offered.

As is well put by Rogerson, Shimer and Wright (2005, page 976), a pure theory of wage dispersion is of interest for two reasons. “First, the early literature suggested that search is relevant only if the distribution from which you are sampling is nondegenerate, so theorists were naturally led to study models of endogenous dispersion. Second, many people see dispersion as a fact of life, and for them the issue is empirical rather than theoretical.” A pure theory of wage dispersion may provide an explanation for the “unexplained” wage differences in the labor market. Mortensen (2003, page 1) reports that “Observable worker characteristics that are supposed to account for productivity differences typically explain no more than 30 percent of the variation in compensation.”

Burdett and Mortensen (1998) (hereafter BM) develop a first pure theory of wage dispersion. They show that adding on-the-job search to Diamond (1971) would produce a non-degenerate distribution of equilibrium wage offers. Their idea is that, relative to lower wages, higher wages, while imposing greater costs of labor compensation on the firm, also offer the benefits of lower employment turnover – the probability with which the worker quits from his current job decreases in the pay of that job. This trade off between compensation and job turnover results in differential wages offered in equilibrium by identical firms to identical workers.

In BM, an employment contract is a promise of constant wage until the worker finds a better outside offer and quits his current job. Thus the dispersion in wages is essentially a dispersion in the compensation contracts offered. Burdett and Coles (2003) (hereafter BC) generalize BC, allowing the contract to optimize on two dimensions: the initial wage offer, and the profile of continuation wages as a function of the worker’s tenure at the firm. Their environment produces not only a non-degenerate equilibrium distribution of initial wages offered, but also continuation wages that are monotonically increasing in the worker’s tenure.1

1Stevens (2004) considers a similar model where workers are risk neutral. There is a pure wage dispersion with a two-point support, and there is no job turnover. Like BC, Stevens (2004) assumes that firms post dynamic contracts to which they can commit, but does not allow the contracts to respond, ex post, to the worker’s outside offers.
In both BM and BC, however, the continuations of the initial contract the firm offers, whether it be a constant wage until the worker quits voluntarily, or a wage-tenure profile where wage increases in the worker’s tenure, are not allowed to be dynamically contingent on the outside offers the worker receives ex post. Once the job starts, the firm never responds to the worker’s outside offers. Obviously, if one stipulates that the worker’s outside offer is observable and the labor contract is allowed to be written as a plan that is fully contingent on the worker’s history with the firm, including all offers he receives on the job, past and current, then the contracts in BM and BC may not be fully optimal. If so, then would a non-degenerate wage distribution still emerge as an equilibrium outcome of their labor market?

The answer is no, and the logic is as follows. In both papers discussed above, since the contract does not allow for ex post adjustments in its continuation to the worker’s outside offers, higher costs of labor compensation necessarily imply lower probabilities of termination and thus lower costs of labor turnover. Once this restriction is lifted and the firm is free to match, ex post, the worker’s outside offers, the optimal contract, as we show in this paper, is then able to achieve a given probability of worker retention with a lower expected utility promised to the worker. Put differently, lower probabilities of termination need not necessarily be enforced with higher promised utilities for the worker. Instead of relying on a higher fixed wage for achieving a lower probability of job quitting, a lower initial wage plus a promise to match the worker’s outside offers may achieve the same probability of continuation.

More specifically, we show that, in the unique equilibrium of the model, all workers are paid the same monopsony wage. That is, when firms are allowed to compete with outside offers, by way of making counteroffers, workers get paid less, not more, in equilibrium. What happens, obviously, is that allowing for counteroffers discourages outside offers from being made in the first place and, through that, reduces competition in the labor market, rather than increase it. In the absence of counteroffers, any firm need only offer a slightly higher wage – deviating from the monopsony wage – in order to steal workers from other firms. With counteroffers, such a deviation would be countered in time and thus is never used.

Perhaps even more surprisingly, the idea that allowing for counteroffers in the dynamic contract destroys the equilibrium dispersion and restores the monopsony wage goes father than having force only with identical firms. Suppose firms are not identical. Suppose some firms can make (identical) workers more productive – but not too much more productive – than other firms. Then the same logic applies and it continues to hold that the firm’s ability to counter the worker’s outside offers could deter the offers from being made in the first place, rendering a unique equilibrium of the labor market where only the monopsony wage is offered.
This brings out the second question we ask in the paper: Is the idea of BM, which is so appealing intuitively, still good for generating equilibrium dispersions in the labor market? The answer is yes. In the second half of the paper we show that if the worker’s outside offers are private, not observed to the firm that currently employs him, then a non-degenerate distribution of contract offers would arise in equilibrium. That is, search and private information combined could support equilibrium contract dispersion. Why?

Remember, in the case of complete information, it is the firm’s state-contingent responses to the worker’s outside offers – the counteroffers – that break BM’s mechanism for supporting the equilibrium dispersion. Now the counteroffers are easier to implement when the worker’s outside offers are publicly observed. In the case of private outside offers, incentive compatibility requires that the worker’s next period expected utility be constant across all states of his outside offer with which he is retained. Incentive compatibility rules out the possibility of making the retained worker’s expected utility contingent on the state of his outside offer. In other words, counteroffers are not incentive compatible. Thus, with private information, the optimal contract never matches the worker’s outside offers and, as a result, the negative relationship between the worker’s promised utility and the probability of termination in BM is restored. In fact, we show that with private information, the optimal contract is exactly the wage-tenure contract in BC.

Since firms are identical and workers are homogeneous, the equilibrium contract dispersion we derive under private information is pure, in the language of the literature.

1.1 More on the Literature

In modelling on-the-job search and how firms react to their employees’ outside offers and how that interacts with the wages/contracts offered in equilibrium, our work is related to Postel-Vinay and Robin (2002) who emphasize the competition between firms with heterogenous productivities. When an employed worker is matched with a poaching firm, the poaching firm and the incumbent firm engage in a Bertrand competition for the worker - they bid optimally for him. If all firms have the same productivity, then the equilibrium wage distribution has a two point support between the monopsony and Walrasian wages, even though all intitial wage offers are the same monopsony wage. In contrast to BM, Postel-Vinay and Robin (2002) allow for state dependent offers and counter-offers. The contract nevertheless is still not fully optimal. Specifically, it is imposed that the firm pays the worker a fixed wage until the next poaching firm appears, and then the firm bids against the poaching firm to offer a new wage to the worker.
Moscarini (2005) also emphasizes the interaction between the incumbent and poaching firms. In his model, when an employed worker is matched with a poaching firm, the poaching firm and the incumbent firm, with whom the worker has differential productivities, would engage in a first-price auction in which they offer a lump-sum transfer for the right to employ the worker. The winning firm and the worker would then engage in Nash bargaining to determine a new wage for the worker. Assuming a small cost of bidding in the auction game though, the less productive firm would simply give up the bidding, and the more productive firm would then get to employ the worker with a zero lump-sum transfer to him. Moscarini (2005) is not a pure theory of wage dispersion, for the equilibrium wage distribution is degenerate if firms are identical.

The papers discussed above, like ours, both model on-the-job search and both allow the incumbent employer to respond optimally to the worker’s outside offers. The difference is in the strategies each use in modeling the interaction between the incumbent and the poaching firms. In Postel-Vinay and Robin (2002) and Moscarini (2005), the incumbent and poaching firms engage in ex post and face to face competition for the worker where they play strategically against each other. Our strategy is to take the worker’s stochastic outside offers as exogenous and let the firm react to the offers through an ex ante optimally designed contract. Obviously, behind our modeling strategy is the traditional search/matching idea where jobs (contracts) are publicly posted, they are matched randomly with workers searching for them, and whoever accepts the offer gets the job. The firms who post the jobs do not engage in ex post interactions with the current employers of the workers they are matched with. In short, what we do in this paper is to work with the most standard model of on-the-job search while getting the contract right – making it fully optimal. 2

The recent work of Moscarini and Postel-Vinay (2013) also studies an equilibrium model of the labor market with on-the-job search where firms post and commit to dynamic employment contracts. There are major differences. First, Moscarini and Postel-Vinay (2013) is not a pure theory of contract dispersion. Their objective is to understand how wage and firm size distributions move dynamically with the business cycle. For that purpose, they impose an “equal treatment constraint” on the firm to pay equal wage to all of its employees. Such a constraint, while greatly reducing the dimensionality in the optimal contracting problem and, through which, facilitating the analysis of the model, reduces also the firm’s ability

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2In the bidding games of Postel-Vinay and Robin (2002) and Moscarini (2005), the outcome of the competition between the incumbent and poaching firms is independent of the worker’s history of employment. Thus, risk sharing, which is an important part of many employment relationships, is not considered. The optimal contract in our model, in contrast, is designed to achieve the most efficient combination of incentives and risk sharing between the risk neutral firm and the risk averse worker.
to condition the worker’s compensation on the history of the relevant states of the world. Relative to Moscarini and Postel-Vinay (2013), we take a step back in the direction of including aggregate uncertainty and firm heterogeneity in the analysis. We then take a step up in three directions that are essential for our objective. First, we allow firms to offer contracts that are contingent on all relevant states of the world, especially the full history of the employment relationship. Second, unlike in Moscarini and Postel-Vinay (2013) where jobs end exogenously, we require the termination of any job be endogenous, as part of the optimal contract. Third, in contrast to Moscarini and Postel-Vinay (2013) which assumes global risk neutrality, workers are risk averse in our model and the optimal contracts are designed to achieve the most efficient combination of incentives and risk sharing.

Here we must emphasize that jobs (contracts) end endogenously is an especially important aspect of our analysis. Indeed it should be an important assumption in any theory of pure wage/contract dispersion that admits on-the-job search. For BM, it is exactly the trade off between compensation and termination that gives identical firms incentives to offer differential wages. In our model, termination again plays a key role in shaping the firms’ trade off between contracts that offer higher and lower initial values.

To conclude the section, note that two important features distinguish our work from those in the existing literature. First, firms in our model post and commit to fully optimal dynamic employment contracts that prescribe efficient compensation and termination policies that are history dependent. Second, we allow for both publicly and privately observed outside offers. None of the existing studies assume fully optimal employment contracts with endogenous termination. None of the existing papers have studies the role of private information in a pure theory of wage (contract) dispersion.

2 Model

The basic structure of the model is almost exactly Burdett and Coles (2003) except that time is discrete in our model but continuous in theirs. The model differs from Burdett and Mortensen (1998) in that we assume risk averse workers and their workers are risk neutral.

Let $t$ denote time: $t = 1, 2, \cdots$ There is a single perishable consumption good in the model. The economy has a continuum, with unit mass, of identical workers. The economy also has a continuum of $m(\geq 0)$ units of identical firms.\footnote{If there is free entry and exit of firms into and from the economy, then $m$ may be determined endogenously, but that wouldn’t affect the analysis.} Firms live forever, their objective is to maximize expected discounted profits. Workers belong to an infinite sequence of overlapping
generations. Each worker, when alive, has a constant probability \(1 - \delta \in [0, 1]\) to survive into the next period (that is, \(\delta\) is the constant mortality rate). Workers who die are replaced immediately by an identical young worker. Workers have the following preferences:

\[
\mathbb{E}_\tau \left[ \sum_{t=\tau}^{\infty} (\beta(1-\delta))^{t-\tau} u(c_t) \right],
\]

where \(\mathbb{E}_\tau\) denotes the worker’s expectation conditional on information available at the beginning of period \(\tau, \tau \geq 1\); \(\beta \in (0, 1)\) is the discount factor which is shared by the firms and the workers; \(c_t\) and \(u(c_t)\) denote, respectively, the worker’s consumption and utility in period \(t\). Assume \(c_t \in \mathbb{R}_+\) for all \(t\). That is, consumption must be non-negative.\(^4\) Last, assume the utility function \(u\) is bounded, strictly increasing, strictly concave, twice differentiable, and satisfies the Inada conditions.

Each period, each firm can hire one worker to produce. Each worker, when employed, produces a constant output of \(\theta \geq 0\). Each period, unemployed workers each receive from outside the labor market a constant compensation which is normalized to be zero.

There is a labor market that opens at the beginning of each period where workers and firms are matched to form productive partners. All workers, employed and unemployed, can participate in this market for free. Vacant firms must pay a fixed cost \(k \geq 0\) each period in order to post a vacancy in the market. Any posting of vacancy is an employment contract the firm would offer to the potential worker it will be matched with. Whoever the firm is matched with gets offered the contract.\(^5\)

Matchings are random. Specifically, each worker, employed or unemployed, is matched with a vacant firm with the same probability \(p_w \in [0, 1]\); and each vacant firm is matched with a worker with an equal probability \(p_f \in [0, 1]\). Each period, the total number of matches the labor market generates is determined by \(M(1, v)\), where \(1\) is the measure of all workers and \(v\) is the measure of all vacant firms.

Once a vacant firm is matched with a worker, it offers him the employment contract it has posted. If the worker is employed, then his employer has the option to respond to the offer. The workers stays with his current employer if the latter provides a counter offer that weakly dominates his outside offer; otherwise he quits to pursue the outside offer. In this process,

\(^4\)What is important is that the worker’s consumption is bounded from below, but not specifically by zero.

\(^5\)An alternative specification for the structure of the market is to assume that the vacant firm, instead of posting a fixed contract for both employed and unemployed workers, offers differential contracts to employed and unemployed workers. How this specification would affect the model’s equilibrium outcome will be discussed later in the paper.
each party gets to move once, the poaching firm first, the incumbent firm next, the worker last.

The contract can be fully dynamic with which the employment relationship ends in two scenarios. One, the worker dies. Two, the worker is terminated according to the terms of the contract. The latter scenario includes two cases: the worker quits voluntarily to take a better outside offer he receives on the job which the firm refuses to match; and the firm terminates the worker to send him to unemployment. In each case, the firm goes immediately back to the labor market to look for a new worker.

Any termination imposes a non-negative cost $C_0 \geq 0$ on the firm. Imagine the firm who terminates a worker (at the end of period $t$) to go back to the labor market (at the beginning of period $t + 1$) to post a vacancy. Then the total cost he pays is $C_0 + \beta k$.

We make the following assumptions on contracting.

**Assumption 1 (Limited Liability)** Compensation to the worker must be non-negative. In other words, it is only feasible that the firm pays the worker (a non-negative amount of consumption), the worker cannot pay the firm.

**Assumption 2 (Limited Commitment)** In each period, the worker is free to walk away from the contract, before and after receiving his outside offer. The firm can fully commit to the terms of any contract it offers.

Assumption 2 implies that, if the firm wants to retain a worker who has an outside value of $\xi$, then the continuation of the contract must promise the worker expected utility of at least $\xi$. Note that Assumptions 1 and 2 are both imposed, explicitly or implicitly, in Burdett (1978), Burdett and Mortensen (1998), and Burdett and Coles (2003).

In the remainder of the paper, we first study, in Section 3, the model under the assumption of complete information. That is, the worker’s outside offers are publicly observed by the worker and the firm. We then study the model assuming that the worker’s outside offers are private to himself, not observable to the firm that currently employs him.

### 3 Complete Information: Degenerate Dispersion

**Assumption 3 (Public Outside Offers)** Any outside offer is public information between the worker who receives it and the firm who employs him.
Proposition 1 The economy has a unique stationary equilibrium where all firms post the same contract which offers expected utility $V_{\text{min}}$, and in equilibrium all employed workers are paid the same monopsony wage $w = 0$.

That is, under Assumption 3, the economy does not have a stationary equilibrium with a non-degenerate distribution of contract offers. All workers are offered their reservation utility and, once employed, no worker would quit his current job, as in Diamond (1971). This is in constrast with the results of Burdett and Mortensen (1998) and Burdett and Coles (2003).

The formal proof of the proposition will be given in Sections 3.1-3.4. Here, let us first discuss the economic intuition behind the result.

Suppose the economy does have a stationary equilibrium with a non-degenerate distribution of differential contract offers. Remember these contracts are offered indiscriminately to the employed and unemployed workers whom the firms are randomly matched with. Consider first the problem of an incumbent firm whose worker has received an offer (an employment contract) from a vacant firm. Now observe, importantly, that the vacant firm (who extends the outside offer to the worker) and the incumbent firm (who must now respond optimally to the vacant firm’s offer) face exactly the same optimization problem, except that the incumbent firm, if it loses the worker, must incur an extra cost $C_0 \geq 0$ for terminating an existing employment relationship. Thus if the contract being offered to the worker is optimal for the vacant firm - which is, by assumption - it must also be optimal for the incumbent firm to use as a counter offer to retain the worker. Thus in equilibrium no outside offers will be accepted by employed workers. Any outside offer that promises a higher expected utility for the worker would be matched by the worker’s current employer. And of course any offer that promises a lower expected utility will be disregarded by the incumbent firm and again the worker stays with his existing job. To summarize, in equilibrium only unemployed workers would accept any offer any vacant firm posts. Note that unemployed workers are identical and have the same reservation utility.

Consider then the problem of the vacant firm who is deciding what contract to post/offer before the market opens. Given the logic in above paragraph, what the vacant firm should offer, (which, remember would only be accepted by unemployed workers,) should be the contract that maximizes the firm’s value subject to giving the unemployed worker an expected utility which is weakly better than his reservation utility. In fact, the starting expected utility the vacant firm offers will just be equal to the unemployed worker’s reservation utility, given that the firm’s maximum value is attained at and only at the unemployed worker’s reservation utility. The rest of the proposition then follows immediately.
In this environment, as in Burdett and Mortensen (1998), an offer of higher expected utility potentially has three effects on the value of the firm. (a) Higher compensation costs to the firm. (b) A higher probability with which the contract is accepted by a matched worker, employed or unemployed. (c) A lower probability with which the employed worker quits (the effect of on-the-job search). In our model, an offer of higher expected utility does not imply a higher equilibrium probability of job acceptance, because in equilibrium jobs are never accepted by employed workers (reason given in the above paragraph) and unemployed workers have the same reservation utility. Question then is, would (a) and (c) exist in our model to generate the trade-off that is necessary for the equilibrium dispersion, as they do in Burdett and Mortensen (1998)?

As discussed earlier, in Burdett and Mortensen (1998), since the contract does not allow ex post adjustment in its continuation to the worker’s outside offers, a smaller probability of termination necessarily implies higher costs of compensation. Now the inability of the contract to respond optimally to the worker’s outside offers is important. Once this restriction is lifted, this link between the costs of compensation and the probability of termination no long exists. That is, once the contract is allowed to respond optimally to the worker’s outside offers, a lower expected utility promised to the worker need not imply a higher probability of termination - the contract always has the ability to match the worker’s outside offer in order to enforce continuation. In other words, a lower probability of termination need not be enforced by a higher expected utility promised to the worker. A contract that responds more aggressively to the worker’s outside offers may attain a lower probability of termination with a relatively low expected utility promised to the worker.

3.1 The Labor Market

To start the analysis, we first describe the aggregate variables of the labor market whose values will be taken as given, in equilibrium, by firms and workers in their individual decision making.

Let $u \in [0, 1]$ denote the measure of unemployed workers in equilibrium and $1 - u \in [0, 1]$ that of employed workers. Let

$$p_w = M(1, m - (1 - u)) \in [0, 1] \quad (1)$$

denote the probability with which a worker, employed or unemployed, is matched with a firm in equilibrium. Remember once a firm is matched with a worker, employed or unemployed, the
firm automatically offers the worker the contract it has publicly posted. Let \( p_f \in [0, 1] \) denote the probability with which an individual vacant firm is matched with a worker, employed or unemployed, in equilibrium. That is,

\[
p_f = \frac{M(1, m - (1 - u))}{m - (1 - u)} \in [0, 1].
\] (2)

At the beginning of each period, in the labor market there is a distribution of vacant firms in the starting expected utility they post for new hires. The support of this distribution is denoted \( \Phi^* \), the set of expected utilities that vacant firms are able to deliver and offer in equilibrium. Then for each \( \xi \in \Phi^* \), let \( F^*(\xi) \) denote the fraction of vacant firms that post a job that offers the worker an expected utility no greater than \( \xi \). Assume \( F^* \) has a density denoted \( f^* : \Phi^* \rightarrow \mathbb{R}_+ \).

Note that all \( \xi \)s in \( \Phi^* \) may not be offered in equilibrium with positive probability, but we require that all offers be feasible for the vacant firm to deliver. In other words, for any \( \xi \in \Phi^* \), there exists a feasible contract, the notion of which to be given in the next subsection, that gives the worker expected utility \( \xi \). Note also that we allow firms to use symmetric but mixed strategies for job posting. As such, each expected utility posted, and subsequently offered, is simply a random draw from the distribution \( F^* \), with \( F^* \) being the equilibrium mixed strategy used for job posting by all vacant firms.

At the beginning of any period, there is also a distribution of employed workers in the expected utility their employer has promised to deliver. Let \( G(V) \) denote the equilibrium fraction of employed workers who are promised by their current employer an expected utility no greater than \( V \), for all \( V \in \Phi^* \).

Finally, let \( V_0 \) denote the expected utility for an unemployed worker at the beginning of a period in equilibrium. We have \( V_0 \in [V_{\text{min}}, V_{\text{max}}] \) and

\[
V_0 = u(0) + \beta(1 - \delta) \left[ p_w \int_{\Phi^*} \max\{\xi, V_0\} dF^*(\xi) + (1 - p_w)V_0 \right].
\] (3)

Remember the unemployed worker’s consumption is normalized to zero. With probability \( p_w \) the unemployed worker is matched with a vacant firm to receive a random offer of expected utility \( \xi \in \Phi^* \). He would take this offer if the value of this offer is above his reservation utility, which is \( V_0 \), and reject it to remain unemployed otherwise. With probability \( 1 - p_w \) he is not matched with a vacant firm and he then remains unemployed moving into the next period.

\footnote{We could allow the firm to post/offer differential contracts for workers of differential employment statuses. This case is discussed in Section 3.4.}
3.2 Equilibrium Contracting

In the labor market, each vacant firm posts an expected utility, together with a contract which delivers that expected utility, that it promises to offer to any worker, employed or unemployed, whom it expects to be randomly matched with. In choosing the starting expected utility and the corresponding contract, the firm takes as given, in addition to the parameters of the physical environment, the aggregate states of the labor market, including in particular the contract used in equilibrium which we denote as $\sigma^{*}$, and the distribution $F^{*} : \Phi^{*} \rightarrow [0, 1]$ of the starting expected utilities posted/offered by individual firms in equilibrium (through the equilibrium contract $\sigma^{*}$).

In formulating the individual vacant firm’s contracting problem, we take the stand that the contract must be able to respond in each period to any outside offer that the firm perceives to be feasible for other vacant firms to offer and hence its worker to receive in equilibrium. That is, each individual contract must and need only consider the $\xi$s in $\Phi^{*}$, the set expected utilities that are feasible for the equilibrium contract to deliver. With this, in equilibrium, for any individual firm, an employment contract, formulated recursively, is as follows:

$$\{c(V), \Omega(V), V_r(\xi; V), I(V), V_n(V) : \xi \in \Omega(V) \subseteq \Phi^{*}, V \in \Phi\},$$

where

(i) $V$, the “state variable”, denotes the expected utility of the worker that the continuation of the contract promises to deliver at the beginning of the period, and the set $\Phi \subseteq \Sigma \equiv [\frac{u(0)}{1-\beta(1-\delta)}, \frac{u(\infty)}{1-\beta(1-\delta)}]$ denotes the set of all $V$s that are feasible for this contract to deliver - the state space. Note also that at this stage of individual contracting, $\Phi$ is not $\Phi^{*}$, although later the $\Phi$ for the optimal contract will be required to be consistent with $\Phi^{*}$ in equilibrium. At this stage, $\Phi$ is an endogenous part of the individual contract which takes $\Phi^{*}$, the set of outside offers that it perceives for his worker to receive in equilibrium, as given.

(ii) $\xi$ denotes the worker’s current outside offer. Each $\xi$ is drawn from the set $\Phi^{*}$, the set of all possible outside offers in equilibrium.

(iii) $c(V)$ denotes the worker’s compensation in the period.

(iv) $\Omega(V) \subseteq \Phi^{*}$ denotes the set of current outside offers (the $\xi$s in $\Phi^{*}$) with which the worker is retained. That is, the worker is retained if $\xi \in \Omega(V)$ and terminated if $\xi \in \Phi^{*} \setminus \Omega(V)$.

(v) $V_r(\xi; V)$ denotes the worker’s next period promised utility if he is retained, i.e., if $\xi \in \Omega(V)$. Note if the worker is terminated upon $\xi$, then his next period expected utility is simply $\max\{\xi, V_0\}$. 

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(vi) \( I(V) \in \{0, 1\} \) indicates the worker’s status with the firm after receiving no outside offer. Specifically, if \( I(V) = 1 \), then the worker is retained and given expected utility \( V_n(V) \). If \( I(V) = 0 \), then the worker is terminated and his value is \( V_0 \). In other words, the value of the worker conditional on not being matched with a firm is \( I(V)V_n(V) + (1 - I(V))V_0 \).

We now formulate optimality. For each \( V \in \Phi \), let \( U(V) \) denote the maximum expected value of the firm given that the worker it currently employs is promised expected utility \( V \). Next, for all \( \xi \in \Phi \), let \( U(\xi) \) be the expected value for the vacant firm who posts a contract that offers expected utility \( \xi \). It is straightforward to calculate that for all \( \xi \in \Phi \),

\[
U(\xi) = \frac{-k + p_f \gamma(\xi) U(\xi)}{1 - (1 - p_f \gamma(\xi))}\beta, \tag{5}
\]

where remember \( p_f \) is the probability with which vacant firms are matched with a worker (employed or unemployed), and \( \gamma(\xi) \in [0, 1] \) is the probability with which a contract that offers expected utility \( \xi \) is accepted, upon being offered to a randomly matched worker.

We call \( \gamma(\xi) \) the acceptance probability for the offer \( \xi \). We call \( \gamma(\cdot) \) the acceptance probability function. Note, importantly, that the firm takes both \( p_f \) and \( \gamma(\cdot) \) as given. Note also that since the set \( \Phi \) is a choice variable for the firm, we take as given that domain for the function \( \gamma(\cdot) \) is \( \Sigma \). We provide later in the section an explicitly assessment for \( \gamma(\xi) \) for all \( \xi \in \Sigma \) in equilibrium.

The value function \( U : \Phi \to \mathbb{R} \) and the optimal contract must then solve the following Bellman equation: For all \( V \in \Phi \),

\[
U(V) = \max_{c, \Omega, V_r(\cdot), I, V_n} (\theta - c) + \beta (\pi - C_0) + \beta(1 - \delta)p_w \left( \int_{\Omega} U(V_r(\xi)) \, dF^*(\xi) + \int_{\Phi^* \setminus \Omega} (\beta \pi - C_0) \, dF^*(\xi) \right)
+ \beta(1 - \delta)(1 - p_w) [IU(V_n) + (1 - I) (\beta \pi - C_0)] \tag{6}
\]

subject to

\[
u(c) + \beta(1 - \delta)p_w \left( \int_{\Omega} V_r(\xi) \, dF^*(\xi) + \int_{\Phi^* \setminus \Omega} \max\{\xi, V_0\} \, dF^*(\xi) \right)
+ \beta(1 - \delta)(1 - p_w)[IV_n + (1 - I)V_0] = V, \tag{7}
\]

\[
c \geq 0, \tag{8}
\]

\[
\Omega \subseteq \Phi^*, \tag{9}
\]

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\[ V_r(\xi) \in \Phi, \forall \xi \in \Omega, \]  
(10)

\[ V_r(\xi) \geq \max\{\xi, V_0\}, \forall \xi \in \Omega, \]  
(11)

\[ I \in \{0, 1\}, V_n \in \Phi, V_n \geq V_0, \]  
(12)

where \( \Phi \) is the largest self-generating set with respect to the constraints (7)-(12) and \( \Phi \subseteq \Sigma \),\(^7\) and

\[ \pi \equiv \max_{\xi \in \Phi} U(\xi) \]  
(13)

where the values \( U(\xi) \) are given by (5).

In (6), \( \pi \) is the value of being vacant for the firm. Being vacant, the firm picks an optimal \( \xi \) from the set of deliverable values \( \Phi \) to post, as equation (13) describes. Notice that in any case of termination, the firm incurs immediately the cost \( C_0 \) and then move into the next period to have value \( \pi \). Equation (7) is the promise-keeping constraint. Notice that in the case of termination, the worker is free to take the outside offer or to become unemployed. So any outside offer below \( V_0 \) would never be taken, and thus should never be offered. Put differently, the probability that the worker receives an outside offer below \( V_0 \) is zero. Equation (10) says that what the firm promises to the retained worker must be what the contract could deliver. Equation (11) is the self-enforcing constraint: the contract must give the worker better than his outside offer in order to retain him.

To complete the formulation of optimal contracting, we must determine how the individual firm would assign values to \( \gamma(\xi) \), for all \( \xi \in \Phi \) and all \( \Phi \subseteq \Sigma \). Note it is important that the values of \( \gamma(\xi) \), as the other equilibrium objects, including \( \sigma^*, F^*, V_0, p_w, p_f, \) and \( u \), are taken as given by individual firms in their optimal decision making.

Observe first that \( \gamma(\xi) = 0 \) for all \( \xi < V_0 \). In words, no worker, employed or unemployed, would ever accept a contract with expected utility \( \xi \) strictly less than the reservation utility \( V_0 \).

Suppose then \( \xi \geq V_0 \). An unemployed worker would always accept the offer. Whether an employed worker would accept the offer depends upon how the worker’s current employer would respond to that offer. On this, there are two cases.

Case one Suppose $\xi \in \Phi^*$. This is an on the equilibrium path offer to which the incumbent firm has prepared to react with the contract it posts. Specifically, it would retain the worker if $\xi \in \Omega^*(V)$ and terminate the worker if $\xi \in \Phi^* \setminus \Omega^*(V)$, according to what the optimal contract prescribes. Thus,

$$\gamma(\xi) = u + (1 - u) \int \mathbb{I}(\xi \in \Phi^* \setminus \Omega^*(V))dG(V)$$

where $\mathbb{I}$ is the indicator function, with $\mathbb{I}(\xi \in \Phi^* \setminus \Omega^*(V)) = 1$ if $\xi \in \Phi^* \setminus \Omega^*(V)$ and 0 otherwise.

Case two Suppose $\xi \in \Phi \setminus \Phi^*$. That is, the offer the worker receives is something his current employer did not anticipate to occur (outside the set $\Phi^*$). How then would the firm react to this $\xi$, an off equilibrium path offer that came as a surprise? Here we take the stand that such an offer would be viewed as a zero probability incidence and thus the worker and his current employer, upon observing it, would not change their beliefs about the distribution from which any future outside offer would be drawn (that is, they believe that any future outside offer would still be drawn randomly from the (rationally perceived) equilibrium distribution $F^*: \Phi^* \to [0, 1]$). As such, upon the draw of the $\xi$, if the worker and his incumbent employer would enter into a continuation of their initial contract which promises the worker a new expected utility of $V \in \Phi$, then the value for the firm would just be $U(V)$ - the value function $U(\cdot)$ remains valid for calculating values for the firm. Given these, if

$$\max_{V \in \Phi \text{ and } V \geq \xi} U(V) \geq \beta \pi - C_0,$$  \hspace{1cm} (14)

then the incumbent firm would retain the worker by counteroffering a contract with expected utility $V \geq \xi$. As such, $\xi$ is accepted only if it is offered to an unemployed worker. Therefore $\gamma(\xi) = u$. Otherwise, the incumbent firm would just terminate the worker and so $\gamma(\xi) = 1$. Note the left hand side of (14) is the maximum value it could obtain if it retains the worker, the right hand side is the value from terminating the worker.

To summarize, for any $\xi \in \Phi$,

$$\gamma(\xi) = \begin{cases} 
0, & \text{if } \xi < V_0 \\
u + (1 - u) \int \mathbb{I}(\xi \in \Phi^* \setminus \Omega^*(V))dG(V), & \text{if } \xi \geq V_0 \text{ and } \xi \in \Phi^* \\
u, & \text{if } \xi \geq V_0, \xi \in \Phi \setminus \Phi^*, \text{ and (14) holds} \\
1, & \text{if } \xi \geq V_0, \xi \in \Phi \setminus \Phi^*, \text{ and (14) does not hold} 
\end{cases} \hspace{1cm} (15)$$

---

\textsuperscript{8}The consistency criterion as in Fudenberg and Tirole (1991) is satisfied by assuming that the incumbent firm believes that with a very small probability $\varepsilon > 0$, a vacant firm will post a contract with expected utility $\xi$ drawn randomly from $\Sigma$. 

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3.3 Equilibrium Definition

We now define a stationary rational expectations equilibrium of the model, requiring the aggregate stocks and distributions, the contracts used, and the starting expected utilities offered by firms be time invariant, and that individual decisions be consistent with aggregate outcomes. Formally,

**Definition 1** A stationary rational expectations equilibrium of the economy consists of

(i) An equilibrium contract \( \sigma^* = \{ c^*(V), \Omega^*(V), V^*_r(\xi; V), I^*(V), V^*_n(V) : \xi \in \Omega^*(V) \text{ and } V \in \Phi^* \} \) for firms to use in equilibrium;

(ii) An equilibrium (mixed) strategy \( F^* : \Phi^* \to [0, 1] \) of vacant firms for posting a starting expected utility for new hires;

(iii) An equilibrium distribution of expected utilities for employed workers (those currently employed under \( \sigma^* ) \) \( G : \Phi^* \to [0, 1] \);

(iv) An expected utility of unemployed workers \( V_0 \in \Sigma \);

(v) An unemployment rate \( u \in [0, 1] \);

(vi) An expected value for vacant firms \( \pi \geq 0 \);

(vii) The matching probabilities \( p_w \) and \( p_f \) for workers and vacant firms respectively, and the offer acceptance function \( \gamma : \Sigma \to [0, 1] \);

such that

(a) The equilibrium contract \( \sigma^* \) is optimal for each individual firm. That is, \( \sigma^* \) solves the problem (6)-(13).

(b) \( F^* \) is the vacant firm’s optimal strategy for expected utility posting: it solves

\[
\max_{F : \Phi^* \to [0, 1]} \int_{\Phi^*} U(\xi) dF(\xi),
\]

(16)

taking as given the optimal contract \( \sigma^* \) (hence \( \Phi^* \)), and the equilibrium \( G, V_0, \gamma(\cdot) \), and \( p_f \), with \( U(\cdot) \) given by (5).

(c) Unemployed workers accept an offer \( \xi \in \Phi^* \) if and only if \( \xi \geq V_0 \), \( V_0 \) given by (2).

(d) \( p_w = M(1, m - (1 - u)) \); \( p_f \) satisfies (1); and \( \gamma(\cdot) \) satisfies (15).

(e) The distribution \( G \) of the employed workers’ expected utilities are consistent with \( F^* \), the equilibrium distribution of starting expected utility offers, the dynamics the equilibrium contract \( \sigma^* \) generates, and the constant mortality rate \( \delta \).

(f) The equilibrium unemployment rate \( u \) is consistent with \( \sigma^* \) and \( F^* \) (particularly the termination policies they dictate), as well the total measure of firms in the economy \( m \) and the matching function \( M \).
3.4 Proof of Proposition 1

We first prove that, for a vacant firm, the problem of choosing the optimal starting expected utility $\xi$ to maximize its value of $U(\xi)$ is equivalent to choosing the optimal $\xi$ to maximize $U(\xi)$.

**Lemma 1** In the stationary equilibrium, it holds for all $\xi \in \Phi^*$ with $f^*(\xi) > 0$ that

$$\xi \in \arg \max U(\xi') \text{ s.t. } \xi' \in \Phi \text{ and } \xi' \geq V_0.$$ 

**Proof.** Observe first that in equilibrium no vacant firm would offer a starting expected utility below the reservation utility of unemployed workers $V_0$, since no worker, employed or unemployed, would accept such an offer. Given this, we need only consider initial outside offers with $\xi \geq V_0$. And of course any unemployed worker would accept any outside offer higher than his reservation utility $V_0$. The rest of the proof takes 3 steps.

**Step 1** We show that for all $\xi \in \Phi^*$ with $f^*(\xi) > 0$, $U(\xi) \geq \pi$.

Take $\xi \in \Phi^*$ with $f^*(\xi) > 0$ as given. Then (b) of Definition 1 implies $U(\xi) = \pi \geq 0$, which, given $k \geq 0$, in turn implies $U(\xi) \geq U(\xi) = \pi$ by (5).

**Step 2** We show $\gamma(\xi) = u$ for all $\xi \in \Phi$ with $U(\xi) \geq \pi$.

Take $\xi \in \Phi$ with $U(\xi) \geq \pi$ as given. Take $V \in \Phi$ as given. Note that since in equilibrium $\Phi = \Phi^*$ by (c) of Definition 1, any offer $\xi \in \Phi$ is anticipated by the incumbent firm. If the incumbent firm terminates the worker at $\xi$, then its expected value is $\beta \pi - C_0$. If the incumbent firm retains the worker by counteroffering a contract with expected utility $V_r(\xi; V) = \max\{\xi, V_0\} = \xi$, then its expected value is $U(\xi)$. Note that the worker is indifferent between being terminated and being retained. Given

$$U(\xi) \geq \pi \geq \beta \pi - C_0,$$

it is optimal for the incumbent firm to retain the worker with $\xi \in \Omega(V)$, which implies $\gamma(\xi) = u$ by (15).

**Step 3** We show that Lemma 1 holds.

---

9Given $\beta < 1$, the incumbent firm strictly prefers retaining the worker with $\xi \in \Omega(V)$ than terminating him except when $k = 0$, $C_0 = 0$, and $\pi = 0$. However, it is straightforward to show that there does not exist such an equilibrium. The logic goes as follows: if it does, then all firms must earn zero profits by offering the same fixed wage of $\theta$. Thus, a firm can earn strictly positive profits by offering $\theta - \varepsilon$ instead, which is only acceptable to unemployed workers.
For all $\xi \in \Phi$ with $U(\xi) \geq \pi$ (including all $\xi \in \Phi^*$ with $f^*(\xi) > 0$ by Step 1), $\gamma(\xi) = u$ by Step 2, which implies
\[
U(\xi) = \frac{-k + p_f u U(\xi)}{1 - (1 - p_f u) \beta}
\]
by (5). The desired result then follows from (b) of Definition 1.

What Lemma 1 says is that the vacant firm, when deciding which starting expected utility to offer, need only consider what value the contract itself, after it is accepted, brings to the firm, but not how the offer would affect the probability with which the contract is accepted. The reason is that in equilibrium no outside offers are accepted by employed workers (they are either matched, or no better than the contract the worker currently has), while all unemployed workers would accept any outside offer that is better than his reservation utility $V_0$. That is, in equilibrium, $\gamma(\xi)$ is constant in $\xi \in \Phi^*$ with $f^*(\xi) > 0$.

We now prove the theorem by showing that in equilibrium
\[
\{\xi \in \Phi^* : f^*(\xi) > 0\} = \{V_0\} = \left\{ \frac{u(0)}{1 - \beta(1 - \delta)} \right\}.
\]
This takes five steps.

**Step 1** We show that for all $V \in \Phi$ with $c(V) > 0$, $U''(V) \leq -\frac{1}{u'(c(V))} < 0$.

Take $V \in \Phi$ with $c(V) > 0$ as given. Then there exists $\varepsilon > 0$ such that for all $\tilde{V} \in [V - \varepsilon, V]$, \{u^{-1}(u(c(V))) - (V - \tilde{V}), \Omega(V), V_r(\xi; V), I(V), V_n(V)\} satisfies constraints (7)-(12). We then have
\[
U(\tilde{V}) - U(V) \geq c(V) - u^{-1}(u(c(V)) - (V - \tilde{V})),
\]
which implies
\[
U''(V) = \lim_{\tilde{V} \to V} \frac{U(\tilde{V}) - U(V)}{\tilde{V} - V} \leq \lim_{\tilde{V} \to V} \frac{c(V) - u^{-1}(u(c) - (V - \tilde{V}))}{\tilde{V} - V} = -\frac{1}{u'(c(V))},
\]
where the first inequality follows from (17) and $\tilde{V} - V \leq 0$, and the last equality follows from L’Hopital’s rule.

**Step 2** We show that for all $V \in \Phi^*$ with $f^*(V) > 0$, either $V = V_0$ or $c(V) = 0$.  

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Take $V \in \Phi^*$ with $f^*(V) > 0$ as given. Suppose $V > V_0$ and $c(V) > 0$. Then, $c(V) > 0$ implies $U'_r(V) < 0$ by Step 1, which in turn implies that there exists $\tilde{V} \in (V_0, V)$ such that $U(\tilde{V}) > U(V)$, which contradicts with Lemma 1. The result then follows.

**Step 3** We show $\mathcal{V} \equiv \max\{V \in \Phi: c(V) = 0\} \leq V_0$.

Suppose $\mathcal{V} > V_0$. Take $V \in \Phi$ with $c(V) = 0$ as given.

First, we show that for all $\xi \in \Omega(V)$, $V_r(\xi; V) \leq \max\{\xi, \mathcal{V}\}$. Suppose otherwise. Then there exists $\tilde{\xi} \in \Omega(V)$ such that $V_r(\tilde{\xi}; V) > \max\{\tilde{\xi}, \mathcal{V}\} = \max\{\tilde{\xi}, V_0\}$, which implies $V_r(\tilde{\xi}; V) > \mathcal{V} > V_0$, which in turn implies $c(V_r(\tilde{\xi}; V)) > 0$ by the definition of $\mathcal{V}$. Given $V_r(\tilde{\xi}; V) > \max\{\tilde{\xi}, V_0\}$, we can construct a deviation from the supposedly optimal contract by changing $c(V)$ by a small amount $dc > 0$ and $V_r(\tilde{\xi}; V)$ by a small amount $dV_r < 0$ with

$$u'_+(c(V))dc + \beta(1 - \delta)p_w f(\tilde{\xi})dV_r = 0$$

while leaving everything else in the optimal contract unchanged. With this deviation, the firm’s expected profit changes by

$$-dc + \beta(1 - \delta)p_w U'_r(V_r(\tilde{\xi}; V)) f(\tilde{\xi})dV_r$$

$$= -[1 + u'_+(c(V))U'_r(V_r(\tilde{\xi}; V))]dc$$

$$\geq - \left[1 - \frac{u'_+(c(V))}{u'(c(V_r(\tilde{\xi}; V)))}\right] dc$$

$$> 0,$$

where the equality follows from (18), the first inequality follows from $U'_r(V_r(\tilde{\xi}; V)) \leq -\frac{1}{u'(c(V_r(\tilde{\xi}; V)))}$ by $c(V_r(\tilde{\xi}; V)) > 0$ and Step 1, and the last inequality follows from the fact $u'_+(c(V)) > u'(c(V_r(\tilde{\xi}; V)))$, which holds because $u$ is strictly concave and $c(V) = 0 < c(V_r(\tilde{\xi}; V))$. This is a contradiction. The result then follows.

Second, we show $V_n(V) \leq \mathcal{V}$. This can be done applying an argument that is similar to the one used above. We leave this for the reader.

Third, we show $\mathcal{V} \leq V_0$. Given that for all $V \in \Phi$ with $c(V) = 0$, $V_r(\xi; V) \leq \max\{\xi, \mathcal{V}\}$ for all $\xi \in \Omega(V)$ and $V_n(V) \leq \mathcal{V}$, the promise keeping constraint (7) implies

$$V \leq \mathcal{V} \leq u(0) + \beta(1 - \delta) \left[ p_w \int_{\Phi^*} \max\{\xi, \mathcal{V}\} dF^*(\xi) + (1 - p_w)\mathcal{V} \right],$$

which implies $V \leq \mathcal{V} \leq V_0$ by (3), which contradicts with $\mathcal{V} > V_0$. 

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**Step 4** We show \( \{ \xi \in \Phi^* : f^*(\xi) > 0 \} = \{ V_0 \} \).

Given \( V \leq V_0 \), \( V \leq V_0 \) for all \( V \in \Phi^* \) with \( f^*(V) > 0 \) by Step 2. The result then follows from the fact that no vacant firms would post a contract with expected utility strictly less than \( V_0 \) since no worker (employed or unemployed) would ever accept such an offer.

**Step 5** We show \( V_0 = \frac{u(0)}{1 - \beta(1 - \delta)} \).

Given \( \{ \xi \in \Phi^* : f^*(\xi) > 0 \} = \{ V_0 \} \) by Step 4, (3) implies

\[
V_0 = u(0) + \beta(1 - \delta) \left[ p_w \int_{\Phi^*} \max\{\xi, V_0\} dF^*(\xi) + (1 - p_w)V_0 \right] \\
= u(0) + \beta(1 - \delta) \left[p_w V_0 + (1 - p_w)V_0 \right],
\]

which in turn implies \( V_0 = \frac{u(0)}{1 - \beta(1 - \delta)} \). This completes the proof of the proposition.

To summarize, two elements are important in the construction of the proof.

First, because in equilibrium all outside offers would be countered, in equilibrium no outside offers are accepted by employed workers (Lemma 1). This has two implications. One, in equilibrium any offer of higher expected utility cannot be used as a means for attaining a higher probability of contract acceptance. Two, in equilibrium a higher expected utility for the worker does not imply a lower probability of job turnover. Unlike in Burdett and Mortensen (1998), in the equilibrium of our model the employed worker never quits to take a better outside offer.

Second, the firm’s value is strictly decreasing in the expected utility it promises to the unemployed worker, and thus the optimal contract attains its maximum value for the firm if and only if it gives the unemployed worker his reservation utility \( V_0 \).

### 3.5 An Extension of the Argument

The insight from the analysis so far is that the threat to match an outside offer that exceeds the worker’s current promised utility lowers the value of such an offer. This, in turn, renders such an offer not being offered in the first places and destroys the dispersion of expected utilities received by new hires in equilibrium.

The argument was presented with the assumption of homogeneous firms. The essence of the argument, however, does not hinge essentially upon that assumption. In this section, we take a step up to show that even with firms that differ in productivity – they make workers more or less productive – counteroffers in the dynamic contract can destroy the equilibrium...
dispersion in worker compensation and restore the monopsony wage. In other words, firms with differential productivities may offer identical wages in equilibrium.

The argument goes as follows. Suppose, for the sake of simplicity, that there are two firm types, one being more productive than the other; but workers, again, are all identical. The more productive firms are able, and may be willing, to offer higher expected utilities to their new hires. Now from the logic of the above analysis, any expected utility a more-productive vacant firm is willing to offer is an outside offer that any more-productive incumbent firm is willing to counter. In other words, in equilibrium any job offer from a more-productive firm would not be taken by a worker employed at another more-productive firm. Now, would the more-productive vacant firm be offering anything that a worker employed at a less-productive firm would take? It depends on whether the less productive firm is willing to counter which, in turn, depends on whether the offer is sufficiently higher – higher than the threshold above which the less-productive firm is not willing to counter. And, of course, the more-productive firm is willing to make such an offer if the threshold is sufficiently low, or the difference in productivity between the two productivity types is sufficiently small.

So consider a modification of the model. Assume the worker’s period output with the more-productive firm is $\theta_h$, and with the less-productive firm $\theta_l$, and $\theta_l < \theta_h$. Suppose the fraction of firms with the low productivity $\theta_l$ is $q \in (0,1)$, and with the high productivity, $\theta_h$, $1 - q$. We also assume $k = C_0 = 0$. When it is costly to terminate an old worker ($C_0 > 0$) or to recruit a new worker ($k > 0$), the firm would have more incentives to match an existing worker’s outside offer in order to retain him. This would strengthen, instead of weakening, the case.

**Proposition 2** Suppose $k = C_0 = 0$. The model has a stationary equilibrium of the model in which all vacant firms post a contract offering expected utility $V_{\text{min}}$ if and only if

$$\frac{\theta_h}{\theta_l} \leq \frac{u + (1 - u)q}{(1 - u)q},$$

where the unemployment rate $u$ is defined by

$$(1 - \delta)[(1 - u) + uM(1, m - (1 - u))] = 1 - u.$$ 

**Proof.** Suppose the equilibrium is such that all vacant firms, with $\theta_l$ or $\theta_h$, post a contract that offers expected utility $V_{\text{min}}$. That is, $F^*(V_{\text{min}}) = 1$. By (3) then, $V_0 = V_{\text{min}}$.

**Step 1** We solve for the optimal contract for an individual firm.
Since the worker’s outside value is \( V_{\min} \) whether he receives an outside offer or not, he never has incentives to quit any ongoing contract. Hence, all expected utility \( V \in [V_{\min}, V_{\max}] \) can be delivered by a contract that offers a constant wage of \( u^{-1}((1 - \beta(1 - \delta))V) \). In turn, this implies that the state space of the a feasible contract is \( \Phi = [V_{\min}, V_{\max}] \).

Let \( \bar{U}_\theta(\xi) \) denote the expected value of a vacant firm with productivity \( \theta \in \{\theta_l, \theta_h\} \) who posts a contract offering expected utility \( \xi \in \Phi \). Let \( U_\theta(V) \) denote the expected value of a firm with productivity \( \theta \in \{\theta_l, \theta_h\} \) who employs a worker at expected utility \( V \in \Phi \). Given \( k = C_0 = 0 \), we have

\[
\bar{U}_\theta(\xi) = p_f \gamma(\xi)U_\theta(\xi) + (1 - p_f \gamma(\xi))\beta \bar{U}_\theta(\xi),
\]

\[
U_\theta(V) = (\theta - u^{-1}((1 - \beta(1 - \delta))V)) + \beta[\delta \bar{U}_\theta(V_{\min}) + (1 - \delta)U_\theta(V)], \forall V \in \Phi.
\]

Next, since \( V_0 = V_{\min} \), only unemployed workers would accept a contract offering expected utility \( V_{\min} \), which implies \( \gamma(V_{\min}) = u \). Thus (20) and (21) can be rewritten as

\[
\bar{U}_\theta(V_{\min}) = p_f u U_\theta(V_{\min}) + (1 - p_f u)\beta \bar{U}_\theta(V_{\min}),
\]

\[
U_\theta(V_{\min}) = \theta + \beta[\delta \bar{U}_\theta(V_{\min}) + (1 - \delta)U_\theta(V_{\min})].
\]

Solving these equations gives

\[
\bar{U}_\theta(V_{\min}) = \frac{p_f u \theta}{[1 - \beta(1 - \delta)][1 - \beta(1 - p_f u)] - \beta \delta p_f u},
\]

which, by (21), implies

\[
U_\theta(V) = \frac{[1 - \beta(1 - p_f u)]\theta}{[1 - \beta(1 - \delta)][1 - \beta(1 - p_f u)] - \beta \delta p_f u} - \frac{u^{-1}((1 - \beta(1 - \delta))V)}{1 - \beta(1 - \delta)}, \forall V \in \Phi.
\]

With the above value calculations, we then know that the firm, with productivity \( \theta \in \{\theta_l, \theta_h\} \) and is currently employing a worker at expected utility \( V_{\min} \), would match an outside offer \( \xi \in \Phi \) if and only if

\[
U_\theta(\xi) \geq \beta \bar{U}_\theta(V_{\min}),
\]

which, given (22) and (23), is equivalent to

\[
\xi \leq \frac{1}{1 - \beta(1 - \delta)} u \left( \frac{(1 - \beta)[1 - \beta(1 - \delta)]\theta}{[1 - \beta(1 - \delta)][1 - \beta(1 - p_f u)] - \beta \delta p_f u} \right) \equiv \bar{\xi}(\theta).
\]

That is, it would match its worker’s outside offer up to \( \bar{\xi}(\theta) \) in order to retain him. Notice that \( \bar{\xi}(\theta) \) is strictly increasing in \( \theta \) and, in particular, \( \bar{\xi}(\theta_l) < \bar{\xi}(\theta_h) \).
Step 2 We calculate the function $\gamma(\cdot)$, taking as given that all (other) vacant firms post a contract offering expected utility $V_{\text{min}}$ as solved in Step 1.

Consider an individual vacant firm in the labor market, with any $\theta$. Suppose it posts a contract offering expected utility $\xi \in [V_{\text{min}}, \bar{\xi}(\theta)]$. Then the contract would only be accepted by unemployed workers, implying $\gamma(\xi) = u$. Suppose it posts a contract offering expected utility $\xi \in (\bar{\xi}(\theta_l), \bar{\xi}(\theta_h)]$. Then the contract would be accepted by those who are unemployed and those employed at a firm with the low productivity $\theta_l$. This implies $\gamma(\xi) = u + (1 - u)q$.\(^{10}\) Suppose it posts a contract offering expected utility $\xi \in (\bar{\xi}(\theta_h), V_{\text{max}})$. Then the contract would be accepted by all workers, employed and unemployed, implying $\gamma(\xi) = 1$. To summarize,

$$
\gamma(\xi) = \begin{cases} 
  u, & \text{if } \xi \in [V_{\text{min}}, \bar{\xi}(\theta)] \\
  u + (1 - u)q, & \text{if } \xi \in (\bar{\xi}(\theta_l), \bar{\xi}(\theta_h)] \\
  1, & \text{if } \xi \in (\bar{\xi}(\theta_h), V_{\text{max}}) 
\end{cases}
$$ (25)

Step 3 We derive the desired result. Now in order for there to be a stationary equilibrium where all vacant firms offer the same expected utility $V_{\text{min}}$ to the worker it is matched with, it is necessary and sufficient that

$$
V_{\text{min}} \in \arg\max_{\xi \in \Phi} \{ U_{\theta}(\xi) \}, \; \theta \in \{\theta_l, \theta_h\}
$$ (26)

where, by (20),

$$
U_{\theta}(\xi) = \frac{p_f \gamma(\xi)}{1 - \beta(1 - p_f \gamma(\xi))} U_{\theta}(\xi),
$$

with $\gamma(\xi)$ given by (25).

It is straightforward to show that the low productivity firm never has incentives to offer an expected utility higher than $V_{\text{min}}$. For the high productivity firm, to make (26) hold we need only guarantee that it has no incentives to post a contract offering expected utility $\xi \in (\bar{\xi}(\theta_l), \bar{\xi}(\theta_h)]$. Such $\xi$, once offered, would induce workers who are currently employed at a low productivity firm to take it. That is, (26) holds if and only if for all $\xi \in (\bar{\xi}(\theta_l), \bar{\xi}(\theta_h)]$,

$$
U_{\theta_h}(V_{\text{min}}) \geq U_{\theta_h}(\xi) = \frac{\frac{p_f \gamma(\xi)}{1 - \beta(1 - p_f \gamma(\xi))} U_{\theta_h}(\xi)}{\frac{p_f[u + (1 - u)q]}{1 - \beta[1 - p_f[u + (1 - u)q]]} U_{\theta_h}(\xi)},
$$

\(^{10}\)Note that the fraction of employed workers who are with a firm with productivity $\theta_l$ is $q$. When all firms offer the same expected utility $V_{\text{min}}$, they have the same probability $p_f u$ to hire a new worker, and the same probability $\delta$ of losing the worker each period after having hired him. Hence, the fraction of employed workers with low productivity is equal to that with high productivity, which is $q$. 

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where the first equality follows from (20) and the second equality follows from (25). Finally, given that $U_{\theta_h}(\xi)$ is a decreasing function of $\xi$ by (23), the above condition holds if and only if
\[U_{\theta_h}(V_{\text{min}}) \geq \frac{p_f[u + (1-u)q]}{1-\beta\{1-p_f[u + (1-u)q]\}} U_{\theta_h}(\xi(\theta_l)),\]
which, given (22), (23) and (24), can be rewritten as
\[p_f u_{\theta_h} \geq \frac{p_f[u + (1-u)q]}{1-\beta\{1-p_f[u + (1-u)q]\}} \left\{ [1-\beta(1-p_f u)]\theta_h - (1-\beta)\theta_l \right\},\]
which can be rearranged to read as (19) and the proposition is proved. ■

When (19) holds, that is, when firms differ but not by much in productivity, in equilibrium all employed workers would be offered the same expected utility $V_{\text{min}}$ in the labor market, are paid the same monopsony wage after employment starts. Note, however, the contracts the firms offer in equilibrium do differ, in that the more-productive firm would specify to counter higher outside offers – in case they arise which they don't in equilibrium – than the less-productive firm. In other words, although differential contracts are offered, they all offer, in equilibrium, the same monopsony wage.

4 Incomplete Information: Non-degenerate Dispersion

In this section, we consider the case of private outside offers.

Assumption 4 (Private Outside Offers) All outside offers any employed worker receives are his private information, not observable to the firm that currently employs him.

Under Assumption 4, if the firm wishes to make the terms of the contract contingent on the outside offers the worker receives, it must induce the worker to report his outside offers truthfully. This would change the structure of the optimal contract and the distribution of the expected utilities offered in equilibrium.

The labor market in equilibrium can be described similarly as is for the case of complete information. We start the analysis here with the vacant firm’s problem of what expected utility and contract to post in the labor market. We take as given that the firm operates in an equilibrium of the model where there is a non-degenerate distribution of expected utilities being offered by a set of vacant firms. Specifically, assume the expected utilities offered in equilibrium are such that any random match generates an offer $\xi \in \Phi^*$, where $\Phi^*$, as in the case of complete information, is the set of all expected utilities that a vacant firm is able to offer.
and deliver. The set $\Phi^*$ and the distribution of the expected utilities offered, $F^* : \Phi^* \to [0, 1]$, are known (rationally perceived) to all workers and firms in the economy.

Under Assumption 4, the terms of the contract cannot be made directly contingent on the worker’s outside offers, but could instead be contingent on the reports of the worker’s outside offers. Using again the worker’s beginning-of-period expected utility, denoted $V$, as a state variable, a dynamic contract, defined recursively, is

$$
\sigma = \{c(V), \Omega(V), V_r(\xi;V), I(V), V_n(V) : \xi \in \Omega(V) \subseteq \Phi^* \text{ and } V \in \Phi\}.
$$

The definition for the variables is the same as for that in the case of complete information, except that the $\xi$ is the now the worker’s report of his current outside offer, privately observed by himself.

A contract $\sigma$ is feasible if it satisfies the following constraints: for all $V \in \Phi$,

$$
\begin{align*}
    u(c(V)) + \beta(1 - \delta)p_w \left( \int_{\Omega(V)} V_r(\xi;V) dF^*(\xi) + \int_{\Phi^* \setminus \Omega(V)} \max\{\xi, V_0\} dF^*(\xi) \right) \\
    + \beta(1 - \delta)(1 - p_w)[I(V)V_n(V) + (1 - I(V))V_0] &= V,
\end{align*}
$$

(27)

$$
V_r(\xi;V) \geq \begin{cases} 
    V_r(\xi';V), & \forall \xi' \in \Omega(V) \\
    \max\{\xi, V_0\}, & \forall \xi \in \Omega(V),
\end{cases}
$$

(28)

$$
\max\{\xi, V_0\} \geq \begin{cases} 
    V_r(\xi';V), & \forall \xi' \in \Omega(V) \\
    \max\{\xi, V_0\}, & \forall \xi \in \Phi^* \setminus \Omega(V),
\end{cases}
$$

(29)

$$
I(V)V_n(V) + (1 - I(V))V_0 \geq \begin{cases} 
    V_r(\xi';V), & \forall \xi' \in \Omega(V) \\
    V_0
\end{cases},
$$

(30)

$$
c(V) \geq 0,
$$

(31)

$$
\Omega(V) \subseteq \Phi^*,
$$

(32)

$$
V_r(\xi;V) \in \Phi, \forall \xi \in \Omega(V),
$$

(33)
\[ V_r(\xi; V) \geq \max\{\xi, V_0\}, \forall \xi \in \Omega(V), \quad (34) \]

\[ I(V) \in \{0, 1\}, V_n(V) \in \Phi, V_n(V) \geq V_0, \quad (35) \]

where \( \Phi \) is the largest self-generating set with respect to the above constraints (27)-(35).

In the above, equation (27) is the promise-keeping constraint that requires the choices of the current variables be consistent with the definition of \( V \). Equation (28)-(30) are the incentive constraints which require that the worker report truthfully whether he receives an outside offer and what the outside offer he receives is. Specifically, upon receiving any \( \xi \in \Phi^* \), the worker could report an outside offer in the retention region, \( \xi' \in \Omega(V) \), or in the termination region, \( \xi' \in \Phi^* \setminus \Omega(V) \), or he could report not receiving any outside offer. In the second case, upon termination of his current job the worker could choose to accept the outside offer \( \xi \) or to become unemployed to obtain expected utility \( V_0 \).

Equations (31) and (32) require, respectively, that the compensation to the worker and the termination policy be feasible. Equation (33) requires that if the worker is retained, then he must be promised an expected utility that is feasible for the contract to deliver. Equation (34) is the self-enforcing constraint which says that if the firm retains the worker, then the worker must be given no less than his outside options, \( \xi \) and \( V_0 \). Finally, equation (35) requires that the policies in the state of no outside offer be feasible and self-enforcing.

Note that in formulating the incentive constraint, we assume that after reporting \( \xi' \in \Omega(V) \) and the firm offers to retain him with expected utility \( V_r(\xi'; V) \), the worker cannot quit his current job to pursue the \( \xi \) he received any more. Alternatively, we could assume that the worker can still quit after he reports \( \xi' \in \Omega(V) \) and the firm offers \( V_r(\xi'; V) \) to retain him. This would not change the worker’s incentives to report truthfully.\(^{11}\)

Requiring truth-telling has an immediate implication for termination. Observe that if termination occurs at any current outside offer \( \xi \), then it must occur at any current outside offer \( \xi' > \xi \). This is easy to see. Suppose not. Then, upon receiving \( \xi \), the worker would report \( \xi' \) and will receive an expected utility weakly higher than \( \xi' \) (because of the self-enforcing constraint) and strictly higher than \( \xi \). This breaks incentive compatibility. Similarly, if retention occurs at any \( \xi \), then it must also occur at any \( \xi' < \xi \). For otherwise the worker who receives \( \xi' \) would report \( \xi \) to obtain a higher expected utility. These suggest that incentive compatibility

\(^{11}\)The worker with outside offer \( \xi \) has incentives to quit even after he reports \( \xi' \in \Omega(V) \) and the firm offers \( V_r(\xi'; V) \) to retain him only if \( V_r(\xi'; V) < \max\{\xi, V_0\} \), which implies that the worker never has incentives to report \( \xi' \) in the first place.
requires that workers who receive higher outside offers (above a cutoff) be terminated and lower outside offers (below the cutoff) be retained.

Truth-telling also imposes a constraint on the worker’s compensation across the states in which he is retained. First, truth-telling implies that the worker’s expected utility must be a constant across all states of his outside offer with which he is retained. In other words, incentive compatibility rules out the possibility of making the worker’s expected utility contingent on the state of his outside offer. Next, given the lack of commitment from the worker, the constant expected utility in the states of retention must not be lower than the cutoff for retention/termination. And finally, in order to induce truth-telling between the states of retention and termination, the constant expected utility the retained worker receives must just be equal to the cutoff for retention/termination. And these, of course, give the incentive compatible contract the features which are essential for supporting a non-degenerate wage dispersion in BM and BC.

The optimality of the contract and a stationary rational expectations equilibrium of the model can be formulated in a way that is parallel to that for the case of complete information. We leave that for the reader to save space. We now present our main results in this section.

**Proposition 3** The following holds for the optimal contract offered in equilibrium.

(i) \( \Phi^* = [V_0, V_{\text{max}}] \).

(ii) For all \( V \in \Phi^* \), \( \Omega(V) = [V_0, V_n(V)] \), \( V_r(\xi; V) = V_n(V) \) for all \( \xi \in \Omega(V) \), and \( I(V) = 1 \).

(iii) For all \( V \in \Phi^* \), \( c(V_n(V)) \geq c(V) \) and \( V_n(V) \geq V \); if \( f^*(V) > 0 \), then \( V_n(V) > V \).

By Proposition 3 then, the contract offered in equilibrium takes exactly the same form as does the optimal wage-tenure contract derived in Burdett and Coles (2003). Specifically, each period, the firm sets a bar (strictly above the worker’s current expected utility) for the worker’s outside offer, below which the worker’s next period expected utility is at that bar and above which the worker quits to pursue his outside offer. The firm never responds to the worker’s outside offers, even though it is feasible for it to do so. We therefore conclude that with unobservable outside offers, a non-degenerate distribution of contracts offered arises in the stationary equilibrium of the model, as in Burdett and Coles (2003). \(^{12}\)

\(^{12}\)The argument, which is well known, goes as follows. Suppose all vacant firms post the same contract in equilibrium. Then the equilibrium contract can be shown to be a fixed wage contract. It then follows that it is profitable for a vacant firm to post a deviating contract which offers a slightly higher fixed wage to steal workers from other firms.
4.1 Proof of Proposition 3

The proof is organized in Lemmas 2-5 which we state and prove in the following. Specifically, (i) follows from Lemma 2, (ii) follows from Lemmas 3-4, and (iii) follows from Lemma 5.

Lemma 2 $\Phi^* = [V_0, V_{\text{max}}]$.

Proof. We need only show that the optimal contract for the individual firm, which takes $\Phi^*$ as given, has $\tilde{\Phi} = [V_0, V_{\text{max}})$, where we use $\tilde{\Phi}$ to denote the state space for the optimal contract.

Step 1 We show that if $\tilde{V} \in \tilde{\Phi}$, then $\tilde{V} \geq V_0$.

Suppose $\tilde{V} \in \tilde{\Phi}$. Then there exists a feasible contract $\sigma = \{c(V), \Omega(V), V_r(\xi; V), I(V), V_n(V) : \xi \in \Omega(V) \text{ and } V \in \Phi\}$ satisfying (27)-(35) such that

$$\tilde{V} = u(c(\tilde{V})) + \beta(1 - \delta)p_w \left( \int_{\Omega(\tilde{V})} V_r(\xi; \tilde{V})dF^*(\xi) + \int_{\Phi^* \setminus \Omega(\tilde{V})} \max\{\xi, V_0\}dF^*(\xi) \right)$$

$$+ \beta(1 - \delta)(1 - p_w)[I(\tilde{V})V_n(\tilde{V}) + (1 - I(\tilde{V}))V_0]$$

$$\geq u(0) + \beta(1 - \delta) \left[ p_w \int_{\Phi^*} \max\{\xi, V_0\}dF^*(\xi) + (1 - p_w)V_0 \right]$$

$$= V_0,$$

where the first equality follows from (27), the inequality follows from $c(\tilde{V}) \geq 0$ by (31), $V_r(\xi; \tilde{V}) \geq \max\{\xi, V_0\}$ for all $\xi \in \Omega(\tilde{V})$ by (34), and $V_n(\tilde{V}) \geq V_0$ by (35), and the last equality follows from (3).

Step 2 We show that there is a feasible contract that attains all $V \in [V_0, V_{\text{max}})$.

Construct the contract as follows. For all $V \in [V_0, V_{\text{max}})$, let $\Omega(V) = \{\xi \in \Phi^* : \xi \leq V_n(V)\}$, $V_r(\xi; V) = V_n(V)$ for all $\xi \in \Omega(V)$, and $I(V) = 1$; furthermore, choose $c(V) \geq 0$ and $V_n(V) \in [V_0, V_{\text{max}})$ to satisfy

$$u(c(V)) + \beta(1 - \delta) \left[ p_w \int_{\Phi^*} \max\{\xi, V_n(V)\}dF^*(\xi) + (1 - p_w)V_n(V) \right] = V.$$

It is straightforward to show that such $c(V)$ and $V_n(V)$ exist, and the contract such constructed satisfies (27)-(35). \[\blacksquare\]

Lemma 3 For any feasible contract $\sigma = \{c(V), \Omega(V), V_r(\xi; V), I(V), V_n(V) : \xi \in \Omega(V), V \in \Phi\}$, the following holds for all $V \in \Phi$.

(a) Suppose $\Omega(V) \neq \emptyset$. Then
(i) There exists $\nabla_r(V) \in \Phi$ with $\nabla_r(V) \geq V_0$ such that

$$V_r(\xi; V) = \nabla_r(V), \forall \xi \in \Omega(V);$$

(ii) $\Omega(V) = \{\xi \in \Phi^* : \xi \leq \nabla_r(V)\}$;

(iii) If $I(V) = 1$, then $\nabla_r(V) = V_n(V)$;

(iv) If $\nabla_r(V) > V_0$, then $I(V) = 1$.

(b) Suppose $\Omega(V) = \emptyset$. Then $I(V) = 1$ implies $V_n(V) \leq \max\{\xi, V_0\}$ for all $\xi \in \Phi^*$.

Proof. Let $\sigma$ be given and satisfies (27)-(35). Fix $V \in \Phi$.

(a) Suppose $\Omega(V) \neq \emptyset$.

(i) Suppose there exist $\xi, \xi' \in \Omega(V)$ with $V_r(\xi; V) < V_r(\xi'; V)$. Then the worker with outside offer $\xi$ strictly prefers reporting $\xi'$ to get expected utility $V_r(\xi'; V)$ than reporting $\xi$ truthfully to get expected utility $V_r(\xi; V)$, violating incentive compatibility. Thus (36) holds for some $\nabla_r(V) \in \Phi$ and, by the self-enforcing constraint (25), $\nabla_r(V) \geq V_0$.

(ii) We first show $\{\xi \in \Phi^* : \xi \leq \nabla_r(V)\} \subseteq \Omega(V)$. Equivalently, we need only show that for all $V \in \Phi^*$, if $\xi < \nabla_r(V)$ then $\xi \in \Omega(V)$. Suppose not. Suppose there exists $\xi \in \Phi^* \setminus \Omega(V)$ with $\xi < \nabla_r(V)$. That is, the worker receiving an outside offer lower than $\nabla_r(V)$ is not retained. Then the worker with outside offer $\xi$ strictly prefers reporting some $\xi' \in \Omega(V)$ to get $V_r(\xi'; V) = \nabla_r(V)$ than reporting $\xi$ truthfully to get $\max\{\xi, V_0\}$, violating the incentive constraint (29).

Next, we show $\Omega(V) \subseteq \{\xi \in \Phi^* : \xi \leq \nabla_r(V)\}$. For this, we need only show $\xi \in \Phi^* \setminus \Omega(V)$ for all $\xi > \nabla_r(V)$. Suppose there exists $\xi \in \Omega(V)$ with $\xi > \nabla_r(V)$. Then the expected utility of the worker with outside offer $\xi$ is $V_r(\xi; V) = \nabla_r(V) < \max\{\xi, V_0\}$ if he reports $\xi$ truthfully, violating the self-enforcing constraint (34).

(iii) Suppose $I(V) = 1$. Suppose $V_n(V) > \nabla_r(V)$. Then the worker with outside offer $\xi \in \Omega(V)$ strictly prefers reporting not receiving any outside offer to get $V_n(V)$ than reporting $\xi$ truthfully to get $V_r(\xi; V) = \nabla_r(V)$. This violates constraint (28). Suppose $V_n(V) < \nabla_r(V)$. Then the worker with no outside offer strictly prefers reporting some $\xi' \in \Omega(V)$ to get $V_r(\xi'; V) = \nabla_r(V)$ than reporting not receiving any outside offer truthfully to get $V_n(V)$, violating (30). To summarize, in order for the incentive constraints to hold, it must hold that $\nabla_r(V) = V_n(V)$.

(iv) Suppose $\nabla_r(V) > V_0$ and $I(V) = 0$. Then the worker who has not received any outside offer strictly prefers reporting some $\xi' \in \Omega(V)$ to get $V_r(\xi'; V) = \nabla_r(V)$ than reporting not receiving any outside offer truthfully to get $V_0$. This violates the incentive constraint (30).
(b) Suppose $\Omega(V) = \emptyset$. Suppose $I(V) = 1$. Suppose $V_n(V) > \max\{\xi, V_0\}$ for some $\xi \in \Phi^*$. Then the worker with outside offer $\xi$ strictly prefers reporting not receiving any outside offer to get $V_n(V)$ than reporting $\xi$ truthfully to get $\max\{\xi, V_0\}$, violating the incentive constraint (29).

**Lemma 4** In equilibrium the following holds:

(i) For all $V \in (V_0, V_{\text{max}})$, $\Omega(V) \neq \emptyset$, $I(V) = 1$, and $V_n(V) > V_0$.

(ii) $I(V_0) = 1$ and $V_n(V_0) = V_0$.

**Proof.** Let a feasible contract $\{c(V), \Omega(V), V_r(\xi; V), I(V), V_n(V) : \xi \in \Omega(V) \text{ and } V \in \Phi\}$ satisfying (27)-(35) be given.

(i) Let $V \in (V_0, V_{\text{max}})$.

We first show that $\Omega(V) \neq \emptyset$. Suppose otherwise. Then there are two cases: (i) $I(V) = 0$, which implies $V = V_0$ by (27); (ii) $I(V) = 1$, which implies $V_n(V) \leq \max\{\xi, V_0\}$ for all $\xi \in \Phi^*$ by (b) of Lemma 3, which, given $\Phi^* = [V_0, V_{\text{max}})$ by Lemma 2 and $V_n(V) \geq V_0$ by (35), in turn implies $V_n(V) = V_0$, which in turn implies $V = V_0$ by (27).

We next show $I(V) = 1$. Suppose $I(V) = 0$. Then given that $\Omega(V)$ is not empty as shown above, $V_r(V) = V_0$ by (iv) of (a) of Lemma 3, which implies $V = V_0$ by (27).

We now show $V_n(V) > V_0$. Suppose $V_n(V) = V_0$. Then given that $\Omega(V)$ is not empty and $I(V) = 1$ as shown above, $V_r(V) = V_n(V) = V_0$ by (iii) of (a) of Lemma 3, which implies $V = V_0$ by (27).

(ii) Let $V = V_0$.

After the worker reports not receiving any outside offer, the incumbent firm either terminates the worker ($I(V_0) = 0$) or retain the worker ($I(V_0) = 1$) with expected utility $V_n(V_0) = V_0$. Note that the worker is indifferent between being terminated and being retained. The expected profit for the incumbent firm is $\beta \pi - C_0$ in the case of termination and $U(V_0)$ in the case of retention. Thus, it suffices to show $U(V_0) > \beta \pi - C_0$. Construct a feasible contract at $V = V_0$ as follows: $c(V_0) = 0$, $\Omega(V_0) = \emptyset$, $I(V_0) = 0$, and $V_n(V_0) = V_0$ such that

$$U(V_0) \geq (\theta - 0) + \beta (1 - \delta)(\beta \pi - C_0) + \beta \delta (\pi - C_0) > \beta \pi - C_0$$

given $\frac{\theta}{1-\beta} \geq \pi \geq \beta \pi - C_0$ in which at least one inequality holds strictly.
Lemma 5 The following holds for the optimal contract: For all $V$,

(i) $c(V_n(V)) \geq c(V)$;

(ii) $V_n(V) \geq V$ where the strict inequality holds if $f^*(V) > 0$.

Proof. Given (i) and (ii) of the proposition which were already proven, the firm’s problem of optimal contracting can be rewritten as: For all $V \in [V_0, V_{\max})$,

$$U(V) = \max_{c, V_n} (\theta - c) + \beta \delta (\pi - C_0) + \beta(1 - \delta)p_w[F^*(V_n)U(V_n) + (1 - F^*(V_n)) (\beta \pi - C_0)]$$

subject to

$$u(c) + \beta(1 - \delta)p_w \left[F^*(V_n)V_n + \int_{V_n}^{V_{\max}} \xi dF^*(\xi)\right] + \beta(1 - \delta)(1 - p_w)V_n = V,$$

$$c \geq 0,$$

$$V_n \geq V_0,$$

where $U(\cdot)$ is the firm’s value function.

Let $\alpha$, $\mu$ and $\beta(1 - \delta) \gamma$ be the Lagrangian multipliers for the constraints (37)-(39), respectively. Then the Kuhn-Tucker conditions for the above Bellman equation are as follows:

$$-1 + \alpha u'(c) + \mu = 0,$$

$$p_w f^*(V_n) [U(V_n) - (\beta \pi - C_0)] + [p_w F^*(V_n) + (1 - p_w)][U'(V_n) + \alpha] + \gamma = 0,$$

$$\mu c = 0,$$

$$\gamma (V_n - V_0) = 0,$$

$$\mu, \gamma \geq 0.$$

In addition, the Envelope Theorem gives

$$U'(V) = -\alpha, \ \forall V.$$

Suppose $c(V) > c(V_n(V)) \geq 0$. Then,

$$U'(V_n(V)) + \alpha(V) = -\alpha(V_n(V)) + \alpha(V)$$

$$= -\frac{1 - \mu(V_n(V))}{u'(c(V_n(V)))} + \frac{1 - \mu(V)}{u'(c(V))}$$

$$= -\frac{1 - \mu(V_n(V))}{u'(c(V_n(V)))} + 1$$

$$> 0$$

30
where the first equality follows from (45), the second from (40), the third from \(c(V) > 0\) which implies \(\mu(V) = 0\) by (42), and the inequality follows from \(c(V) > c(V_n(V))\) and \(\mu(V_n(V)) \geq 0\) by (44). Hence, (41) implies

\[
p_w f^*(V_n(V))[U(V_n(V)) - (\beta \pi - C_0)] + \gamma(V) < 0,
\]

which, given \(\gamma(V) \geq 0\) by (44), in turn implies

\[
f^*(V_n(V)) > 0 \text{ and } U(V_n(V)) - (\beta \pi - C_0) < 0.
\]

That is, there are some vacant firms which offer an expected utility \(V_n(V)\) with

\[
\overline{U}(V_n(V)) \leq U(V_n(V)) < \beta \pi - C_0 \leq \pi,
\]

where the first inequality follows from (5), which contradicts with the equilibrium definition. Therefore, we conclude that \(c(V_n(V)) \geq c(V)\) for all \(V\), which implies \(V_n(V) \geq V\) for all \(V\) by (37).

Take \(V \in [V_0, V_{\text{max}}]\) with \(f^*(V) > 0\) as given. Suppose \(V_n(V) = V\). Then,

\[
\begin{align*}
p_w f^*(V_n)[U(V_n) - (\beta \pi - C_0)] + [p_w F^*(V_n) + (1 - p_w)](U'(V_n) + \alpha) + \gamma \\
= p_w f^*(V)[U(V) - (\beta \pi - C_0)] + [p_w F^*(V) + (1 - p_w)](U'(V) + \alpha) + \gamma \\
= p_w f^*(V)[U(V) - (\beta \pi - C_0)] + \gamma \\
> 0,
\end{align*}
\]

where the second equality follows from (45), and the inequality follows from \(f^*(V) > 0\) which implies \(U(V) \geq \overline{U}(V) = \pi \geq \beta \pi - C_0\) in which at least one inequality holds strictly\(^{13}\), and \(\gamma \geq 0\) by (44). This contradicts with (40). Hence, we conclude \(V_n(V) > V\).

5 Conclusion

The search for a pure theory of wage dispersion goes back to Diamond (1971) where, in a wage-posting game with identical firms and workers, the only equilibrium wage is the monopsony wage. Diamond (1971) does not model on-the-job search. Burdett and Mortensen (1998) shows that adding on-the-job search to Diamond (1971) would intensify competition among firms, resulting in differential wage offering strategies being used in the model’s stationary equilibrium. What we show in this paper is that, if contracts are fully optimal, then Burdett

\(^{13}\)\(U(V) = \beta \pi - C_0\) only if \(k = 0, C_0 = 0,\) and \(\pi = 0\). However, there does not exist such an equilibrium as argued in footnote 12.
and Mortensen (1998) should give the same monopsony wage of Diamond (1971). Neverthe-
less, adding private information to Burdett and Mortensen (1998) would generate the pure
contract dispersion their economic intuition points out.

We now conclude the paper with two remarks. First, in the paper we have followed the
literature to assume that the structure of the labor market is such that vacant firms would post
a fixed contract for both employed and unemployed workers, and then whoever it is matched
with gets offered this contract. An alternative specification is to assume that vacant firms,
upon being match with a worker, employed or unemployed, could make the offer dependent
on the worker’s employment status. This would not change the essence of our argument
and it is straightforward to verify that Proposition 1 continues to hold with this alternative
specification. Second, in the paper we have assumed that the cost of making an offer is zero
and vacant firms would make an offer to employed workers even though it anticipates that it
would be matched. Alternatively, one could assume that there is a (small) cost involved in
making an offer. This would simply discourage vacant firms from making unsuccessful offers.
Again, Proposition 1 would continue to hold.

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