Insulated Platform Competition*†

Alexander White‡ E. Glen Weyl§

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Abstract

The externalities that operating system users receive from software developers are among the leading features of those 'platform' industries but are rarely incorporated into applied models of imperfect competition. We argue this omission arises from the difficulty of collapsing the dynamic pricing characterizing such industries into a static policy analysis model. Given the role these pricing strategies play in coordinating consumer behavior, a theory ignoring them quickly becomes intractable and indeterminate. Postulating that platforms *identify* and then *robustly implement* best response allocations, we show platforms play an *Insulated Equilibrium* that eliminates the need for consumers to coordinate their behavior. This facilitates the analysis of an oligopoly model without unrealistic restrictions imposed for tractability. We use this to illustrate the additional distortion, analogous to that identified by Spence's (1975) study of a quality-choosing monopolist, arising when platforms determine both their prices and their (externality-driven) level of quality.

Keywords: Two-Sided Markets, Multi-Sided Platforms, Oligopoly, Insulated Equilibrium, Robust Implementation, Antitrust and Mergers in Network Industries

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‡Harvard University Department of Economics, Tsinghua University School of Economics and Managment; alexanderwhite@fas.harvard.edu

§University of Chicago Department of Economics & Toulouse School of Economics; weyl@uchicago.edu

1 Introduction

A prominent aspect of competitive strategies in industries with network or 'consumption externalities' (e.g. software and media platforms) is the tendency to charge low prices when participation is low, cashing in only once the service has matured, in order to avoid a 'chicken-and-egg' problem. Pioneering theoretical work on these 'two-sided' or 'platform' markets by Rochet and Tirole (2003) (RT03) and Armstrong (2006) (A06) relies implicitly on a static reduced-form representation of such dynamic pricing. In particular, in their papers platforms charge prices to each group of 'users' or 'consumers' which *depend on realized levels of demand* on the opposite 'side of the market', so as to make any given user's decision about which platform(s) to join independent of other users' behavior. Taken literally, however, such pricing strategies are feasible only in special cases that are too stylized for applied analysis. This paper develops the concept of *Insulation*, an extension of the robust implementation to which RT03 and A06 appeal to environments of rich consumer heterogeneity and uniform prices. It thus collapses coordination dynamics into a canonical static model of imperfect competition amenable to policy analysis.[1]

The literature on platforms has highlighted the pervasiveness of consumption externalities in many industries and has persuasively argued that their influence on pricing can be of first-order importance.[2] For example, it has helped account for operating system subsidies to application development, credit card point systems and the free availability of almost all websites. In fact, such externalities have become the centerpiece of prominent public policy debates, such as that over network neutrality regulation where it is often argued that such regulations benefit consumers by expanding their choice of websites. However, while quantitative evaluation of such claims requires a model flexible enough to incorporate rich structures of consumer preferences and firm heterogeneity, rich models of 'one-sided' competition, *à la* Berry, Levinsohn, and Pakes (1995) (BLP), have been considered intractable in the platform context.[3]

In Section 3, we build such a model and illustrate that, in the absence of platform pricing policies that depend on realized levels of demand, the possibility of multiple equilibria in which consumers either succeed or fail to coordinate on joining a platform makes general analysis intractable. However, as A06 shows, when platforms can charge arbitrary conditional (reduced-form dynamic) tariffs, a separate form of indeterminacy arises, in the spirit of Klemperer and Meyer (1989). We argue that both of these problems are resolved by observing that platforms' demand-contingent pricing strategies are, as

[1] The connection between the RT03 analysis and robust implementation was first made by Gomes and Pavan (2011).

[2] An excellent recent survey of this literature is given by Rysman (2009).

[3] A06 says of the extension of a rich two-sided monopoly model, 'A full analysis of this case is technically challenging in the case of competing platforms' (p. 671).

suggested by Becker (1991), *designed* to coordinate consumers on their desired outcome. Thus, we explicitly assume that each platform, taking as given other platforms' pricing strategies, *identifies* and then seeks to *robustly implement* its best response levels of consumer participation. Our approach preserves the simplicity of a static model while simultaneously capturing the dynamics of coordination.

In order for platforms to implement their desired allocation, they must first be able to identify them. This is possible under a relatively mild assumption of *No Sunspots*, developed in Section 4. This assumption posits that only changes in platform strategies that affect the *level of participation of consumers on that platform*, which we refer to as its *coarse allocation* or 'allocation' for short, can impact the coarse allocations of other platforms. Thus we rule out platforms' strategies playing a pure 'sunspot' coordination role among consumers participating on other platforms, in the spirit of Cass and Shell (1983). Furthermore, because the participation levels of users on one side of the market tie down the utility profiles of users on the other side we can apply the results from our recent work on exchange economies with a continuum of consumers (Azevedo, Weyl, and White, 2011), which guarantee that the coarse allocation ties down the prices of all platforms and thus their profits. Consequently, platforms need only identify their best response coarse allocation.

This motivates our fundamental hypothesis, *Insulation*, that we develop in Section 5, inspired jointly by observations about dynamic pricing above and the recent theoretical literature on robust implementation. Platforms' dynamic pricing strategies, and sometimes even their static tariffs (e.g. in the case of advertising pricing), typically adjust to shifts in the attractiveness of the platform, with prices dipping or rising as the platform becomes less or more attractive because of changes in participation on the other side of the market. Our static model mirrors such processes through platforms' attempts to robustly implement (Bergemann and Morris, 2005) their respective desired allocations by providing consumers with, as near as possible, a dominant strategy (Chung and Ely, 2007) to either participate or not in the platform. When users are heterogeneous only along a single dimension, such dominant strategy implementation is literally possible, a fact that the foundational RT03 exploits. However, as argued by Weyl (2010), when consumers are richly heterogenous a uniform price cannot induce dominant strategies. We thus, instead, assume that platforms adopt the 'next best thing', an extension to competition of that paper's notion of an *Insulating Tariff* that Cabral (2011) has shown (in a special case) is a valid reduced-form of optimal dynamic pricing.

A firm charging a *Residual Insulating Tariff* (RIT) ensures that, regardless of the realized outcome on the opposite side of the market, the equilibrium allocation on this side of the market is insulated and thus remains at the level the platform desires to implement.

Whenever it is feasible to implement in dominant strategies, the RIT does so; when it is not, the RIT makes the desired allocation a dominant strategy for the representative consumer on each side to choose this allocation. When all platforms use RITs, platforms' tariffs are said, jointly, to constitute an *Insulating Tariff System* (ITS), 'anchored' at whatever allocation it implements. We call an equilibrium in which all platforms optimize, given the behavior of other platforms, by charging their part of an ITS an *Insulated Equilibrium* (IE). Because such a system of tariffs, anchored at any given allocation, is unique, platform incentives are well-defined at an IE. Thus, whether a given allocation constitutes and IE can be checked based on primitives or, equivalently, marginal costs may be recovered from the demand system *à la* Rosse (1970).

In Section 6, we apply this solution concept to analyze first-order conditions characterizing Insulated Equilibria and show that there are two distortions created by market power. One of these forces is the classical Cournot (1838) *market power distortion*; the other is the *Spence distortion*, owing its name to the seminal analysis in Spence (1975) of a monopolist's choice of quality. While, as a general matter, the effect of intensified competition on the market power distortion is well-known, its effect on the Spence distortion, and the consequences of this distortion for social welfare, depend, in an intuitive manner that we clarify, on the nature of consumer and platform heterogeneity.

As a result, it is crucial that, unlike previous models in the literature, ours accommodates significant richness in the environment. We make no specific assumptions on (i) functional forms for firm costs or distribution of user preferences, (ii) the dimensions of heterogeneity of consumer preferences, (iii) the number and symmetry of platforms or (iv) consumption patterns (i.e., single versus multi-homing). Instead we assume only mild 'regularity' conditions.

While the model we consider throughout most of the paper has exactly two sides and no externalities within sides, in Subection 7.2 we show how these restrictions can be easily relaxed, at the cost of some notational complexity. It should thus be possible to use our framework to evaluate models of competition among firms in markets *with* consumption externalities that are no more restrictive than the models typically used to study competition in markets *without* such externalities. We therefore believe that our approach has the potential to enrich the applied analysis of platform competition and inform competition and regulatory policy in such markets. We illustrate this by extending the principles behind the new United States and United Kingdom Horizontal Merger Guidelines to platform mergers in Subsection 7.1 and sketching a path towards structural estimation of our model in 7.3. Section 8 concludes with a discussion of directions for future research.

2 Contribution in Context

2.1 Basic pricing incentives

While generality is a primary goal of our analysis, to see how our solution concept works it is useful to begin with a simple example based on A06's canonical 'two-sided single-homing' setup. Consider a setting where two platforms compete for two groups of users, each of whom 'joins' at most one platform. Suppose that, while having an idiosyncratic 'membership value', $\epsilon_i^{\mathcal{I},\mathtt{j}}$, for each platform *itself*, all users of a given group have the same 'interaction value', $\beta^{\mathcal{I}}$, per user of the opposite side, $\mathcal{J} \neq \mathcal{I}$, who joins the same platform. Thus, the gross surplus user i on side $\mathcal{I}$ derives from joining platform $\mathtt{j}$ can be written $\epsilon_i^{\mathcal{I},\mathtt{j}} + \beta^{\mathcal{I}} N^{\mathcal{J},\mathtt{j}}$.

If platforms are assumed (as in this section of A06) to simply charge 'flat prices' that are not contingent on opposite-side participation levels, then there is a possibility of multiple equilibria among users, due to the 'chicken and egg' problem. Thus, an alternative assumption is natural: that platforms charge tariffs that adjust as a function of the number of opposite-side users that participate so as to guarantee each user a fixed utility level for joining. Indeed, by charging such tariffs, and only by doing so, platforms can render it a *dominant strategy* for each of their desired users to join (and for all of their undesired users not to join). In this setting with homogenous interaction values, our hyposthesized *Residual Insulating Tariffs* act in precisely this way, thus robustly guaranteeing that each platform receives its best-response payoff.

At the ensuing *Insulated Equilibrium* (IE), we obtain an equally natural pricing formula. Let $P^{\mathcal{I},\mathtt{j}}$ denote the total price platform $\mathtt{j}$ charges users on side $\mathcal{I}$, let $C_{\mathcal{I}}^{\mathtt{j}}$ denote $\mathtt{j}$'s marginal cost of serving an additional side $\mathcal{I}$ user, and let $\mu^{\mathcal{I},\mathtt{j}} \equiv N^{\mathcal{I},\mathtt{j}} / \left(-\frac{N^{\mathcal{I},\mathtt{j}}_{P^{\mathcal{I},\mathtt{j}}}}{}\right)$ denote $\mathtt{j}$'s *market power*, i.e., its standard markup under 'one-sided' differentiated Bertrand competition, where $N^{\mathcal{I},\mathtt{j}}$ denotes $\mathtt{j}$'s demand on side $\mathcal{I}$. The IE pricing formula is

$$P^{\mathcal{I},\mathtt{j}} = C_{\mathcal{I}}^{\mathtt{j}} + \mu^{\mathcal{I},\mathtt{j}} - N^{\mathcal{I},\mathtt{j}} \beta^{\mathcal{J}}. \tag{1}$$

Note that, *because users' interaction values are homogeneous*, the last term in this formula, $N^{\mathcal{I},\mathtt{j}} \beta^{\mathcal{J}}$, is *both* (i) the marginal gross surplus derived by platform $\mathtt{j}$'s side $\mathcal{J}$ users from the participation of each side $\mathcal{I}$ user *and* (ii) the marginal revenue that platform $\mathtt{j}$ can extract from these side $\mathcal{J}$ users due to each side $\mathcal{I}$ user's participation. Thus, in this setting, one-sided market power is the only force that distorts platforms' equilibrium prices away from their socially optimal levels.

In a more general environment where users have heterogenous valuations for both membership and interaction, (i) and (ii) do not coincide. While (i) depends on the interaction values of all of platform $\mathtt{j}$'s users on side $\mathcal{J}$, (ii) depends on the interaction values only of those users who are *marginal* for $\mathtt{j}$. Moreover, in such an expanded preference space, the set of marginal users driving (ii) and their immediate neighbors have interaction values that are themselves heterogenous. As a result, it ceases to be feasible for $\mathtt{j}$ to set a tariff that gives all users a dominant strategy: any reduction in $\mathtt{j}$'s tariff on side $\mathcal{I}$, in response to, say, a drop in opposite-side participation, inevitably under-compensates some users that were previously 'just inside' the margin and attracts some that were 'just outside'.

Residual Insulating Tariffs thus generalize the above perfectly compensating tariffs to environments with rich preference heterogeneity in what we believe to be the most natural feasible way. They adjust to variation in opposite-side participation so as to perfectly compensate the *representative marginal consumer*, drawing in as many consumers as drop out, holding fixed the overall level of demand. When this demand level is held fixed, *the utility associated with joining* $\mathtt{j}$ *for average marginal users on the opposite side of the market is held fixed*, and the chicken and egg problem is robustly eliminated.

The IE pricing formula in (1) generalizes in a correspondingly natural way, becoming

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} + \mu^{\mathcal{I},\mathtt{j}} - N^{\mathcal{I},\mathtt{j}}\gamma, \tag{2}$$

where γ is a combination of the interaction values of the various groups of marginal users on side $\mathcal{J}$. Under IE, platforms' objectives depart from social objectives in a manner that is qualitatively different from the the way they depart in conventional markets when (and only when) users' interaction values are heterogenous. The general formula in (2) thus accounts for, in a logically calibrated way, *the* crucial source of distortion that distinguishes the analysis of platform pricing from that of one-sided markets.

Due to the focus in Spence's (1975) article on the tendency of firms to over-cater to their marginal consumers, we refer to this as the *Spence distortion*. We explore this distortion in Section 6, where we show that the impact IE predicts consumer heterogeneity to have is consistent with economic intuition and at the same time quite rich. Moreover, IE allows for the types of analysis typically performed using conventional oligopoly models to be extended in a straightforward way to the study of platforms. We demonstrate this in Section 7.1, where we consider the effects on consumer surplus of a merger between platforms, extending the one-sided analysis of Jaffe and Weyl (2011). In a hypothetical merger between platforms, an important but difficult issue is whether the increase in externalities the platforms offer offsets their increased prices. As we show, IE provides a precise basis for analyzing this question, as it allows us to quantify changes in the Spence distortion, in addition to standard price effects, resulting from the merger.

2.2 Coordination and prices

Since the work of Rohlfs (1974), the literature on imperfect competition with consumption (or 'network') externalities has explored many paths in modeling the process by which consumers coordinate their actions.[4] For instance, in Katz and Shapiro (1985), firms take consumer beliefs as given, while Becker (1991) argued that, in the context of monopoly, platforms will typically find a way to coordinate consumers on their preferred outcome and Caillaud and Jullien (2003) assumed that, under competition, coordination would favor an 'incumbent' platform. Ellison and Fudenberg (2003), Ellison, Fudenberg, and Möbius (2004), Hagiu (2006) and Anderson, Ellison, and Fudenberg (2010) study the coordination problem in detail and find a large multiplicity of equilibria. Ambrus and Argenziano (2009) and Lee (2010) offer alternative notions of consumers coordinating always in their collective best interest to refine these equilibria. While each approach offers its own formal justification, they all leave open the question, in the spirit of Weinstein and Yildiz (2007), of how coordination occurs among a large and diffuse population of consumers.

A particularly salient answer to this question is provided by RT03. This model exploits the fact that, if all consumers' values are *proportional* to the number of consumers on the other side, differing only in the constant of proportionality, and prices are denominated *per-interaction*, then every consumer has a dominant strategy to join one platform or another, given the prices. This eliminates the scope for mis-coordination and squares with the intuition we discuss in the introduction about the use of prices as an instrument to coordinate demand. The monopoly model in A06 follows a strategy that is similar in this respect. It assumes that consumers have homogeneous valuation for externalities, which allows for platforms to promise *utility* levels, as in Dybvig and Spatt (1983) and Armstrong and Vickers (2001). However, both of these strategies rely on the particular preference structure assumed. Weyl (2010) generalizes these, in the context of monopoly, to *Insulating Tariffs*, which apply in settings with rich heterogeneity.

However, A06 also points out that in the context of competition, allowing platforms to charge prices that vary with the number of users on the other side of the market creates a paradox in the spirit of Klemperer and Meyer (1989). If one platform believes its rivals will lower prices dramatically when it loses consumers, this acts as a deterrent to intense competition. Thus, as we discuss in Section 5, *some* structure of tariffs can support nearly any outcome as an equilibrium among platforms. Approaches to refine these equilibria, such as assuming platforms' prices do not depend on the number of consumers on the other side, which A06 does in its sections on competition, or by reference to price discriminatory

[4]We prefer the phrase 'consumption externality' over 'network effect' as the latter is easily confused with a number of other related by not equivalent concepts in the literature on airline networks (Borenstein, 1989) and economies on graphs (Jackson, 2008).

motives (Reisinger, 2011) revive the chicken and egg problem.

Our approach takes seriously the role of prices as coordinating devices, formalizing Becker's (1991) intuition that platforms will typically be able to coordinate users on their desired outcome. Indeed, we take *this* to be the motivation behind the RT03 tariffs, rather than to give firms, *per se*, the strategic flexibility that is well known to eliminate the predictive power of theory. However, not all conditional pricing schemes achieve this goal: in the RT03 context it is specifically proportional pricing that does, while in the A06 setting, it is determined by the shape of consumers' homogeneous interaction values. What these share is that they both eliminate the scope for coordination failures by giving consumers dominant strategies. In Section 5 we maintain that platforms will use generalizations of these strategies, and of Weyl's (2010) notion of Insulating Tariffs, to avoid mis-coordination. This eliminates both the scope for coordination problems and ties down tariffs to resolve 'Armstrong's Paradox'.

2.3 Scope

Our model generalizes with respect to existing literature by accommodating arbitrary

1. preference heterogeneity among consumers, which, as discussed in Subsection 2.1, drives many comparative statics and welfare properties, in contrast to standard assumptions of restricted preference structures such as A06's single-homing model;

2. functional forms for demand, in contrast to the common assumption, for tractability, of two-sided Hotelling (1929) demand;

3. numbers of asymmetric firms, in contrast to the standard assumption of two, usually symmetric, firms;

4. consumption patterns, rather than imposing, as nearly all theoretical and empirical models have, that users either "single-home" (i.e. join at most one platform) or "multi-home" (i.e. view all platforms as independent and typically join either all or none of them).

However, while our model generalizes in the dimensions listed above, it retains two important assumptions that are typical of models in the literature on multi-sided platforms. First, we take as exogenous the interaction among the consumers of different sides of the market once they join platforms and assume that consumer payoffs from joining a set of platforms depends only on the number of consumers participating and the payment to the platform(s). This brings useful generality when the interventions one considers are unlikely to affect the microstructure of interactions, but is obviously problematic when such effects are significant. Second, we assume all consumers on a given side are homogenous in the externalities they cause. While some progress has been made on allowing for heterogeneous externalities, work that does so in a manner consistent with general heterogeneity is still preliminary (Veiga and Weyl, 2011). Other issues that we do not consider include explicit dynamics and price discrimination within sides. While all of these issues have been considered in specific settings, none has been treated in a way that makes it feasible to add to our model without substantially limiting its generality and empirical relevance.

3 The Model

There is a set $\mathscr{M} = \{\mathtt{1}, ..., \mathtt{m}\}$ of 'two-sided platforms', with elements indexed by $\mathtt{j}$. These firms serve two separate groups of 'consumers' or 'users', each of measure 1, said to be on opposite 'sides of the market', $\mathcal{A}$ and $\mathcal{B}$, indexed by $\mathcal{I}$. Consumers on each side of the market can choose to 'join' any combination of platforms, i.e., they pick an element in the power set of the set of platforms, $\wp(\mathscr{M})$. We denote the particular subset or 'bundle' of platforms that consumer i on side $\mathcal{I}$ chooses by $\mathscr{M}_i^{\mathcal{I}} \in \wp(\mathscr{M})$.

To capture consumption externalities, or 'cross-network effects', we assume that the payoff to a consumer on side $\mathcal{I}$ from joining a given set of platforms depends, in some way, on the number of consumers of the opposite side of the market, $\mathcal{J} \equiv -\mathcal{I}$, that join each of the platforms in this bundle.[5] Intuitively, one may think of the number of side $\mathcal{J}$ consumers participating on each platform in a bundle as, from the standpoint of a consumer on side $\mathcal{I}$, a *characteristic* of that bundle, partially determining its perceived quality. We now introduce a statistic that keeps track of these characteristics.

Definition 1. *A* Coarse Allocation, $\boldsymbol{N} \equiv \left(\boldsymbol{N}^{\mathcal{A}}, \boldsymbol{N}^{\mathcal{B}}\right) \in [0,1]^{2m}$, *specifies the* total *fraction or 'number' of consumers participating on each side of each platform. We denote a generic element by* $N^{\mathcal{I},\mathtt{j}}$.

Demand. Consumers have quasi-linear utility, and their optimization problem takes the form of a discrete choice over bundles of platforms. We write the payoff to user i on side $\mathcal{I}$ from joining bundle of platforms $\mathscr{X}$ as

$$v^{\mathcal{I}}\left(\mathscr{X}, \mathbf{N}^{\mathcal{J}}, \boldsymbol{\theta}_i^{\mathcal{I}}\right) - \sum_{\mathtt{j} \in \mathscr{X}} P^{\mathcal{I},\mathtt{j}},$$

where $\boldsymbol{\theta}_i^{\mathcal{I}} \in \Theta^{\mathcal{I}}$ denotes consumer i on side $\mathcal{I}$'s 'type' or 'characteristics'. The set of side $\mathcal{I}$ types, $\Theta^{\mathcal{I}} = \mathbb{R}^{L^{\mathcal{I}}}$, $2^m - 1 \leq L^{\mathcal{I}} \in \mathbb{N}$, does not impose any particular restrictions on the

[5] As we discuss in Section 2.3, we do not explicitly model the 'interaction' that may take place among users on opposite sides.

dimensions in which consumers can be heterogenous.[6] The function $v^{\mathcal{I}} : \wp(\mathscr{M}) \times [0,1]^m \times \Theta^{\mathcal{I}} \to \mathbb{R}$ is thus a map to a consumer's willingness to pay from each possible consumption choice, the characteristics of the available goods and the user's individual characteristics. $P^{\mathcal{I},\mathtt{j}}$ denotes the total price a user on side $\mathcal{I}$ must pay to join platform $\mathtt{j}$, the details of which we discuss below, when defining platforms' strategies. Let $f^{\mathcal{I}} : \Theta^I \to \mathbb{R}$ denote the probability density function of user types on side $\mathcal{I} = \mathcal{A}, \mathcal{B}$, satisfying $\int_{\Theta^{\mathcal{I}}} f^{\mathcal{I}}(\boldsymbol{\theta})d\boldsymbol{\theta} = 1$.

Assumption 1 further characterizes the demand system.

Assumption 1. *The functions $v^{\mathcal{I}}$ and $f^{\mathcal{I}}$, $\mathcal{I} = \mathcal{A}, \mathcal{B}$, jointly with their domains, have the following properties:*

1. Full Support*: For all $\boldsymbol{N}^{\mathcal{J}} \in [0,1]$, the corresponding $(2^m - 1)$-dimensional distribution of gross utility profiles $v^{\mathcal{I}}(\cdot, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta})$ has full support over $\mathbb{R}^{2^m-1}$.*

2. Smoothness*: For all $\boldsymbol{N}^{\mathcal{J}} \in [0,1]$, the corresponding $(2^m - 1)$-dimensional distribution of gross utility profiles is non-atomic and twice continuously differentiable. Moreover, $v^{\mathcal{I}}(\cdot,\cdot,\cdot)$ is twice continuously differentiable in all dimensions of its second argument.*

3. Normalization*: For all $\boldsymbol{\theta} \in \boldsymbol{\Theta}^{\mathcal{I}}$, $v^{\mathcal{I}}\left(\emptyset, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}\right) = 0$. For all consumers the 'outside option' gives a payoff normalized to zero.*

Before further specifying the strategic aspects of the game, we turn off the 'two-sided' feature of the model in order to establish useful properties that such a demand system exhibits in a conventional one-sided setting.

Lemma 1 (Invertibility)**.** *Hold fixed the coarse allocation on the opposite side of the market, $\boldsymbol{N}^{\mathcal{J}}$, and consider the demand system on side $\mathcal{I}$. For any interior side $\mathcal{I}$ coarse allocation, $\boldsymbol{N}^{\mathcal{I}} \in (0,1)^m$, there exists a unique vector, $\boldsymbol{P}^{\mathcal{I}} \in \mathbb{R}^m$, of platforms'* total *prices, that supports this allocation.*

Proof. Holding fixed $\boldsymbol{N}^{\mathcal{J}}$, the demand system in this model is a special case of the demand system in Azevedo, Weyl, and White's (2011) exchange economy with a continuum of agents. Therefore, Theorem 1 of that paper implies this result. □

Lemma 1 ensures that demand on side $\mathcal{I}$, $\boldsymbol{N}^{\mathcal{I}}\left(\boldsymbol{P}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right)$, can be inverted with respect to the vector of total prices on side $\mathcal{I}$ and that this inverse demand function, $\boldsymbol{P}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right)$, is well-defined over the domain $(0,1)^m \times [0,1]^m$. Also note that our assumptions guarantee that the functions $\boldsymbol{N}^{\mathcal{I}}$ and $\boldsymbol{P}^{\mathcal{I}}$ are twice continuously differentiable with respect to all of their arguments.

[6] See Subsection 6.2 for examples.

Lemma 2 (Consumers' Marginal Value of Supply)**.** *Let $V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}\right)$ denote the gross utilitarian surplus to consumers on side $\mathcal{I}$ associated with coarse allocation $\left(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}\right)$. It holds that*

$$\frac{\partial V^{\mathcal{I}}}{\partial N^{\mathcal{I},\mathtt{j}}} = P^{\mathcal{I},\mathtt{j}}. \tag{3}$$

Moreover, $V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}\right)$ is concave in all elements of $\boldsymbol{N}^{\mathcal{I}}$.

Proof. This follows from Proposition 1 of Azevedo et al. *op. cit.*. □

Lemma 3 formalizes, in a setting with rich complementary preferences, the familiar intuition that a good's marginal consumers have valuations that are equal to its price.

Supply. Turning to platforms, firm $\mathtt{j}$'s profits are given by

$$\Pi^{\mathtt{j}} \equiv P^{\mathcal{A},\mathtt{j}}N^{\mathcal{A},\mathtt{j}} + P^{\mathcal{B},\mathtt{j}}N^{\mathcal{B},\mathtt{j}} - C^{\mathtt{j}}\left(N^{\mathcal{A},\mathtt{j}},N^{\mathcal{B},\mathtt{j}}\right),$$

where $C^{\mathtt{j}}\left(N^{\mathcal{A},\mathtt{j}},N^{\mathcal{B},\mathtt{j}}\right)$ denotes platform $\mathtt{j}$'s costs as a function of the number of users on each side and is assumed to be twice continuously differentiable in both arguments.

Equilibrium. Platforms move first, simultaneously. Then, having observed the platforms' moves, all consumers simultaneously choose which platforms to join. We allow each platform to charge a tariff to consumers on side $\mathcal{I}$ that is a function of the *entire coarse allocation* on side $\mathcal{J}$. A (pure) strategy for platform $\mathtt{j}$, $\sigma^{\mathtt{j}} \equiv \left(\sigma^{\mathcal{A},\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{B}}\right),\sigma^{\mathcal{B},\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{A}}\right)\right)$, is a pair of such functions. These conditional strategies are, as we discuss further below, static reduced-forms for platforms' ability to condition their price at any given time on market conditions, so as to avoid coordination problems.

Formally, $\sigma^{\mathcal{I},\mathtt{j}} : [0,1]^m \to \mathbb{R}$. In order to ensure differentiability of each platform's residual profits, we assume that all platforms' price functions, $\sigma^{\mathcal{I},\mathtt{j}}$, are twice continuously differentiable. While this assumption facilitates our approach, it will become clear that, under IE, platforms never have an incentive to deviate to charging tariffs that violate this assumption. Let Σ denote the set of all pairs of C^2 functions, and let Σ^m denote the m^{th} Cartesian power of this set. We denote the profile of strategies of the entire set of platforms by $\boldsymbol{\sigma} \in \Sigma^m$. A profile of platform strategies is a function, $\boldsymbol{\sigma} : [0,1]^{2m} \to \mathbb{R}^{2m}$, which we assume can be written $\boldsymbol{\sigma}(\boldsymbol{N}) \equiv \left(\boldsymbol{\sigma}^{\mathcal{A}}\left(\boldsymbol{N}^{\mathcal{B}}\right),\boldsymbol{\sigma}^{\mathcal{B}}\left(\boldsymbol{N}^{\mathcal{A}}\right)\right)$. Under this notation, $\boldsymbol{\sigma}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{J}}\right) : [0,1]^m \to \mathbb{R}^m$ maps from the coarse allocation on side $\mathcal{J}$ to the vector of prices charged by all platforms on side $\mathcal{I}$.

Consumers react to platforms' announcement of price functions. Thus, a pure strategy for consumer i on side $\mathcal{I}$ is a *functional*,[7] which we denote by $\mathscr{M}_i^{\mathcal{I}}[\sigma]$, where $\mathscr{M}_i^{\mathcal{I}} : \Sigma^m \to$

[7] By 'functional', we mean a function that takes a function as at least one of its input-arguments. Hereafter, we surround arguments of functionals with square brackets when the entire function is to be taken as the

$\wp(\mathscr{M})$. To denote a *Side Strategy Profile*, for the set of consumers on side $\mathcal{I}$, we define the correspondence $\mathscr{M}^{\mathcal{I}}\left(\boldsymbol{\theta}^{\mathcal{I}},[\sigma]\right)$. With our atomless continuum of consumers, there will always be sets of consumers of measure zero who are indifferent between bundles. For definiteness, we tie down the behavior of such agents by assuming that consumers' strategy profiles are (a) symmetric, i.e. in every subgame, each consumer takes some action with probability one and (b) pure, i.e. all agents sharing a common type adopt the same strategy. It thus follows that $\mathscr{M}^{\mathcal{I}}$ is a functional, where $\mathscr{M}^{\mathcal{I}}:\Theta^{\mathcal{I}}\times\Sigma^m\to\wp(\mathscr{M})$ identifies all side $\mathcal{I}$ consumers' behavior in response to all $\boldsymbol{\sigma}\in\Sigma^m$. We denote the *Marketwide* consumer strategy profile by $\widehat{\mathscr{M}}(\boldsymbol{\theta},[\sigma])$, where $\widehat{\mathscr{M}}:\left\{\Theta^{\mathcal{A}}\times\Theta^{\mathcal{B}}\right\}\times\Sigma^m\to\wp(\mathscr{M})$.

Above, we defined a coarse allocation to be the number of consumers participating on each platform, on each side of the market. We can now derive this statistic as a function of the strategy profiles of consumers and platforms. To do so, we define the functional $\boldsymbol{N}:\left\{\widehat{\mathscr{M}}\right\}\times\Sigma^m\to[0,1]^{2m}$, mapping from marketwide consumer strategy profile and platform strategy profile to coarse allocation. $\boldsymbol{N}\left[\widehat{\mathscr{M}},\boldsymbol{\sigma}\right]$ has generic elements

$$N^{\mathcal{I},\mathrm{j}}\left[\widehat{\mathscr{M}},\boldsymbol{\sigma}\right]=\int_{\left\{\theta^{\mathcal{I}}\in\Theta^{\mathcal{I}}:\mathrm{j}\in\widehat{\mathscr{M}}\left(\theta^{\mathcal{I}},[\sigma]\right)\right\}}f^{\mathcal{I}}(\boldsymbol{\theta})\,d\boldsymbol{\theta}.$$

Finally, we denote by $\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}},[\sigma]\right)$ a *best response* strategy profile for consumers on side $\mathcal{I}$, where $\mathscr{M}^{\mathcal{I}*}:\Theta^{\mathcal{I}}\times[0,1]^m\times\Sigma^m\to\wp(\mathscr{M})$ specifies an optimal bundle for every consumer on side $\mathcal{I}$, given a coarse allocation on side $\mathcal{J}$ and platforms' strategy profile.

4 The Allocation Approach

In this section, we establish a simple framing of platforms' profit maximization problem, in the spirit of Myerson (1981) and Riley and Samuelson's (1981) approach to solving for optimal auctions based on allocation and implied revenue. First, we explain the motivation for such an approach in our oligopoly setting with consumption externalities. We then define a *No Sunspots* property of consumers' strategies, which, if satisfied, ensures the validity of this approach.

In the standard analysis of Nash-in-prices equilibrium of a differentiated products industry, each firm takes as given other firms' prices and chooses the price(s) for its own good(s) that maximizes profits. The presence of consumption externalities complicates matters. Consider the *Consumer Game* that takes place in the second stage, after platforms have announced their strategies. As we discuss in Section 2.3, such games can have

input. E.g., $z[f]$ indicates that z depends on the entire shape of the function f. In contrast, when the input to a function is a function *evaluated* at a particular point, we surround the argument with ordinary parentheses. E.g., $Z(f(x))$ indicates that Z depends on the value $f(\cdot)$ takes when evaluated at x.

multiple Nash Equilibria, since the optimal bundle for consumers on one side of the market depends on the actions of consumers on the other side. (See Figure 1.) Thus, a platform's optimal strategy may depend sensitively on the particular way it expects consumers to coordinate. This potential for multiple equilibria among consumers makes each platform's optimization problem *over prices* potentially complex.

Rather than considering the large space of potential price functions, the platform (and we, as modelers) can focus on the much smaller space of residual allocations. In order for this framing of the problem to be fully useful, it must be the case that, given the strategies of the other platforms, there is a function that maps from the number of consumers that platform $\mathtt{j}$ serves on each side of the market, $\left(N^{\mathcal{A},\mathtt{j}}, N^{\mathcal{B},\mathtt{j}}\right)$, to the profit that $\mathtt{j}$ realizes. Such a mapping exists, so long as consumers' strategy profile exhibits the No Sunspots property, the term for which we borrow from Cass and Shell (1983) and which we now define.

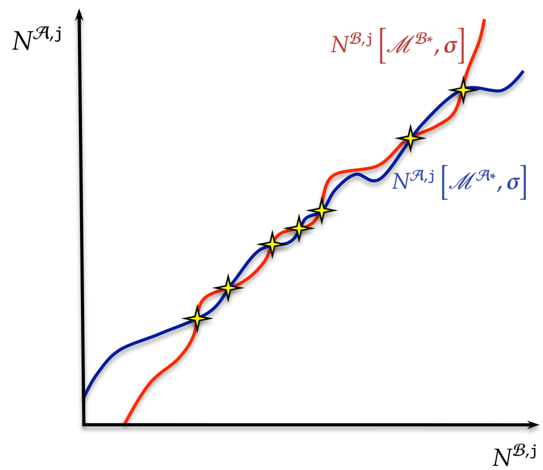

Figure 1: A hypothetical case with multiple equilibria in the second stage, for a given profile of platform strategies, $\boldsymbol{\sigma}$.

Definition 2 (No Sunspots)**.** *Given the strategy profile of the other platforms, $\boldsymbol{\sigma}^{-\mathtt{j}}$, a consumer strategy profile, $\widehat{\mathscr{M}}$, perceives* No Sunspots *from platform* $\mathtt{j}$ *if, for all $\boldsymbol{\sigma}^{\mathtt{j}}, \widehat{\boldsymbol{\sigma}}^{\mathtt{j}}$,*

$$\boldsymbol{N}^{\mathtt{j}}\left[\widehat{\mathscr{M}},\left(\boldsymbol{\sigma}^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)\right] = \boldsymbol{N}^{\mathtt{j}}\left[\widehat{\mathscr{M}},\left(\widehat{\boldsymbol{\sigma}}^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)\right] \text{ implies } \boldsymbol{N}\left[\widehat{\mathscr{M}},\left(\boldsymbol{\sigma}^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)\right] = \boldsymbol{N}\left[\widehat{\mathscr{M}},\left(\widehat{\boldsymbol{\sigma}}^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)\right].$$

To appreciate the meaning of this condition, suppose that it is violated. Then, platform $\mathtt{j}$ could, through changes in its policies, affect *other platforms'* allocations without affecting its own. In the context of our model, we believe this condition to be very weak, because it seems highly implausible that the prices of one platform would serve purely as a coordination device mediating the decisions of consumers to join *other* platforms, solely by influencing their beliefs. We thus posit that this condition is fulfilled in Assumption 2.

Assumption 2. *We restrict attention to consumer strategy profiles that perceive No Sunspots from any platform.*

Under Assumption 2, it is possible to assign a unique profit level, for platform $\mathtt{j}$, to every (interior) level of demand that it serves on each side of the market. We state this formally in Lemma 3.

Lemma 3. *There exists a profit function for platform* $\mathtt{j}$, $\Pi^{\mathtt{j}}\left(N^{\mathcal{A},\mathtt{j}}, N^{\mathcal{B},\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)$*, that is well-defined over the domain* $(0,1)^2 \times \Sigma^{m-1}$.

Proof. When No Sunspots holds for platform $\mathtt{j}$, each value of the coarse allocation for platform $\mathtt{j}$, $\boldsymbol{N}^{\mathtt{j}} = \left(N^{\mathcal{A},\mathtt{j}}, N^{\mathcal{B},\mathtt{j}}\right)$, implies a unique value for the entire coarse allocation, $\boldsymbol{N} = \left(\boldsymbol{N}^{\mathcal{A}}, \boldsymbol{N}^{\mathcal{B}}\right)$. By Lemma 1, it holds that $\boldsymbol{P}^{\mathcal{I}}$ is a function of $\boldsymbol{N}^{\mathcal{A}}$ and $\boldsymbol{N}^{\mathcal{B}}$, for $\mathcal{I} = \mathcal{A}, \mathcal{B}$. Therefore, platform $\mathtt{j}$'s prices, $\left(P^{\mathcal{A},\mathtt{j}}, P^{\mathcal{B},\mathtt{j}}\right)$, are uniquely determined by $N^{\mathcal{A},\mathtt{j}}$, $N^{\mathcal{B},\mathtt{j}}$ and $\boldsymbol{\sigma}^{-\mathtt{j}}$. □

Lemma 3 says that, in the absence of sunspots, platform $\mathtt{j}$ has a coherent residual inverse demand system. Thus, given the strategies of other platforms, each can identify a *best response allocation*. As in any optimization problem in a rich environment, a best-response is generically unique. Once this best response allocation is identified, the platform must determine how to implement this allocation using its available strategies. This question forms the basis of the following section.

5 Robust Implementation and Insulation

Section 4 casts platforms' problem as the choice of a best response allocation. Here, we address the question of how each platform induces or 'implements' this. We identify strategies called *Residual Insulating Tariffs* which, in a sense to be precisely specified, most robustly do so. We then posit our fundamental hypothesis – that platforms use RITs – forming the basis for our solution concept of *Insulated Equilibrium*, which generates testable predictions of prices based on cost and demand primitives.

Suppose that, given the strategy profile of other firms, $\boldsymbol{N}^{\mathtt{j}*}$ uniquely maximizes platform $\mathtt{j}$'s profits. Then, in order to be a best response to $\boldsymbol{\sigma}^{-\mathtt{j}}$, platform $\mathtt{j}$'s strategy, $\boldsymbol{\sigma}^{\mathtt{j}}$, must 'weakly implement' $\boldsymbol{N}^{\mathtt{j}*}$. More formally, for $\boldsymbol{\sigma}^{\mathtt{j}}$ to be a best response to $\boldsymbol{\sigma}^{-\mathtt{j}}$, there must be a Nash Equilibrium in the Conumer Game defined by $\left(\boldsymbol{\sigma}^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}\right)$ that features $\boldsymbol{N}^{\mathtt{j}*}$.

There are, however, *infinitely many* strategies for platform $\mathtt{j}$ that fulfill this criterion. To see this, let $\boldsymbol{N}^{*}$ denote the entire coarse allocation corresponding to $\boldsymbol{N}^{\mathtt{j}*}$. It is straightforward to see that $\boldsymbol{\sigma}^{\mathtt{j}}$ is a best response to $\boldsymbol{\sigma}^{-\mathtt{j}}$ if and only if $\boldsymbol{\sigma}^{\mathtt{j}}\left(\boldsymbol{N}^{*}\right) = \left(P^{\mathcal{A},\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{B}*}\right), P^{\mathcal{B},\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{A}*}\right)\right)$. In other words, the value of $\boldsymbol{\sigma}^{\mathtt{j}}(\cdot)$ is tied down by the demand system *only when evaluated at coarse allocation* $\boldsymbol{N}^{*}$. As Figure 2 illustrates, the value $\boldsymbol{\sigma}^{\mathtt{j}}(\cdot)$ takes on at other allocations is unconstrained, and thus the slope of platform $\mathtt{j}$'s tariff with respect to the allocation on the other side of the market is unconstrained.

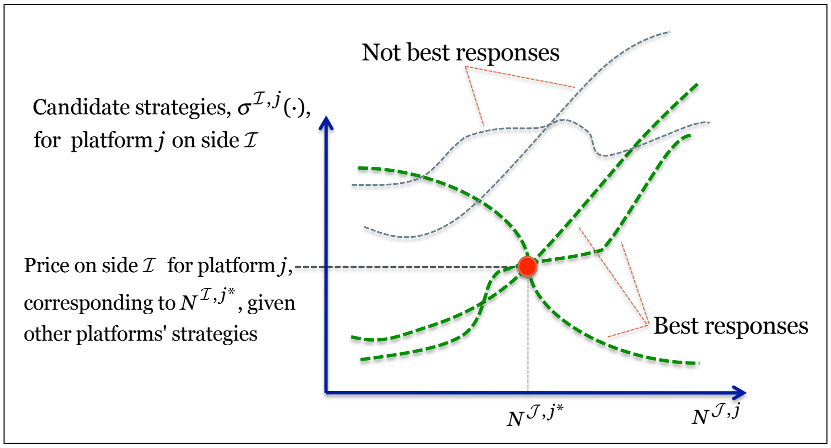

Figure 2: The best-response criterion does not tie down a platform's *strategy*.

As pointed out by A06's Proposition 3, this leads to a great multiplicity of *Platform Equilibria*, holding fixed consumers' strategy profile. This issue, which we refer to as *Armstrong's Paradox*, mirrors the multiplicity of supply function equilibria in a deterministic setting, analyzed by Klemperer and Meyer (1989). In particular, even with respect to those parts of the pricing functions directly relevant to first-order incentives, the two full $m \times m$ Jacobian matrices of $\sigma^{\mathcal{I}}$ and $\sigma^{\mathcal{J}}$ are free, where m denotes the number of platforms. In order to satisfy the first-order equilibrium conditions at any point these need only satisfy $2m \le 2m^2$ first-order conditions at the conjectured equilibrium point. As a result, we conjecture that it is possible to construct a set of platform strategies that support, as an SPE, any coarse allocation in which all platforms make positive profits. However, regardless of whether this is precisely true, the set of SPEs is very large. One clear manifestation of this problem is the fact that it is impossible, based solely on first-order conditions as is commonly done in standard one-sided markets (following Rosse, 1970), to identify firms' marginal cost from a measurement of the demand system and an observation of prices. Thus, if a solution concept for this class of games is to have significant predictive power, it must be stronger than SPE.

Our solution to Armstrong's Paradox relies on the following observation: the ***entire motivation*** for introducing allocation-dependent tariffs into models with consumption externalities was to allow firms, in the spirit of Becker (1991), to solve coordination problems among consumers. After all, despite the analysis of Klemperer and Meyer, most static empirical analysis in industrial organization uses the Nash-in-prices solution concept. Thus the motivation for allowing richer tariffs was not simply to expand the generality of the analysis, but rather, it was to allow firms to condition their prices in order to prevent the mis-coordination among consumers discussed in Section 4. As we now discuss, not all tariffs do this equally well, providing a rationale for our assumption that platforms choose tariffs that minimize the scope for such mis-coordination.

5.1 Mis-coordination and robust implementation.

Many tariffs available to platform $\mathtt{j}$ implement $\boldsymbol{N}^{\mathtt{j}*}$ as a unique Nash Equilibrium. However, the platform might wish to achieve something stronger than this form of uniqueness. At least ever since the work of Wilson (1987), a large literature in mechanism design has stressed the importance of implementing desired outcomes in ways robust to players' higher-order beliefs (Bergemann and Morris, 2005; Chung and Ely, 2007). In a quasi-linear, private values environment like the one considered here, doing so would require platforms to make it a dominant strategy for each consumer either to join or not join. Such robust implementation of its best response allocation seems a compelling strategy for a platform to adopt.

In fact this is precisely the technique that RT03 uses to solve its model of competition and that A06 uses to solve its general model of monopoly. In RT03, the joint assumption that both consumers' utility and platforms' prices are proportional to the number of users on the other side of the market implies that each user has a dominant strategy. In A06's monopoly model, the assumption that consumers are heterogenous only in the value of their outside option plays the same role in this regard: following Dybvig and Spatt (1983) and Armstrong and Vickers (2001), the setup allows platforms to commit to offering consumers a particular level of utility by adjusting their price in response to changes in the number of users on the other side, fully insuring users against any change in their utility, thus giving them a dominant strategy.

However, note that both of these modeling techniques rely on a special assumption about consumer heterogeneity, namely that it is one-dimensional and has a simple ordering in the spirit of Spence (1973) and Mirrlees (1971) in terms of user valuations of externalities. When this is not the case, marginal consumers are heterogeneous and thus it is not possible, using a uniform price, to give all consumers a dominant strategy. We now illustrate this point with a simple example.

A monopolist platform faces consumers on side $\mathcal{A}$ that have heterogenous 'participation benefits', $B_i^{\mathcal{A}}$, and heterogeneous 'interaction benefits', $b_i^{\mathcal{A}}$, per consumer on side $\mathcal{B}$, thus deriving gross utility from joining the platform given by $B_i^{\mathcal{A}} + b_i^{\mathcal{A}} N^{\mathcal{B}}$. Suppose the platform wishes to implement $N^{\mathcal{A}} = N^{\mathcal{B}} = \frac{1}{2}$. In the absence of price discrimination, there must be some threshold, $P^{\mathcal{A}}$, such that all consumers i with $B_i^{\mathcal{A}} + \frac{1}{2} b_i^{\mathcal{A}} > P^{\mathcal{A}}$ participate. For concreteness, suppose the threshold above which half of the population satisfies this is $P^{\mathcal{A}} = 5$. Then, any implementation in dominant strategies $\sigma^{\mathcal{A}}\left(N^{\mathcal{B}}\right)$ must provide an incentive for all users with $B_i^{\mathcal{A}} + \frac{1}{2} b_i^{\mathcal{A}} > 5$ to participate for every $N^{\mathcal{B}} \in (0,1)$ and those with $B_i^{\mathcal{A}} + \frac{1}{2}b_i^{\mathcal{A}} < 5$ a similar incentive not to participate. In particular a $\left(B^{\mathcal{A}}, b^{\mathcal{A}}\right) = (5.1, 0)$ consumer would have to have a dominant strategy to participate, while a $(4.4, 1)$ consumer would need to have a dominant strategy not to participate. Among other things, this would require that

$$5.1 > \sigma^{\mathcal{A}}(.9) > 4.4 + .9 \cdot 1 = 5.3,$$

an obvious contradiction.

5.2 Residual Insulating Tariffs

Given that dominant strategy implementation is infeasible with rich heterogeneity, one may consider a weaker notion of robust implementation that encompasses implementation in dominant strategies whenever it is feasible. To this end, we now define *Residual Insulating Tariffs* (RITs).

Definition 3. *Given a profile of strategies of other platforms, $\boldsymbol{\sigma}^{-\mathtt{j}}$, platform $\mathtt{j}$ is said to charge a* Residual Insulating Tariff *on side $\mathcal{I}$ if, for all $\boldsymbol{N}^{\mathcal{J}}, \widetilde{\boldsymbol{N}^{\mathcal{J}}} \in [0,1]$,*

$$N^{\mathcal{I},\mathtt{j}}\left[\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}, [\sigma]\right), \boldsymbol{\sigma}\right] = N^{\mathcal{I},\mathtt{j}}\left[\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \widetilde{\boldsymbol{N}^{\mathcal{J}}}, [\sigma]\right), \boldsymbol{\sigma}\right].$$

For a firm $\mathtt{j}$ to charge an insulating tariff on side $\mathcal{I}$, it must choose a price function, $\sigma^{\mathcal{I},\mathtt{j}}(\boldsymbol{N}^{\mathcal{J}})$, that, given the strategies of the other platforms, preserves its allocation on side $\mathcal{I}$, regardless of the strategy profile adopted by side $\mathcal{J}$ consumers. To see how such a function operates, consider the demand for platform $\mathtt{j}$ among side $\mathcal{I}$ consumers, $N^{\mathcal{I},\mathtt{j}}$. It can be written

$$N^{\mathcal{I},\mathtt{j}} = N^{\mathcal{I},\mathtt{j}}\left(\sigma^{\mathcal{I},\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right), \boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right), \boldsymbol{N}^{\mathcal{J}}\right).$$

An insulating tariff, charged by firm $\mathtt{j}$ on side $\mathcal{I}$ is thus a function, $\sigma^{\mathcal{I},\mathtt{j}}(\cdot)$, that takes into account the shape of $N^{\mathcal{I},\mathtt{j}}(\cdot,\cdot,\cdot)$ and the shape of other firms' side $\mathcal{I}$ price functions, $\boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}(\cdot)$, in order to ensure that the output of $N^{\mathcal{I},\mathtt{j}}$ is constant. Note that, whenever dominant strategy implementation is feasible, it is exactly the insulating tariff: any tariff that leads all consumers' choices to be independent of others' choices will, *a fortiori*, lead the aggregate quantity to be independent of the allocation on the other side. Thus, the RT03 and A06 dominant strategy tariffs are special cases of the insulating tariff, under the preference structures assumed in those settings. More generally, as Lemma 4 establishes, a unique insulating tariff exists, given the strategies of other platforms.

Lemma 4 (Existence and Uniqueness of a Residual Insulating Tariff)**.** *There exists a unique* *function,* $\overline{P^{\mathcal{I},\mathrm{j}}}\left(\boldsymbol{N}^{\mathcal{J}};\widetilde{N},\left[\boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right)\right]\right)$*, such that,* $\forall \boldsymbol{N}^{\mathcal{J}}$, $\forall \widetilde{N} \in (0,1)$, $\forall \boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}$,

$$N^{\mathcal{I},\mathrm{j}}\left(\overline{P^{\mathcal{I},\mathrm{j}}}\left(\boldsymbol{N}^{\mathcal{J}};\widetilde{N},\left[\boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right)\right]\right),\boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right),\boldsymbol{N}^{\mathcal{J}}\right)=\widetilde{N},$$

where $\widetilde{N}$ *denotes the 'anchor allocation' that* $\overline{P^{\mathcal{I},\mathrm{j}}}$ *implements. Moreover,* $\overline{P^{\mathcal{I},\mathrm{j}}}$ *is* C^2 *in all dimensions of its first argument.*

Proof. For existence, note that (i) $N^{\mathcal{I},\mathrm{j}}(\cdot,\cdot,\cdot)$ is continuous in its first argument, since it is the integral of a smooth set, and (ii) $\forall \boldsymbol{N}^{\mathcal{J}},\ \forall \boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}$, $\lim_{P^{\mathcal{I},\mathrm{j}}\to-\infty} N^{\mathcal{I},\mathrm{j}}\left(P^{\mathcal{I},\mathrm{j}},\boldsymbol{P}^{\mathcal{I},-\mathbf{j}},\boldsymbol{N}^{\mathcal{J}}\right)=1$ (and $\lim_{P^{\mathcal{I},\mathrm{j}}\to\infty} N^{\mathcal{I},\mathrm{j}}\left(P^{\mathcal{I},\mathrm{j}},\boldsymbol{P}^{\mathcal{I},-\mathbf{j}},\boldsymbol{N}^{\mathcal{J}}\right)=0$), since $\forall \boldsymbol{\theta}^{\mathcal{I}},\ \forall \boldsymbol{N}^{\mathcal{J}},\ \forall \boldsymbol{\sigma}^{\mathcal{I},-\mathbf{j}}, \exists P^{\mathcal{I},\mathrm{j}}$ such that

$$\max_{\mathscr{X}:\mathrm{j}\in\mathscr{X}}\left\{v^{\mathcal{I}}\left(\mathscr{X},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\theta}^{\mathcal{I}}\right)-\widehat{P^{\mathcal{I},\mathscr{X}}}\right\}>(<)\max_{\mathscr{Y}:\mathrm{j}\notin\mathscr{Y}}\left\{v^{\mathcal{I}}\left(\mathscr{Y},\boldsymbol{N}^{\mathcal{J}},\boldsymbol{\theta}^{\mathcal{I}}\right)-\widehat{P^{\mathcal{I},\mathscr{Y}}}\right\}.$$

For uniqueness, note that $N^{\mathcal{I},\mathrm{j}}(\cdot,\cdot,\cdot)$ is nonincreasing in its first argument, since it is the sum of a set of nonincreasing functions. To see that it is in fact strictly decreasing, note that by our full support assumption, strictly positive density must always exist on the set of marginal consumers for whom the above relationship holds with equality.

To show that $\overline{P^{\mathcal{I},\mathrm{j}}}$ is C^2 in all dimensions of its first argument, we note that, in response to a change in the value of an arbitrary element of $\boldsymbol{N}^{\mathcal{J}}$, $N^{\mathcal{J},\mathrm{k}}$, in order to be insulating $\overline{P^{\mathcal{I},\mathrm{j}}}$ must be differentiable by the inverse function theorem and have derivative equal to

$$\frac{\sum_{\mathrm{l}\neq\mathrm{j}}\frac{\partial N^{\mathcal{I},\mathrm{j}}}{\partial P^{\mathcal{I},\mathrm{l}}}\frac{\partial \sigma^{\mathcal{I},\mathrm{l}}}{\partial N^{\mathcal{J},\mathrm{k}}}+\frac{\partial N^{\mathcal{I},\mathrm{j}}}{\partial N^{\mathcal{J},\mathrm{k}}}}{-\frac{\partial N^{\mathcal{I},\mathrm{j}}}{\partial P^{\mathcal{I},\mathrm{j}}}}$$

so long as the denominator of this is bounded away from zero, which it is by the above argument that demand is strictly decreasing in own-price. Furthermore, this expression is, itself, differentiable in all elements of $\boldsymbol{N}^{\mathcal{J}}$ by the smoothness assumptions we have imposed. □

Another perspective from which to view RITs is by comparison to Armstrong and Vickers's (2001) notion of competition in utility space. When consumers are homogeneous, it is feasible, with a uniform price, for a platform to make their utility independent of the number of users on the other side of the market, as in A06's monopoly model. Even in the case when consumers are heterogeneous, but the *marginal* consumers are homogenous, the marginal users may be perfectly compensated for changes in the allocation on the other side of the market, as in RT03. On the other hand, when even the marginal consumers are heterogeneous, the most a firm can hope to accomplish is to compensate the *average marginal* consumer for changes in the allocation on the other side. This is equivalent to making the platform's desired allocation the dominant strategy for the *Representative Consumer* (RC) on a given side of the market, who aggregates together the (quasi-linear) preferences of that side's individual consumers.[8] In particular, suppose that on side $\mathcal{I}$ there is a single agent in charge of choosing quantities, or 'slots' on platforms, for his constituent consumers on side $\mathcal{I}$ to efficiently allocate among themselves, and that the RC's objective is to maximize the sum of constituents' utility. The RC's objective function can thus be written as

$$V^{\mathcal{I}}\left(\boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right) - \boldsymbol{\sigma}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{J}}\right) \cdot \boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}}.$$

where '$\cdot$' denotes the inner-product operator and where, as defined in Lemma 2, $V^{\mathcal{I}}$ denotes gross consumer surplus on side $\mathcal{I}$, which, here, can be interpreted as the gross payoff to the representative consumer. Consider the following *Representative Consumer Game*, defined by the strategy profile, $\boldsymbol{\sigma}$, announced by platforms. On side $\mathcal{I}$, the RC chooses coarse allocation $\boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}}$; activity among side $\mathcal{J}$ consumers occurs as before. We can now state Theorem 1.

Theorem 1. *Let* $\overline{\boldsymbol{N}^{\mathcal{I}}} \equiv \boldsymbol{N}^{\mathcal{I}}\left(\left(\overline{P^{\mathcal{I},\mathtt{j}}}\left(\boldsymbol{N}^{\mathcal{J}}; N^{\mathcal{I},\mathtt{j}*}, \left[\boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\right]\right), \boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\left(\boldsymbol{N}^{\mathcal{J}}\right)\right), \boldsymbol{N}^{\mathcal{J}}\right)$ *denote the coarse allocation on side* $\mathcal{I}$ *that results from platform* $\mathtt{j}$ *charging the RIT anchored at* $N^{\mathcal{I},\mathtt{j}*}$*. In an RC game,* $\boldsymbol{\sigma}$*, it is a strictly dominant strategy for the Representative Consumer to select* $\boldsymbol{N}_{RC}^{\mathcal{I}} = \overline{\boldsymbol{N}^{\mathcal{I}}}$ *if and only if platform* $\mathtt{j}$*'s strategy is the RIT, anchored at* $N^{\mathcal{I},\mathtt{j}*}$*. Formally, it holds that*

$$V^{\mathcal{I}}\left(\overline{\boldsymbol{N}^{\mathcal{I}}}, \boldsymbol{N}^{\mathcal{J}}\right) - \boldsymbol{\sigma}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{J}}\right) \cdot \overline{\boldsymbol{N}^{\mathcal{I}}} > V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right) - \boldsymbol{\sigma}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{J}}\right) \cdot \boldsymbol{N}^{\mathcal{I}},$$

for all $\boldsymbol{N}^{\mathcal{J}}$ *and for all* $\boldsymbol{N}^{\mathcal{I}} \neq \overline{\boldsymbol{N}^{\mathcal{I}}}$*, if and only if* $\boldsymbol{\sigma}^{\mathcal{I}} = \left(\overline{P^{\mathcal{I},\mathtt{j}}}\left(\cdot; N^{\mathcal{I},\mathtt{j}*}, \left[\boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\right]\right), \boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\right)$.

Proof. First note that

$$\begin{aligned} & V^{\mathcal{I}}\left(\boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right) - \boldsymbol{\sigma}^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{J}}\right) \cdot \boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}} = \\ & \max_{\mathscr{M}^{\mathcal{I}} \in \left\{\mathscr{M}^{\mathcal{I}} : \mathrm{N}^{\mathcal{I}}\left[\mathscr{M}^{\mathcal{I}}, \sigma\right] = \mathrm{N}_{RC}^{\mathcal{I}}\right\}} \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\boldsymbol{\theta}^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}}\left(\boldsymbol{\theta}^{\mathcal{I}}, [\sigma]\right) = \mathscr{X}} \left(v^{\mathcal{I}}\left(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}\right) - \widehat{P^{\mathcal{I},\mathscr{X}}}\left(\boldsymbol{N}^{\mathcal{J}}\right)\right) f(\boldsymbol{\theta})\, d\boldsymbol{\theta} \\ & \qquad \leq \sum_{\mathscr{X} \in \wp(\mathscr{M})} \int_{\boldsymbol{\theta}^{\mathcal{I}} : \mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}; [\sigma]\right) = \mathscr{X}} \left(v^{\mathcal{I}}(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}) - \widehat{P^{\mathcal{I},\mathscr{X}}}(\boldsymbol{N}^{\mathcal{J}})\right) f(\boldsymbol{\theta}) d\boldsymbol{\theta}, \quad (4) \end{aligned}$$

where, by revealed preference, the inequality in (4) is strict if and only if $\boldsymbol{N}_{\boldsymbol{RC}}^{\mathcal{I}} \neq \overline{\boldsymbol{N}^{\mathcal{I}}}$ as net surplus has a unique maximizer by its concavity established in Lemma 2. Second, note that, by Lemma 4, $\boldsymbol{N}^{\mathcal{I}}\left[\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}; [\sigma]\right), \boldsymbol{\sigma}\right] = \overline{\boldsymbol{N}^{\mathcal{I}}}$, $\forall \boldsymbol{N}^{\mathcal{J}}$, if and only if $\boldsymbol{\sigma}^{\mathcal{I},\mathtt{j}}(\cdot) = \overline{P^{\mathcal{I},\mathtt{j}}}\left(\cdot; N^{\mathcal{I},\mathtt{j}*}, \left[\boldsymbol{\sigma}^{\mathcal{I},-\mathtt{j}}\right]\right)$. This establishes our claim. □

[8] See Anderson, de Palma, and Thisse (1992), particularly Chapter 3, for foundations of the representative consumer approach in a one-sided discrete choice setting.

Thus, in a setting with rich demand, the Residual Insulating Tariff is, in this sense, the closest platform j can get to implementing its desired allocation in dominant strategies, using a uniform price.

5.3 Hypothesis: Insulation

The crucial substantive hypothesis from which we derive our solution concept posits that, in order to minimize the scope for mis-coordination, all platforms use Residual Insulating Tariffs. Because there always exists a (unique) RIT anchored at a platform's best response allocation, this merely refines platforms' choice space without restricting it; that is, platforms can never do better by unilaterally deviating to *any* other tariff.

Hypothesis (Insulation)**.** *All platforms employ Residual Insulating Tariffs.*

While it is common for advertising rates to depend on the number of consumers viewing those advertisements and even the advertisements of rivals, static prices in platform markets almost never *literally* resemble insulating tariffs. On the other hand, a qualitatively obvious and seemingly very common practice of platform businesses is to set prices low when the platform's (endogenous) quality is low to have them rise, over time as quality increases, in order to generate profits. Furthermore, Cabral (2011) has shown that, in the special case of a monopoly with A06 preferences, the optimal price path of a corresponding dynamic game corresponds *exactly* with the insulating tariff. Thus we believe that our hypothesis that platforms will, as far as possible, robustly implement their best response allocation is a reasonable static reduced-form for this inherently dynamic process: the platform aims, relentlessly and consistently, at implementing its best response allocation, regardless of the transitory market conditions that may impede this.

5.4 Insulated Equilibrium

We now introduce vocabulary to describe the world implied by Assumption 5.3, in which all platforms charge insulating tariffs.

Definition 4. *An* Insulating Tariff System (ITS) *on side* $\mathcal{I}$, $\overline{\boldsymbol{P}^{\mathcal{I}}}\left(\boldsymbol{N}^{\mathcal{J}};\widetilde{\boldsymbol{N}^{\mathcal{I}}}\right)$*, is a profile of Residual Insulating Tariffs, parameterized by the coarse allocation it induces,* $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$*. We say that* $\overline{\boldsymbol{P}^{\mathcal{I}}}$ *is 'anchored' at* Reference Allocation $\widetilde{\boldsymbol{N}^{\mathcal{I}}}$*. We denote a marketwide ITS by* $\overline{\boldsymbol{P}}\left(\widetilde{\boldsymbol{N}}\right) \equiv \left(\overline{\boldsymbol{P}^{\mathcal{A}}}\left(\boldsymbol{N}^{\mathcal{B}};\widetilde{\boldsymbol{N}^{\mathcal{A}}}\right),\overline{\boldsymbol{P}^{\mathcal{B}}}\left(\boldsymbol{N}^{\mathcal{A}};\widetilde{\boldsymbol{N}^{\mathcal{B}}}\right)\right)$*.*

Note that, at any anchor allocation, the ITS exists and is unique directly from Lemma 1: it is exactly the unique set of price consistent with the anchor allocation, given the coarse allocation on the opposite side at which it is evaluated. It is also C^2 by the smoothness of the demand system.

We can now define our solution concept. Insulated Equilibria are particular Subgame Perfect Equilibria. We first define the latter in the context of our game and then we state the definition of IE. Given a consumer strategy profile, $\widehat{\mathscr{M}}$, and a profile of strategies adopted by other firms, $\boldsymbol{\sigma}^{-\mathtt{j}}$, denote firm $\mathtt{j}$'s profits by

$$\Pi^{\mathtt{j}}\left[\sigma^{\mathtt{j}}, \boldsymbol{\sigma}^{-\mathtt{j}}; \widehat{\mathscr{M}}\right] \equiv \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} \sigma^{\mathcal{I},\mathtt{j}}\left(N^{\mathcal{J},\mathtt{j}}\left[\widehat{\mathscr{M}}, \boldsymbol{\sigma}\right]\right) N^{\mathcal{I},\mathtt{j}}\left[\widehat{\mathscr{M}}, \boldsymbol{\sigma}\right] - C^{\mathtt{j}}\left(N^{\mathcal{A},\mathtt{j}}\left[\widehat{\mathscr{M}}, \boldsymbol{\sigma}\right], N^{\mathcal{B},\mathtt{j}}\left[\widehat{\mathscr{M}}, \boldsymbol{\sigma}\right]\right). \tag{5}$$

Definition 5. *In a particular platform game, defined by $\widehat{\mathscr{M}}$, a platform strategy profile, $\boldsymbol{\sigma}$, forms a* Platform Nash Equilibrium (PNE) *if* $\sigma^{\mathtt{j}} \in \arg\max_{x\in\Sigma} \Pi^{\mathtt{j}}\left(x, \boldsymbol{\sigma}^{-\mathtt{j}}; \widehat{\mathscr{M}}\right), \ \forall \mathtt{j} \in \mathscr{M}$.

Definition 6. *A set containing a profile of strategies for platforms and for consumers on each side, $\{\boldsymbol{\sigma}^*, \{\mathscr{M}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}\}$, forms a* Subgame Perfect Equilibrium (SPE) *if $\boldsymbol{\sigma}^*$ forms a PNE given $\{\mathscr{M}^{\mathcal{I}}\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$ and, on each side of the market, $\mathscr{M}^{\mathcal{I}}$ is a best response strategy profile for all $\boldsymbol{\sigma} \in \Sigma^m$.*

We now state the definition of an Insulated Equilibrium.

Definition 7. *Let $\{\boldsymbol{\sigma}^*, \widehat{\mathscr{M}^*}\}$ be an SPE with coarse allocation $\boldsymbol{N}^* = \left(\boldsymbol{N}^{\mathcal{A}^*}, \boldsymbol{N}^{\mathcal{B}^*}\right)$. The SPE $\{\boldsymbol{\sigma}^*, \widehat{\mathscr{M}^*}\}$ is an* Insulated Equilibrium (IE) *if platforms' strategy profile is the Insulating Tariff System anchored at $\boldsymbol{N}^*$, i.e. if $\boldsymbol{\sigma}^* = \overline{\boldsymbol{P}}(\boldsymbol{N}^*)$.*

Remark. *By construction, IEs are the only SPEs consistent with Insulation.*

In a Subgame Perfect Equilibrium, platforms select their strategies *as if they had complete certainty* of the outcome of the ensuing Consumer Game, even when the particular Consumer Game that they induce has multiple Nash Equilibria. Thus, one must speak of platforms' profits as functions of *both* platforms' strategies *and* of consumers' strategies. ***Under Insulated Equilibrium, on the other hand, the particular strategy profile adopted by consumers is of no consequence***, since, when the platforms' strategy profile amounts to an Insulating Tariff System, in the subsequent Consumer Game, there is a unique Nash Equilibrium.

5.5 Insulated Equilibrium is identified

Because the ITS anchored at any reference allocation is unique, it defines a unique inverse demand system for each platform at each candidate equilibrium, thus tying down each platform's incentives. Intuitively, as shown in Figure 3, assumption 5.3 ties down each platform's tariff, resolving Armstrong's Paradox. Thus if both costs and demand are known, any candidate allocation either is, or is not, an IE, without any further specification of strategies. Similarly, if costs are unknown, marginal costs may now be recovered *à la* Rosse (1970). Thus the predictive power of IE is equivalent to that of Nash-in-prices equilibrium in a standard industry without consumption externalities. We formalize this statement in Theorem 2.

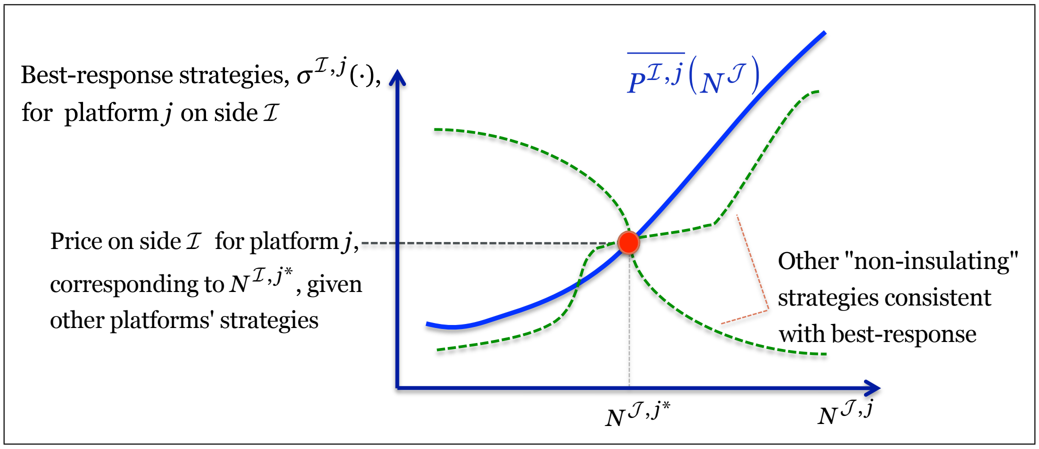

Figure 3: The shape of the Insulating Tariff System is tied down by the demand system.

Theorem 2 (Under Insulated Equilibrium, Marginal Cost is Identified)**.** *Suppose* $\left\{\widehat{\mathscr{M}^*},\boldsymbol{\sigma}^*\right\}$ *is an IE with coarse allocation* $\mathbf{N}^*$*, with generic elements* $N^{\mathcal{I},j*}$*. Then, the vector of platform marginal costs is identified jointly by the vector of prices,* $\left\{\mathbf{P}^{\mathcal{I}}\right\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$*, the coarse allocation, the payoff functions* $\left\{v^{\mathcal{I}}\right\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$ *and the distribution of types* $\left\{f^{\mathcal{I}}\right\}_{\mathcal{I}=\mathcal{A},\mathcal{B}}$.

Proof. Since $\left\{\widehat{\mathscr{M}^*},\boldsymbol{\sigma}^*\right\}$ is an IE with coarse allocation $\boldsymbol{N}^*$, the equilibrium profile of platform strategies is $\boldsymbol{\sigma}^* = \overline{\boldsymbol{P}}(\boldsymbol{N}^*)$. Thus, platform j's profit maximization problem can be written

$$\max_{\left\{N^{\mathcal{A},j},N^{\mathcal{B},j}\right\}} \sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} N^{\mathcal{I},j}\cdot P^{\mathcal{I},j}\left(N^{\mathcal{I},j},N^{\mathcal{J},j}\right) - C^{j}\left(N^{\mathcal{A},j},N^{\mathcal{B},j}\right), \tag{6}$$

where

$$P^{\mathcal{I},j}\left(N^{\mathcal{I},j},N^{\mathcal{J},j}\right) = \overline{P^{\mathcal{I},j}}\left(\mathbf{N}^{\mathcal{J}};N^{\mathcal{I},j},\left[\overline{\mathbf{P}^{\mathcal{I},-j}}\left(\mathbf{N}^{\mathcal{J}},\mathbf{N}^{\mathcal{I}^*}\right)\right]\right)$$

and

$$\mathbf{N}^{\mathcal{J}} = \mathbf{N}^{\mathcal{J}}\left(\overline{P^{\mathcal{J},j}}\left(\mathbf{N}^{\mathcal{I}};N^{\mathcal{J},j},\left[\overline{\mathbf{P}^{\mathcal{J},-j}}\left(\mathbf{N}^{\mathcal{I}};\mathbf{N}^{\mathcal{J}^*}\right)\right]\right),\overline{\mathbf{P}^{\mathcal{J},-j}}\left(\mathbf{N}^{\mathcal{I}};\mathbf{N}^{\mathcal{J}^*}\right),\mathbf{N}^{\mathcal{I}}\right).$$

The values that maximize (6), $N^{\mathcal{A},j*}$ and $N^{\mathcal{B},j*}$, satisfy the first-order condition

$$P^{\mathcal{I},j} + N^{\mathcal{I},j*}\frac{\partial P^{\mathcal{I},j}}{\partial N^{\mathcal{I},j}} + N^{\mathcal{J},j*}\frac{\partial P^{\mathcal{J},j}}{\partial N^{\mathcal{I},j}} = \overline{P^{\mathcal{I},j}} + \frac{N^{\mathcal{I},j*}}{\frac{\partial N^{\mathcal{I},j}}{\partial P^{\mathcal{I},j}}} + N^{\mathcal{J},j*}\frac{\partial \overline{P^{\mathcal{J},j}}}{\partial \mathbf{N}^{\mathcal{I}}}\cdot\frac{\frac{\partial \mathbf{N}^{\mathcal{I}}}{\partial P^{\mathcal{I},j}}}{\frac{\partial N^{\mathcal{I},j}}{\partial P^{\mathcal{I},j}}} = \frac{\partial C^{j}}{\partial N^{\mathcal{I},j}}. \tag{7}$$

By Lemma 1, all of these quantities are well-defined based on the demand and cost systems and the observed allocation. Thus a unique vector of marginal costs is consistent with a given IE. □

6 Pricing and Welfare

In the previous sections, we have defined IE and explained its motivation. The rest of the paper focuses on analyzing the economic predictions of our model, using this solution concept. In this section, we study the prices that arise under IE, comparing them with those that correspond to a socially optimal allocation. It is divided into three parts. In Section 6.1, we consider the benchmark of socially optimal pricing. In Section 6.2 we consider several simple examples, illustrating the *Spence distortion*, caused by differences between the 'interaction values' between platforms' average and marginal consumers, and we study the way this distortion is affected by increased competition. Finally, in Section 6.3, we derive the general formula for pricing under IE and show how it extends this Spencian reasoning. Proofs of all propositions in this section can be found in Appendix A.

In order to afford a clean interpretation of the pricing formulae, we impose, throughout the rest of the paper, the following No Externalities to Outsiders condition.

Assumption 3 (No Externalities to Outsiders)**.** *If* $\mathrm{j} \notin \mathscr{X}$*, then* $v^{\mathcal{I}}\left(\mathscr{X}, \boldsymbol{N}^{\mathcal{J}}, \boldsymbol{\theta}\right)$ *is independent of* $N^{\mathcal{J},\mathrm{j}}$*.*

This is an intuitive assumption reflecting the idea that consumers on opposite sides of the market do not 'interact', unless they join at least one common platform.[9] For instance, it implies that the gross utility a user derives from reading *only* one newspaper does not depend on the number of advertisements in other newspapers.

6.1 Socially optimal pricing

The utilitarian social welfare corresponding to an allocation is equal to

$$\sum_{\mathcal{I}=\mathcal{A},\mathcal{B}} V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right) - \sum_{\mathrm{j}\in\mathscr{M}} C^{\mathrm{j}}\left(N^{\mathcal{A},\mathrm{j}}, N^{\mathcal{B},\mathrm{j}}\right), \tag{8}$$

where $V^{\mathcal{I}}\left(\boldsymbol{N}^{\mathcal{I}}, \boldsymbol{N}^{\mathcal{J}}\right)$ denotes gross consumer surplus on side $\mathcal{I}$, as defined in Lemma (2). In Proposition 1, we state the pricing formula for maximizing this quantity. To do so, let

[9] In this regard, the scope of these results differs from that of Segal (1999), which studies the effects of positive versus negative 'externalities on nontraders'. Relaxing this assumption could allow for an interesting analysis of 'open' versus 'closed' platforms.

us denote by

$$\overline{v_{\mathrm{j}}^{\mathcal{I},\mathrm{j}}} \equiv \frac{\int_{\boldsymbol{\theta}^{\mathcal{I}}:\mathrm{j}\in\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}^{\mathcal{I}})} \frac{\partial v^{\mathcal{I}}\left(\mathscr{M}^{\mathcal{I}*}(\boldsymbol{\theta}),N^{\mathcal{J}},\boldsymbol{\theta}^{\mathcal{I}}\right)}{\partial N^{\mathcal{J},\mathrm{j}}} f(\boldsymbol{\theta})\, d\boldsymbol{\theta}}{N^{\mathcal{I},\mathrm{j}}}$$

the average valuation, among *all* of platform j's side $\mathcal{I}$ consumers, for an additional side $\mathcal{J}$ consumer to join platform j. Also let $C_{\mathcal{I}}^{\mathrm{j}} \equiv \frac{\partial C^{\mathrm{j}}}{\partial N^{\mathcal{I},\mathrm{j}}}$ denote platform j's marginal cost of serving an additional consumer on side $\mathcal{I}$.

Proposition 1. *At a socially optimal allocation, the total price charged to side $\mathcal{I}$ consumers to join platform j satisfies*

$$P^{\mathcal{I},\mathrm{j}} = \underbrace{C_{\mathcal{I}}^{\mathrm{j}}}_{\text{marginal private cost}} - \underbrace{N^{\mathcal{J},\mathrm{j}}\overline{v_{\mathrm{j}}^{\mathcal{J},\mathrm{j}}}}_{\text{externality}}. \tag{9}$$

This rule is a simple application of Pigou's principle that the price of a good should differ from its private cost by an amount equal to the externality its consumption entails. Here, the externality that users on side $\mathcal{I}$ create by participating on platform j is the product of the number of users participating on that platform on the other side of the market, $N^{\mathcal{J},\mathrm{j}}$, and the average marginal utility these users derive from an additional user on side $\mathcal{I}$, $\overline{v_{\mathrm{j}}^{\mathcal{J},\mathrm{j}}}$.

6.2 Pricing at Insulated Equilibrium: examples

In this subsection, we consider a series of special cases of the model. Extending the setup of Rochet and Tirole's (2006) canonical monopoly model, we assume that consumers' preferences consist of a baseline 'membership value', $\epsilon_i^{\mathcal{I},\mathscr{X}} \in \mathbb{R}$, for joining each bundle of platforms as well as a linear 'interaction value', $\beta_i^{\mathcal{I},\mathrm{j}} \in \mathbb{R}$, associated with each platform j. Thus, consumer i on side $\mathcal{I}$'s payoff from joining bundle $\mathscr{X}$ can be written

$$\underbrace{\epsilon_i^{\mathcal{I},\mathscr{X}}}_{\text{membership value for bundle }\mathscr{X}} + \underbrace{\sum_{\mathrm{j}\in\mathscr{X}} \beta_i^{\mathcal{I},\mathrm{j}} N^{\mathcal{J},\mathrm{j}}}_{\text{total value from interaction}} - \sum_{\mathrm{j}\in\mathscr{X}} P^{\mathcal{I},\mathrm{j}}.$$

A crucial feature of A06's monopoly and 'two-sided single-homing' models is consumers' homogeneity in their interaction values and heterogeneity only in the membership values. Consider, first, Generalized Armstrong Preferences (GAPs), which are a special case of those introduced above, exhibiting this feature. Let $\beta^{\mathcal{I}} \in \mathbb{R}$ denote an interaction value common to all side $\mathcal{I}$ users, associated with interaction on any platform. Under GAPs, consumer i's payoff from joining bundle $\mathscr{X}$ is

$$\epsilon_i^{\mathcal{I},\mathscr{X}} + \beta^{\mathcal{I}} \sum_{\mathrm{j}\in\mathscr{X}} N^{\mathcal{J},\mathrm{j}} - \sum_{\mathrm{j}\in\mathscr{X}} P^{\mathcal{I},\mathrm{j}}.$$

In Proposition 2, we state the IE pricing formula under GAPs. Hereafter, let $\mu_{\mathcal{I}}^{\mathtt{j}} \equiv \frac{N^{\mathcal{I},\mathtt{j}}}{-\frac{\partial N^{\mathcal{I},\mathtt{j}}}{p^{\mathcal{I},\mathtt{j}}}}$ denote the conventional 'markup' or 'market power' term that arises in differentiated Bertrand (Nash-in-prices) competition in one-sided markets.

Proposition 2 (Generalized Armstrong Pricing)**.** *When users on side $\mathcal{J}$ have GAPs, then at IE,*

(a) platform $\mathtt{j}$*'s total price on side* $\mathcal{I}$ *satisfies*

$$P^{\mathcal{I},\mathtt{j}} = \underbrace{C_{\mathcal{I}}^{\mathtt{j}}}_{\textit{marginal cost}} + \underbrace{\mu_{\mathcal{I}}^{\mathtt{j}}}_{\textit{market power}} - \underbrace{N^{\mathcal{J},\mathtt{j}}\beta^{\mathcal{J}}}_{\textit{externality}}; \tag{10}$$

(b) platform $\mathtt{j}$*'s Residual Insulating Tariff on side* $\mathcal{J}$ *can be written* $\overline{P^{\mathcal{J},\mathtt{j}}}\left(\mathbf{N}^{\mathcal{I}}\right) = \overline{P^{\mathcal{J},\mathtt{j}}}(\mathbf{0}) + \beta^{\mathcal{J}} N^{\mathcal{I},\mathtt{j}}$. *Given platforms' strategies, in the second stage of the game, all side* $\mathcal{J}$ *consumers have a dominant strategy to select the bundle of platforms that they choose at equilibrium.*

Regarding part (a), when consumers on side $\mathcal{J}$ have homogeneous marginal utilities from externalities, $\overline{v_{\mathtt{j}}^{\mathcal{J},\mathtt{j}}} = \beta^{\mathcal{J}}$. As a result, the only source of distortion causing the equilibrium price given by (10) to differ from the socially optimal price given by (9) is the conventional *one-sided* market power distortion captured by $\mu_{\mathcal{I}}^{\mathtt{j}}$, as conjectured by Liebowitz and Margolis (1994). Regarding part (b), note that by charging a RIT, each platform guarantees to every side $\mathcal{J}$ consumer a fixed incremental utility for joining, mirroring the mechanisms of Dybvig and Spatt (1983) and Armstrong and Vickers (2001); therefore every user has a dominant strategy to select the bundle of platforms where the sum of these utilities, plus her idiosyncratic membership value, is highest.

GAPs are convenient in enabling platforms to implement allocations in dominant strategies. However, they eliminate from consideration the *Spence (1975) distortion*, arising from differences in marginal utilities from externalities between *average* and *marginal* consumers of a platform. We now consider a simple example illustrating the way that such heterogeneity drives this distortion, which is the key normative difference between the platform setting and a standard market without consumption externalities.

Anderson and Coate's (2005) study of the TV and radio industries involves a two-platform setup with the following features, which are also relevant to the (print and online) newspaper and software application markets.[10] On side $\mathcal{A}$, 'advertisers' have independent demand for each platform. That is, each advertiser's decision to join platform $\mathtt{j}$ depends only on $\mathtt{j}$'s price and on the number of side $\mathcal{B}$ 'buyers' (e.g. for viewers, readers, web

[10]Formally, given our Full Support assumption, this description corresponds to a limit case where all but an infinitesimal mass of users have arbitrarily negative membership values that preclude certain consumption patterns. Numerous other articles, including A06's section on multi-homing, study either special cases or variants of this setup.

surfers, etc.) it provides access to. Advertisers, thus, may 'multi-home'. On the opposite side, buyers join only their preferred platform, taking into account the prices and number of advertisers offered by both.

Unlike under GAPs, the set of consumers *on any given margin* can be heterogenous in their interaction values. Denote by $f^{\mathcal{B}}_{\mathtt{j},\emptyset}$ and $\widetilde{\beta^{\mathcal{B}}_{\mathtt{j},\emptyset}}$, respectively, the mass and average interaction value (AIV) (on platform $\mathtt{j}$) of platform $\mathtt{j}$'s *exiting* buyers on the *market expansion margin*, i.e., those who are indifferent between joining platform $\mathtt{j}$ and joining no platform. Denote by $f^{\mathcal{B}}_{\mathtt{j},\mathtt{k}}$ and $\widetilde{\beta^{\mathcal{B}}_{\mathtt{j},\mathtt{k}}}$ the analogous quantities for *switching* buyers on the *cannibalization margin*, i.e., who are indifferent between joining platforms $\mathtt{j}$ and $\mathtt{k}$. Moreover, let $\omega^{\mathcal{B},\mathtt{j}} \equiv \left(1 + \frac{f^{\mathcal{B}}_{\mathtt{j},\emptyset}}{f^{\mathcal{B}}_{\mathtt{j},\mathtt{k}}} + \frac{f^{\mathcal{B}}_{\mathtt{j},\emptyset}}{f^{\mathcal{B}}_{\mathtt{k},\emptyset}}\right)^{-1} \in (0,1)$ denote the side $\mathcal{B}$ *margin weighting* and observe that it is increasing in platform $\mathtt{j}$'s ratio of switching buyers to exiting buyers. Given the independence of demand for the platforms on side $\mathcal{A}$, all of $\mathtt{j}$'s marginal advertisers can effectively be thought of as exiting; denote their AIV by $\widetilde{\beta^{\mathcal{A}}_{\mathtt{j},\emptyset}}$. The following proposition uses these concepts to characterize IE in our media market example.

Proposition 3 (Media Pricing)**.** *In the example with two media platforms described above, at IE, platform* $\mathtt{j}$*'s prices satisfy, for 'buyers',*

$$P^{\mathcal{B},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{B}} + \mu^{\mathcal{B},\mathtt{j}} - N^{\mathcal{A},\mathtt{j}}\widetilde{\beta^{\mathcal{A}}_{\mathtt{j},\emptyset}}, \tag{11}$$

and, for 'advertisers',

$$P^{\mathcal{A},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{A}} + \mu^{\mathcal{A},\mathtt{j}} - N^{\mathcal{B},\mathtt{j}}\left(\omega^{\mathcal{B},\mathtt{j}}\widetilde{\beta^{\mathcal{B}}_{\mathtt{j},\mathtt{k}}} + \left(1-\omega^{\mathcal{B},\mathtt{j}}\right)\widetilde{\beta^{\mathcal{B}}_{\mathtt{j},\emptyset}}\right). \tag{12}$$

Consider first the price charged to buyers, given in (11). The last term reflects the 'discount' that $\mathtt{j}$ offers buyers, with respect to the price it would charge in one-sided Bertrand competition, due to the externalities they create on the opposite side of the market. This discount, however, depends not on the average interaction value of $\mathtt{j}$'s advertisers, as prescribed by (9), the formula for socially optimal pricing. Instead, the discount depends on the AIV of $\mathtt{j}$'s *marginal* advertisers. This is because, when platform $\mathtt{j}$ serves an additional buyer, it can increase the price it charges advertisers by $\widetilde{\beta^{\mathcal{A}}_{\mathtt{j},\emptyset}}$, while holding their number fixed, thus increasing the profits it receives from advertisers by $N^{\mathcal{A},\mathtt{j}} \times \widetilde{\beta^{\mathcal{A}}_{\mathtt{j},\emptyset}}$.

Remark. *The pricing formula in* (11) *holds, more generally, in all cases of our model when platforms' demand on* $\mathcal{A}$ *is independent (i.e. the limit case as* $\frac{\partial N^{\mathcal{A},\mathtt{k}}}{\partial P^{\mathcal{A},\mathtt{j}}}$ *and* $\frac{\partial N^{\mathcal{A},\mathtt{k}}}{\partial N^{\mathcal{B},\mathtt{j}}}$ *approach zero, for all* $\mathtt{k} \neq \mathtt{j}$*). It thus embeds, for example, the monopoly pricing formula of Weyl (2010).*

The price charged to advertisers, given by (12), captures, in the simplest possible way, the extension of the above intuition to cases where there is competition on the opposite side of the market. As it does for buyers, platform $\mathtt{j}$ discounts the price it charges advertisers by an amount equal to the marginal profit each advertiser enables it to extract from its fixed set of buyers. Here, however, since there are two margins of indifferent buyers, the discount given to advertisers depends on a weighted average of the interaction values of buyers on these two margins. A common formalization of an intensification of competition consists in increasing the mass of consumers on the switching margin, while holding fixed the relative size of the two platforms' exiting margins. In such an exercise, the increase in competition on the buyer side of the market decreases $\frac{f^{\mathcal{B}}_{\mathtt{j},\emptyset}}{f^{\mathcal{B}}_{\mathtt{j},\mathtt{k}}}$ while holding fixed $\frac{f^{\mathcal{B}}_{\mathtt{j},\emptyset}}{f^{\mathcal{B}}_{\mathtt{k},\emptyset}}$. Thus, heightening of competition on the buyer side leads to an increase in $\omega^{\mathcal{B},\mathtt{j}}$, causing the discount to advertisers to correspond more closely to the interaction values of the switchers and less to those of the exiters.

Now, let us examine how such an increase in competitiveness affects the Spence distortion. To fix ideas, we consider two extreme subcases of the above example, based on the following assumptions. Regarding buyers' interaction values, assume that, for all i and for $\mathtt{j} = 1, 2$, $\beta_i^{\mathcal{B},\mathtt{j}} = \beta_i^{\mathcal{B}}$. That is, while buyers are heterogenous in their interaction values, each buyer appreciates or dislikes advertisements with the same intensity on either of the two platforms. Regarding buyers' membership values, let $\underline{\epsilon_i^{\mathcal{B}}}$ and $\overline{\epsilon_i^{\mathcal{B}}}$ denote, respectively, the minimum and maximum values for buyer i, for the two platforms (i.e. the min and max of the set $\left\{\epsilon_i^{\mathcal{B},\{1\}}, \epsilon_i^{\mathcal{B},\{2\}}\right\}$), and let $\nu_i^{\mathcal{B}} \equiv \underline{\epsilon_i^{\mathcal{B}}}/\overline{\epsilon_i^{\mathcal{B}}} \leq 1$ denote their ratio. Assume that $\overline{\epsilon_i^{\mathcal{B}}}$ and $\nu_i^{\mathcal{B}}$ are drawn i.i.d across buyers, but that $\overline{\epsilon_i^{\mathcal{B}}}$ may be correlated with $\beta_i^{\mathcal{B}}$ (buyers with high value for the service may value advertising more or less than those with less). However, $\nu_i^{\mathcal{B}}$ is drawn independently of both $\overline{\epsilon_i^{\mathcal{B}}}$ and $\beta_i^{\mathcal{B}}$ (the relative inherent value of platforms is independent of buyer value for advertising).

In the first subcase, competition on the buyer side causes the Spence distortion to decrease. Here, platforms have identical cost functions and are symmetrically differentiated in terms of buyers' membership values. Moreover, suppose that, at IE, the two platforms serve the same number of advertisers, so that $N^{\mathcal{A},1} = N^{\mathcal{A},2} \equiv N^{\mathcal{A}}$, as in Anderson and Coate's model. By symmetry, they will both charge the same price to buyers, which we denote by $P^{\mathcal{B}}$. The set of participating buyers are those for whom

$$\overline{\epsilon_i^{\mathcal{B}}} + \beta_i^{\mathcal{B}} N^{\mathcal{A}} \geq P^{\mathcal{B}} \quad \Leftrightarrow \quad \beta_i^{\mathcal{B}} \geq \frac{P^{\mathcal{B}} - \overline{\epsilon_i^{\mathcal{B}}}}{N^{\mathcal{A}}} \tag{13}$$

holds, while the set of switching buyers are those who participate and for whom $\nu_i^{\mathcal{B}} = 1$. Because $\nu_i^{\mathcal{B}}$ is drawn independently of both $\overline{\epsilon_i^{\mathcal{B}}}$ and $\beta_i^{\mathcal{B}}$, $\mathbb{E}\left[\beta_i^{\mathcal{B}} \mid (13), \nu_i^{\mathcal{B}} = 1\right] = \mathbb{E}\left[\beta_i^{\mathcal{B}} \mid (13)\right] = \overline{v_{\mathtt{j}}^{\mathcal{J},\mathtt{j}}}$, i.e., the AIV among switchers is the same as the AIV among all participating buyers. However, due to the correlation between $\overline{\epsilon_i^{\mathcal{B}}}$ and $\beta_i^{\mathcal{B}}$, it is typically the case that $\mathbb{E}\left[\beta_i^{\mathcal{B}} \mid \beta_i^{\mathcal{B}} = \frac{p^{\mathcal{B}}-\epsilon_i^{\mathcal{B}}}{N^{\mathcal{A}}}\right] \neq \overline{v_{\mathrm{j}}^{\mathcal{J},\mathrm{j}}}$. Therefore, as the distribution of $\nu_i^{\mathcal{B}}$ shifts to place greater density close to one, thereby increasing the weight of the switching margin, the Spence distortion weakens.

While this case is very special, it illustrates the broader idea that when the platforms are differentiated primarily on a dimension orthogonal to externalities, switching buyers are more representative than are exiting buyers of average participating buyers in terms of their interaction values. For example, imagine two competing newspapers differentiated by their political leanings with similar numbers of advertisements. An intensification of their competition for buyers would likely ameliorate the Spence distortion of advertising levels, as well as the standard market power distortion.

In contrast, in the second subcase, in which the platforms are, instead, differentiated by their network effects, competition on the buyer side worsens the Spence distortion. Suppose that, at equilibrium, $N^{\mathcal{A},1} > N^{\mathcal{A},2}$ and that, for all i, $\nu_i = 1$, so the platforms have no *inherent* differentiation for buyers. This endogenous differentiation by externalities may be either vertical, in the spirit of Shaked and Sutton (1982), or horizontal, in the spirit of Hotelling (1929), or it may be a combination of the two. The former description would apply when $\overline{\epsilon_i^{\mathcal{B}}} = 0$ for all buyers, whereas the latter would apply when $\beta_i^{\mathcal{B}}$ is positive for some buyers but negative for others, while $\overline{\epsilon_i^{\mathcal{B}}}$ is a deterministic function of $\beta_i^{\mathcal{B}}$.

Because the platforms are differentiated only by their externalities, any user with $\beta_i^{\mathcal{B}} > \frac{p^{\mathcal{B},1}-p^{\mathcal{B},2}}{N^{\mathcal{A},1}-N^{\mathcal{A},2}}$ will prefer platform 1, over platform 2, and vice versa. Switchers are thus buyers with $\beta_i^{\mathcal{B}} = \frac{p^{\mathcal{B},1}-p^{\mathcal{B},2}}{N^{\mathcal{A},1}-N^{\mathcal{A},2}}$. Note, then, that on platform 1 all infra-marginal buyers and all exiters must have higher interaction values than switchers as they prefer platform 1, and any user preferring platform 1 has a higher interaction value than any switcher. Meanwhile, platform 2's buyers conform to the opposite pattern. Therefore, the average interaction value among each platform's exiters may often be closer than that among switchers to the average $\beta_i^{\mathcal{B}}$ among all of its buyers. For example, if $\beta_i^{\mathcal{B}}$ is (weakly) affiliated with $\overline{\epsilon_i^{\mathcal{B}}}$ then platform 1's average consumers will have higher average interaction values than its exiters, which will in turn be higher than the AIV of switchers. Consequently, platform 1's exiters will be more representative of average users than will switchers.[11]

Thus competition may exacerbate the Spence distortion. The driving mechanism is a manifestation of the phenomenon Hotelling's argument that firms competing in some dimension of product space have an incentive to position themselves too close to one another in this dimension, relative to the social optimum. Despite its specificity, this example thus

[11]Note that in this case platform 2's exiters will have even lower interaction values on average than will the average platform 2 user, which will be less than the average switcher. Thus, it is not clear whether platform 2's exiters or switchers are more representative.

suggests that, whenever externalities are a primary dimension of differentiation among platforms, Hotelling's effect is likely to play an important role. A market matching this intuition may be the one for cable entertainment television stations in the U.S., where basic channels (e.g. TNT, FX) show significant numbers of ads, while premium channels (e.g. HBO, Showtime) offer ad-free programming while charging much higher subscription rates. If this difference in ad intensity were in fact the main dimension of differentiation, intensified competition is likely to exacerbate the Spence distortion.

We now consider two final examples. Unlike the 'media' configuration discussed in Proposition 3 and in the discussion thereafter, we first allow for 'single-homing' competition on both sides of the market. Proposition 4 states platform $\mathtt{j}$'s equilibrium price under this structure of preferences. Given that platforms' conduct is Nash-in-prices *within sides*, in general, when $\mathtt{j}$ changes its price on side $\mathcal{I}$, there will be a change in the entire coarse allocation on this side of the market. Thus, the amount by which platform $\mathtt{j}$ must change its price on side $\mathcal{J}$, in order to hold fixed $N^{\mathcal{J},\mathtt{j}}$, depends on its side $\mathcal{I}$ *diversion ratios*. In the case of two platforms, $\mathtt{j}$'s diversion ratio, $D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}} \equiv \frac{\partial N^{\mathcal{I},\mathtt{k}}}{\partial P^{\mathcal{I},\mathtt{j}}} / \left(-\frac{\partial N^{\mathcal{I},\mathtt{j}}}{\partial P^{\mathcal{I},\mathtt{j}}}\right) = \frac{f^{\mathcal{I}}_{\mathtt{j},\mathtt{k}}}{f^{\mathcal{I}}_{\mathtt{j},\emptyset}+f^{\mathcal{I}}_{\mathtt{j},\mathtt{k}}} \in (0,1)$, is simply the fraction of its side $\mathcal{I}$ demand that goes to platform $\mathtt{k}$ when it increases its own side $\mathcal{I}$ price (holding fixed $\boldsymbol{N}^{\mathcal{J}}$).

Proposition 4 (Two-Sided Single-Homing)**.** *With two competing platforms and single-homing consumers on both sides of the market, platform* $\mathtt{j}$*'s IE price on side* $\mathcal{I}$ *satisfies*

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} + \mu^{\mathcal{I},\mathtt{j}} - N^{\mathcal{J},\mathtt{j}} \times \left(\underbrace{\left(1-\omega^{\mathcal{J},\mathtt{j}}\right)\widetilde{\beta^{\mathcal{J}}_{\mathtt{j},\emptyset}}}_{\textit{same as in } (12)} + \omega^{\mathcal{J},\mathtt{j}} \left(\underbrace{\widetilde{\beta^{\mathcal{J}}_{\mathtt{j},\mathtt{k}}}}_{\textit{switchers' AIV on } \mathtt{j}} + D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}} \left(\underbrace{\widetilde{\beta^{\mathcal{J}}_{\mathtt{k},\mathtt{j}}}}_{\textit{switchers' AIV on } \mathtt{k}} - \underbrace{\widetilde{\beta^{\mathcal{J}}_{\mathtt{k},\emptyset}}}_{\mathtt{k}\textit{'s exiters' AIV}} \right)\right)\right). \quad (14)$$

Similarly to (12), the pricing formula in (14) dictates that the 'externality discount' side $\mathcal{I}$ users receive depends on the AIVs of marginal side $\mathcal{J}$ users. Moreover, as the market on side $\mathcal{J}$ becomes more competitive, through an increase in $\omega^{\mathcal{J},\mathtt{j}}$, the AIV of $\mathtt{j}$'s exiters, $\widetilde{\beta^{\mathcal{J}}_{\mathtt{j},\emptyset}}$, plays a smaller role and the AIVs of marginal consumers for whom platform $\mathtt{k}$ is an optimal choice carry more weight. Note, however, the presence of the term $D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}}\left(\widetilde{\beta^{\mathcal{J}}_{\mathtt{k},\mathtt{j}}} - \widetilde{\beta^{\mathcal{J}}_{\mathtt{k},\emptyset}}\right)$, which does not appear in (12). This enters here, since, when there is competition on side $\mathcal{I}$ and $\mathtt{j}$ changes its price-quantity pair on this side, such a shift affects the number of consumers that $\mathtt{k}$ serves as well. The diversion ratio represents the significance of this shift on side $\mathcal{I}$. This reallocation of side $\mathcal{I}$ consumers influences the perceived quality by side $\mathcal{J}$ consumers of not only platform $\mathtt{j}$ but also of platform $\mathtt{k}$. As a result, in order to hold fixed its quantity on side $\mathcal{J}$, $\mathtt{j}$ must take into account the interaction values of $\mathtt{k}$'s marginal consumers for an additional interaction on that platform.

In particular, the relevant quantity for this purpose is the *difference* between the AIV *on platform* k of switchers and the AIV of k's exiters. Thus, the extent to which platform j distorts the quality it provides to its side $\mathcal{J}$ consumers depends not only on the divergence between the interaction values of its own marginal versus average consumers but also on the distribution of such valuations among consumers on other platforms. As competition on side $\mathcal{I}$ toughens through an increase in $D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}}$, it can bring the platform's incentives closer to or further from the social planner's, according to the heterogeneity issues we discuss above.

When generalized to the case of m symmetric platforms, the symmetric IE pricing formula takes a form that is analogous to that of (14), and an analysis similar to the above applies. We now show this in Proposition 5. Let $f^{\mathcal{J}}_{\mathtt{k}}$ and $\widetilde{\beta^{\mathcal{J}}_{\mathtt{k}}}$ denote, respectively, the mass and AIV of side $\mathcal{J}$ consumers on a given platform's cannibalization margin, and let $f^{\mathcal{J}}_{\emptyset}$ and $\widetilde{\beta^{\mathcal{J}}_{\emptyset}}$ denote the mass and AIV of side $\mathcal{J}$ consumers on a given platform's market expansion margin. Let $D^{\mathcal{I}}$ denote the common diversion ratio between any two platforms on side $\mathcal{I}$, and let $\omega^{\mathcal{J}} \equiv \frac{(m-1)f^{\mathcal{J}}_{\mathtt{k}}}{f^{\mathcal{J}}_{\emptyset}+mf^{\mathcal{J}}_{\mathtt{k}}} \in \left(0, \frac{m-1}{m}\right)$ denote the margin weighting on side $\mathcal{J}$, adapted appropriately to this case.

Proposition 5 (m Symmetric Platforms)**.** *When there are m symmetric platforms, at symmetric IE, each platform's price on side $\mathcal{I}$ satisfies*

$$P^{\mathcal{I}} = C_{\mathcal{I}} + \mu^{\mathcal{I}} - N^{\mathcal{J}}\left(\left(1-\omega^{\mathcal{J}}\right)\widetilde{\beta^{\mathcal{I}}_{\emptyset}} + \omega^{\mathcal{J}}\left(\widetilde{\beta^{\mathcal{I}}_{\mathtt{k}}} + D^{\mathcal{I}}\left(\widetilde{\beta^{\mathcal{I}}_{\mathtt{k}}} - \widetilde{\beta^{\mathcal{I}}_{\emptyset}}\right)\right)\right). \tag{15}$$

6.3 Pricing at Insulated Equilibrium: the general case

Thus far in this section, we have established the benchmark formula for socially optimal pricing and illustrated IE pricing through several examples, drawing special attention to the Spence distortion that arises when consumers have heterogenous interaction values. We now illustrate the way that IE pricing carries through in the unrestricted model. Theorem 3 states the general IE pricing formula. Let $\boldsymbol{D}^{\mathcal{I}}$ denote the matrix of diversion ratios on side $\mathcal{I}$, with generic element $D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}} \equiv \frac{\partial N^{\mathcal{I},\mathtt{k}}}{\partial P^{\mathcal{I},\mathtt{j}}} / \left(-\frac{\partial N^{\mathcal{I},\mathtt{j}}}{\partial P^{\mathcal{I},\mathtt{j}}}\right)$, as defined above.

Theorem 3. *At an IE allocation, the total price platform* j *charges to side $\mathcal{I}$ consumers satisfies*

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} + \mu^{\mathcal{I},\mathtt{j}} - N^{\mathcal{J},\mathtt{j}}\left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right]\right)_{\mathtt{j},\cdot} \cdot \left[-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathtt{j}}\right] \quad . \tag{16}$$

Proof. In view of equation (7), it remains for us to show that $\frac{\partial \overline{P^{\mathcal{J},\mathtt{j}}}}{\partial \boldsymbol{N}^{\mathcal{I}}} = \left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right]\right)_{\mathtt{j},\cdot}$. At IE, platforms' tariffs constitute an Insulating Tariff System. Thus, by definition, in response to changes in the side $\mathcal{I}$ coarse allocation, prices on side $\mathcal{J}$ adjust so as to hold the side $\mathcal{J}$ coarse allocation unchanged. Therefore, the Jacobian of the Insulating Tariff System satisfies

$$\underbrace{\mathbf{0}}_{m\times m} = \left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right] + \left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]\left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right] \qquad \Leftrightarrow \qquad \left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right] = \left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right], \tag{17}$$

and we have our result. □

The first three terms in equation (16) arise in any standard differentiated products Nash-in-prices model, and in all of the special cases we discuss above. It is therefore only the last term that is of significant interest. As we argue in the special cases above, the factors multiplying $N^{\mathcal{J},\mathtt{j}}$ are closely related to Spence's average marginal value of quality to marginal users. We now pursue this analogy in more detail in the general case.

This term consists of two factors. The simpler of the two factors is the negative of the $\mathtt{j}^{th}$ column of the diversion ratio matrix, $-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathtt{j}}$. Under Nash-in-Prices competition, a decrease in price by platform j, on side $\mathcal{I}$, triggers not only an increase in j's quantity, but also a fall in all other firms' quantities (or a rise, for platforms that are complementary to j for most marginal users). Consequently, such a decrease in price by j affects the characteristics on side $\mathcal{J}$, of *all* platforms. $-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathtt{j}}$ measures this change in $\boldsymbol{N}^{\mathcal{I}}$.

The other term, $\left(\left[-\frac{\partial N^{\mathcal{J}}}{\partial P^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial N^{\mathcal{J}}}{\partial N^{\mathcal{I}}}\right]\right)_{\mathtt{j},\cdot}$, is the $\mathtt{j}^{th}$ row of the Jacobian of the Insulating Tariff System on side $\mathcal{J}$. The $\mathtt{k}^{th}$ element in this vector measures the amount by which platform j must change its price on side $\mathcal{J}$, in response to a unit of change in $N^{\mathcal{I},\mathtt{k}}$ and the accompanying price changes on side $\mathcal{J}$ by other platforms, in order to perfectly compensate its average marginal consumer (see Section Section 5.2), thus holding fixed $N^{\mathcal{J},\mathtt{j}}$. To see the way in which this factor generalizes the Spencian ideas discussed above, it is useful to break it into two parts,

$$\underbrace{\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}}_{\text{inverse of density of marginal users (DMU)}} \cdot \underbrace{\left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right]}_{\text{DMU} \times \text{average marginal utilities}} .$$

The inverse of the first component, $-\frac{\partial N^{\mathcal{J}}}{\partial P^{\mathcal{J}}}$, has diagonal elements equal to the density of users indifferent between consuming a bundle including platform k and a bundle not including platform k. Its off-diagonal elements equal the difference between

(a) the density of users who are indifferent between a bundle including platform k but not j and one including j but not k (the density of switchers) and

(b) the density of users indifferent between a bundle including *both* platforms `j` and `k` and a bundle including *neither* `j` nor `k` (the density of marginal consumers for whom `j` and `k` are complements).

Thus the first component in the above expression is the inverse of the matrix representing the density of marginal users. The second component is much like the first, except that the densities of the relevant marginal groups are multiplied by each marginal group's *average marginal utility, conditional on participating on platform* `j`*, for having an additional user on side* $\mathcal{I}$ *of platform* `j`, a natural generalization of the interaction values discussed in the examples of the previous subsection. Thus the product of the two factors is a natural extension of the *average value to the marginal user* of the Spence (1975) and Weyl (2010) analyses.

7 Applications and Extensions

7.1 First-order merger analysis

In this section, we extend the techniques of Jaffe and Weyl (2011) (JW) to consider the effect on consumer surplus of a potential merger of platforms. The key to this extension is that we must take into account not only the potential harms or benefits to consumers from the changes *in* the (insulating) tariffs charged due to the merger, but also the welfare effects of movements *along* these insulating tariffs caused by changes in the degree of externalities generated due to the change in the rate of consumer participation induced by these price changes. As JW argues, so long as the induced change in price is small, the first effect may be measured using the standard Jevons (1871)-Hotelling (1938) rule: $-\Delta p \cdot q$. The second effect consists of two parts: the harms caused by the increased prices charged for increased externalities, and the benefits brought by these increased externalities themselves. Because the former depend on the benefits derived only by marginal users and the latter depend on the benefits delivered only to average users, the difference between these closely resembles the Spence distortion. The total local approximation may thus be written as a sum of Jevons-Hotelling effects and Spence effects, multiplied by the number of consumers experiencing these:

$$\begin{bmatrix} \left(\overline{\boldsymbol{V}^{\mathcal{A}}_{\mathcal{B}}} - \frac{\partial \overline{\boldsymbol{P}^{\mathcal{A}}}}{\partial \boldsymbol{N}^{\mathcal{B}}}\right) \frac{\partial \boldsymbol{N}^{\mathcal{B}}}{\partial \boldsymbol{P}^{\mathcal{B}}} \Delta \boldsymbol{P}^{\mathcal{B}} - \Delta \boldsymbol{P}^{\mathcal{A}} \\ \left(\overline{\boldsymbol{V}^{\mathcal{B}}_{\mathcal{A}}} - \frac{\partial \overline{\boldsymbol{P}^{\mathcal{B}}}}{\partial \boldsymbol{N}^{\mathcal{A}}}\right) \frac{\partial \boldsymbol{N}^{\mathcal{A}}}{\partial \boldsymbol{P}^{\mathcal{A}}} \Delta \boldsymbol{P}^{\mathcal{A}} - \Delta \boldsymbol{P}^{\mathcal{B}} \end{bmatrix}^T \cdot \begin{bmatrix} \boldsymbol{N}^{\mathcal{A}} \\ \boldsymbol{N}^{\mathcal{B}} \end{bmatrix}, \tag{18}$$

where $\Delta \boldsymbol{X}$ denotes the (small) difference between the pre- and post-merger value of vector $\boldsymbol{X}$, under IE. The change in prices counted here is only the *directly* induced change, that is change *in* the insulating tariff, not *along* it. The matrix $\overline{\boldsymbol{V}_{\mathcal{I}}^{\mathcal{J}}}$ is diagonal and has generic diagonal element $\overline{v_{\mathtt{j}}^{\mathcal{I},\mathtt{j}}}$, which, recall from Section 6, denotes the average valuation for an additional interaction among the set of *all* side $\mathcal{I}$ consumers on platform $\mathtt{j}$. From our discussion in Section 6, it should not be surprising that the Jacobian matrix of the insulating tariff $\frac{\partial \overline{\boldsymbol{P}^{\mathcal{I}}}}{\partial \boldsymbol{N}^{\mathcal{J}}}$ plays a role, under oligopoly, that is analogous to that of the average value of marginal consumers, under monopoly.

Calculating the appropriate extension of Farrell and Shapiro (2010) (FS)'s *Upward Pricing Pressure* (UPP) to this context is relatively straightforward.[12] The first term is exactly as in FS, the value (in terms of profits, that is the mark-up) of sales of platform $\mathtt{k}$ diverted as a result of one more slot on platform $\mathtt{j}$ being filled. The second, novel term arises from the fact that, post-merger, platform $\mathtt{j}$ must now consider not only how increasing its participation positively impacts the externalities for which it can charge consumers on the other side of the market but also how it negatively impacts the externalities for which the *merger partner* can charge on the other side. Without loss of generality, we assume the merger occurs between platforms $\mathtt{1}$ and $\mathtt{2}$; in this case the UPP vector is given by

$$\tau^{\mathcal{I},\mathtt{j}} = \begin{cases} D^{\mathcal{I}}_{\mathtt{k},\mathtt{j}}\left(P^{\mathcal{I},\mathtt{k}} - C^{\mathtt{k}}_{\mathcal{I}}\right) + N^{\mathcal{J},\mathtt{k}}\left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J},\mathtt{k}}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right] \times \left[\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathtt{j}}\right], & \text{if } \mathtt{j},\mathtt{k} = \mathtt{1},\mathtt{2},\ \mathtt{k} \neq \mathtt{j} \\ 0, & \text{if } \mathtt{j} \neq \mathtt{1},\mathtt{2} \end{cases}.$$

JW shows that if sufficient technical conditions are satisfied (for example, the pre- and post-merger equilibria must be stable in a strong sense),

$$\Delta \boldsymbol{P} \approx \left(\boldsymbol{\rho}^{-1} - \nabla_P \boldsymbol{\tau}\right)^{-1} \boldsymbol{\tau},$$

where all quantities are evaluated at the pre-merger allocation, $\boldsymbol{\rho}$ is the matrix of *pass-through rates* of marginal costs to prices, and ∇_P is the gradient with respect to price. The error in the approximation is of the order of a quadratic form of $\boldsymbol{\tau}$ (very small if $\boldsymbol{\tau}$ is small) and is also small if the post-merger equilibrium conditions are sufficiently smooth (in prices). This formula is perfectly analogous to the one pertaining to the standard markets that JW considers, with the exception of what enters into the determination of $\boldsymbol{\tau}$. These local approximations of price changes may then be inserted into expression (18), where all other terms in that expression are also evaluated at the pre-merger allocation, to obtain a first-order approximation to the full effect of the merger on consumer welfare. Note that we may also obtain independent approximations of the effect of a merger on each side's welfare by simply evaluating each side independently, rather than summing over the two.

[12]Note that because we assume Bertrand conduct, we need consider only UPP and not JW's generalization of it, GePP, which allows more general conduct.

To summarize, we extend JW's formula (quantities multiplied by pass-through, multiplied by the value of diverted sales) in two ways:

1. The value of diverted sales is extended to include the *full opportunity cost* of those diverted sales in a two-sided setting. This value takes into account both the direct mark-up that diverted sales bring on the side of the market in question and their impact on each merging platform's ability to extract value from externalities, as perceived by marginal consumers, on the opposite side of the market.

2. The effects of the predicted price changes are also accounted for via their impact on participation and, consequently, on the Spence distortion experienced by users on the other side.

7.2 Generalizations

Many sides of the market. Thus far, for expository purposes, we have focused on market configurations with two 'sides' or groups of consumers. The model easily extends to accommodate an arbitrary number of sides. To see this, suppose there are S groups of consumers, indexed by $\mathcal{I} = \mathcal{A}, \mathcal{B}, \mathcal{C}, ...$, and let the gross payoff of joining a bundle of platforms, $\mathscr{X}$, to a consumer of type $\boldsymbol{\theta}^{\mathcal{I}}$ on side $\mathcal{I}$ be $v^{\mathcal{I}}\left(\mathscr{X}, \boldsymbol{N}^{-\mathcal{I}}, \boldsymbol{\theta}^{\mathcal{I}}\right)$, where $v^{\mathcal{I}} : \wp(\mathscr{M}) \times [0,1]^{m(S-1)} \times \Theta^{\mathcal{I}} \to \mathbb{R}$ now depends on $\boldsymbol{N}^{-\mathcal{I}} \in [0,1]^{m(S-1)}$, the coarse allocation on the $S-1$ other sides of the market apart from side $\mathcal{I}$. Also, let platform $\mathtt{j}$'s strategy now be given by $\sigma^{\mathtt{j}} \equiv \left(\sigma^{\mathcal{A},\mathtt{j}}\left(\boldsymbol{N}^{-\mathcal{A}}\right), \sigma^{\mathcal{B},\mathtt{j}}\left(\boldsymbol{N}^{-\mathcal{B}}\right), \sigma^{\mathcal{C},\mathtt{j}}\left(\boldsymbol{N}^{-\mathcal{C}}\right), ...\right)$, where $\sigma^{\mathcal{I},\mathtt{j}} : [0,1]^{m(S-1)} \to \mathbb{R}$ maps from $\boldsymbol{N}^{-\mathcal{I}} \in [0,1]^{m(S-1)}$ to a total price that side $\mathcal{I}$ consumers pay to join platform $\mathtt{j}$.

It is straightforward to see that, when the model is extended in this way, none of the arguments made thus far in the paper depend on the presence of only two sides. In particular, the result of Lemma 1, that a coarse allocation implies a unique price vector, continues to hold. Thus, the simplest way to consider a platform's profit maximization problem continues to be as a choice of allocation, holding fixed the strategies of the other platforms

$$\max_{\left\{N^{\mathcal{A},\mathtt{j}}, N^{\mathcal{B},\mathtt{j}}, N^{\mathcal{C},\mathtt{j}}, ...\right\}} \sum_{\mathcal{I}=\mathcal{A},\mathcal{B},\mathcal{C},...} N^{\mathcal{I},\mathtt{j}} P^{\mathcal{I},\mathtt{j}}\left(N^{\mathcal{I},\mathtt{j}}, \boldsymbol{N}^{-\mathcal{I},\mathtt{j}}\right) - C^{\mathtt{j}}\left(N^{\mathcal{A},\mathtt{j}}, N^{\mathcal{B},\mathtt{j}}, N^{\mathcal{C},\mathtt{j}}, ...\right). \tag{19}$$

Analogously to the results of Section 6.3, the prices that implement the socially optimal allocation satisfy

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} - \sum_{\mathcal{J} \neq \mathcal{I}} N^{\mathcal{J},\mathtt{j}} \overline{v^{\mathcal{J},\mathtt{j}}_{\mathtt{j}}}, \tag{20}$$

and the platforms' prices under IE satisfy

$$P^{\mathcal{I},\mathrm{j}} = C^{\mathrm{j}}_{\mathcal{I}} + \mu^{\mathcal{I},\mathrm{j}} - \sum_{\mathcal{J}\neq\mathcal{I}} N^{\mathcal{J},\mathrm{j}} \left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}} \right]^{-1} \left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}} \right] \right)_{\mathrm{j},\cdot} \cdot \left[-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathrm{j}} \right] . \tag{21}$$

The only difference between these expressions and those discussed in Section 6 is that here, since there are S sides of the market, the number of consumers on side $\mathcal{I}$ affects the payoffs of consumers on all of the $S-1$ other sides. Consequently, the prices charged to side $\mathcal{I}$ consumers under both the socially optimal allocation and the IE allocation take into account the sum of such externalities, with the latter still subject to both the market power and Spence distortions.

Within-side externalities. Until now, we have also assumed that consumers' preferences over platforms are independent of the number of consumers *of the same group* that join each platform. This section extends the model to allow for such within-side network effects, which play a significant role in many industries, such as the provision of mobile phone service and social networking websites. Note that, while our focus is indeed on competition among *multi-sided* platforms, embedded in this generalization is the case, where $S = 1$, of competition among one-sided network providers, as in the literature stemming from the seminal paper of Rohlfs (1974).

When joining a bundle of platforms, $\mathscr{X}$, a consumer of type $\boldsymbol{\theta}^{\mathcal{I}}$ on side $\mathcal{I}$ receives gross payoff $v^{\mathcal{I}}\left(\mathscr{X}, \boldsymbol{N}, \boldsymbol{\theta}^{\mathcal{I}}\right)$, where $v^{\mathcal{I}} : \wp(\mathscr{M}) \times [0,1]^{mS} \times \Theta^{\mathcal{I}} \to \mathbb{R}$ depends on $\boldsymbol{N}$, the *entire* coarse allocation.

We extend platforms' strategy space in the way that allows for the solution concept of IE to be most naturally preserved. Let platform j's strategy be given by $\sigma^{\mathrm{j}} \equiv \left(\sigma^{\mathcal{A},\mathrm{j}}(\boldsymbol{N}), \sigma^{\mathcal{B},\mathrm{j}}(\boldsymbol{N}), \sigma^{\mathcal{C},\mathrm{j}}(\boldsymbol{N}), ...\right)$, where $\sigma^{\mathcal{I},\mathrm{j}} : [0,1]^{mS} \to \mathbb{R}$ maps from $\boldsymbol{N}$, the entire coarse allocation of consumers, including on side $\mathcal{I}$, to a total price.

When there are within-side network externalities, in order to speak of Insulating Tariffs, it becomes convenient to introduce the notion of consumers' *beliefs*, as discussed by Katz and Shapiro (1985),[13] about the strategies of other consumers on the same side. Suppose that prior to choosing their actions, side $\mathcal{I}$ consumers form beliefs about one another's strategies, which, for our purposes, it is not restrictive to assume are common among all consumers. Formally, let $\ddddot{\boldsymbol{N}}^{\mathcal{I}}$ denote the coarse allocation on side $\mathcal{I}$ that side $\mathcal{I}$ consumers believe will prevail in the given consumer game. It is apparent that the best-response strategy profile of side $\mathcal{I}$ consumers depends on $\ddddot{\boldsymbol{N}}^{\mathcal{I}}$. Residual Insulating Tariffs, whose definition we now restate, adapted to this context, pin down a unique value of $\ddddot{\boldsymbol{N}}^{\mathcal{I}}$ and thus a unique value of $\boldsymbol{N}^{\mathcal{I}}$ that is consistent with Consumer Nash Equilibrium.

[13] The object that we refer to as 'beliefs' is, in fact, called 'expectations' by Katz and Shapiro (1985) but is given the former name in more recent literature such as Caillaud and Jullien (2003).

Definition 8. *Given a profile of strategies of other platforms,* $\boldsymbol{\sigma}^{-\mathtt{j}}$*, platform* $\mathtt{j}$ *is said to charge a* Residual Insulating Tariff *on side* $\mathcal{I}$ *if* $\forall\, \overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}, \widetilde{\overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}} \in [0,1]^m$ *and* $\forall\, \boldsymbol{N}^{-\mathcal{I}}, \widetilde{\boldsymbol{N}^{-\mathcal{I}}} \in [0,1]^{m(S-1)}$

$$N^{\mathcal{I},\mathtt{j}}\left[\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \left(\overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}, \boldsymbol{N}^{-\mathcal{I}}\right), [\sigma]\right), \boldsymbol{\sigma}\right] = N^{\mathcal{I},\mathtt{j}}\left[\mathscr{M}^{\mathcal{I}*}\left(\boldsymbol{\theta}^{\mathcal{I}}, \left(\widetilde{\overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}}, \widetilde{\boldsymbol{N}^{-\mathcal{I}}}\right), [\sigma]\right), \boldsymbol{\sigma}\right].$$

As before, all platforms announcing Residual Insulating Tariffs on all sides of the market gives rise to an Insulating Tariff System, $\overline{\boldsymbol{P}}(\boldsymbol{N})$, anchored at a reference allocation. IE thus continues to be defined as in Definition 7, and the shape of the Insulating Tariff System, in response to variation in the own side coarse allocation, is pinned down by the equation

$$0 = \left[\frac{\partial \boldsymbol{N}^{\mathcal{I}}}{\partial \boldsymbol{P}^{\mathcal{I}}}\right]\left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{I}}}}{\partial \overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}}\right] + \left[\frac{\partial \boldsymbol{N}^{\mathcal{I}}}{\partial \overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}}\right] \Leftrightarrow \left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{I}}}}{\partial \overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}}\right] = \left[-\frac{\partial \boldsymbol{N}^{\mathcal{I}}}{\partial \boldsymbol{P}^{\mathcal{I}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{I}}}{\partial \overset{\cdots\cdot}{\boldsymbol{N}^{\mathcal{I}}}}\right]. \tag{22}$$

When platforms' tariffs satisfy equation (22), for *any* beliefs that side $\mathcal{I}$ consumers might have, prices adjust to maintain a given CNE coarse allocation. Therefore, there is a unique coarse allocation that consumers can consistently believe will occur in equilibrium.

Platform $\mathtt{j}$'s profit maximization problem continues to be given by expression (19), and the prices that arise under the socially optimal allocation and the IE allocation are, respectively,

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} - \sum_{\mathcal{J}=\mathcal{A},\mathcal{B},\mathcal{C}\ldots} N^{\mathcal{J},\mathtt{j}}\overline{v^{\mathcal{J},\mathtt{j}}_{\mathtt{j}}} \tag{23}$$

and

$$P^{\mathcal{I},\mathtt{j}} = C^{\mathtt{j}}_{\mathcal{I}} + \mu^{\mathcal{I},\mathtt{j}} - \sum_{\mathcal{J}=\mathcal{A},\mathcal{B},\mathcal{C}\ldots} N^{\mathcal{J},\mathtt{j}}\left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{I}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{N}^{\mathcal{I}}}\right]\right)_{\mathtt{j},\cdot} \cdot \left[-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathtt{j}}\right]. \tag{24}$$

Notice that the only difference between equations (23) and (24), corresponding to the case where there are within-side externalities, and equations (20) and (21), corresponding to the case without such effects, is in the final term. When there are within-side externalities and firm $\mathtt{j}$ changes the number of consumers it serves on side $\mathcal{I}$, this affects the quality of $\mathtt{j}$ as perceived by side $\mathcal{I}$ consumers in addition to consumers of all other sides.

7.3 Application to an affine discrete choice model

An important motivation for our work is to allow the incorporation of consumption externalities into structural empirical research. While much of the remaining work in this direction belongs with empirical applications, such as Song (2011), a few features of an appropriate model can be deduced from theoretical, rather than econometric, considerations. We thus briefly discuss the connections between the characteristics-based approach to demand estimation, which has become popular in recent years, and the key quantities in our model.

Consider the preferences we use in Subsection 6.2, in the case of 'single-homing' typically, which is typically assumed in discrete choice models: $\epsilon_i^{\mathcal{I},\mathrm{j}} + \beta_i^{\mathcal{I}} N^{\mathcal{J},\mathrm{j}}$. If platform j has other (observable and unobservable) characteristics $\boldsymbol{X}^{\mathrm{j}}$ valued by user i on side $\mathcal{I}$ in the standard affine manner with weightings $\boldsymbol{\alpha}_i^{\mathcal{I}}$, then we can rewrite these preferences as

$$\boldsymbol{\alpha}_i^{\mathcal{I}} \cdot \boldsymbol{X}^{\mathrm{j}} + \beta_i^{\mathcal{I}} N^{\mathcal{J},\mathrm{j}} + \epsilon_{i,\mathrm{j}}^{\mathcal{I}},$$

where $\epsilon_{i,\mathrm{j}}^{\mathcal{I}} \equiv \epsilon_i^{\mathcal{I},\mathrm{j}} - \boldsymbol{\alpha}_i^{\mathcal{I}} \cdot \boldsymbol{X}^{\mathrm{j}}$ to emphasize the symmetry with the commonly-employed Berry (1994) preferences if $\epsilon_{i,\mathrm{j}}^{\mathcal{I}}$ is assumed to take a Type I Extreme Value distribution. $\beta_i^{\mathcal{I}}$ is just, then, another standard random coefficient, but plays a particularly prominent role and thus is separated out. Unlike in standard one-sided applications, such as BLP, where characteristics and random coefficients are used primarily to parsimoniously match substitution patterns, here they are of independent economic interest, as in Mazzeo (2002), since they measure the consumption externality of focal interest. Second, allowing for heterogeneity in $\beta_i^{\mathcal{I}}$ (random coefficients) is now not merely a useful way of matching substitution patterns better than would be done by a standard logit model: it is now necessary to allow for heterogeneous valuation of consumption externalities and thus measurement of the Spence distortion. Thus accurate measurement of this heterogeneity becomes particularly crucial.

An important problem in estimating this heterogeneity is the strong likelihood of correlations between $N^{\mathcal{J},\mathrm{j}}$ and unobserved platform characteristics, given that the demand equation on side $\mathcal{J}$ will likely also load on these unobserved characteristics. However, it seems likely that standard instruments for *side* $\mathcal{J}$ prices could be used for side $\mathcal{J}$ *participation* $N^{\mathcal{J},\mathrm{j}}$ as well. Additionally, because of the determinacy leant to first-order conditions by IE, simultaneous estimation of supply and demand relations, as in BLP, is again possible, augmenting identification. Finally, note that from a computational perspective, while the pricing equations have an additional 'two-sided' term, the only substantive requirement for imposing these equations, compared to the one-sided case, is to make use of the derivatives of each platform's market share, not only with respect to own and other platforms' prices, but also with respect to opposite side *participation*. Evaluating such a derivative is a substantively, and thus we suspect computationally, analogous exercise: the price derivative effectively involves computing the average value of $\alpha_i^{\mathcal{I}}$ along the set of marginal consumers, while the participation derivative involves computing the average value of $\beta_i^{\mathcal{I}}$ (both involve also computing the size/density of the marginal set). In contrast, as discussed in Song (2011), when platforms are assumed to charge 'flat' (non-participation-contigent) prices, generically there exist no explicit expressions for equilibrium prices as functions of the demand system and platforms' marginal costs. As a result, the researcher must make additional assumptions in order to approximate the elasticity of demand on one side of the market with respect to *price* on the opposite side.

8 Conclusion

This paper aspires to make three contributions. First, it develops a model with generality comparable to that of standard static industrial organization models, but incorporating the 'multi-sided platforms' features of consumption externalities. Second, it develops a conceptual approach, based on collapsing the inherently dynamic pricing process characteristic of platform markets into the static *Insulated Equilibrium*, that allows this broad model to be analyzed. Finally, it shows how a natural extension of the logic of Spence (1975) can be used to understand both the distortions created by oligopolistic market power and the capacity of competition to remedy these.

While we believe this constitutes one important step forward in the literature on multi-sided platforms, much remains to be done. We therefore now briefly discuss some of the extensions that we consider to be most promising. One interesting avenue would be more detailed treatment of multi-homing and other sources of heterogeneity of externalities across users within a side of the market. If third-degree price discrimination is possible to all groups bringing different externalities either exogenously or endogenously through their choice of platforms (it may be less effective to advertise to a reader who has already seen the advertisement in another paper), it is relatively straightforward to extend our model to allow such externality or bundle-contingent contracts by simply increasing the number of sides. However, we did not explicitly discuss this above because it is not very realistic in many settings. More promising, therefore, is the prospect of combining into our model the analysis of Veiga and Weyl (2011) which allows within-side heterogeneity while still permitting rich preference heterogeneity.

We believe a number of other substantive issues could be usefully analyzed in our framework. These include regulation, such as price and quantity controls that are relevant in, for example, the analysis of network neutrality policies and cases under which symmetric platform equilibria are stable or in which they are likely to tip toward dominance by a single platform. Perhaps most importantly, a workhorse parametric version of the model in the spirit of BLP, as outlined in Section 7.3, would be valuable for applied research. On a more technical level, a more detailed analysis of conditions for existence, stability and uniqueness of IE would be useful.

Our solution concept also seems naturally connected to a number of other problems in economics; elucidating these connections would help unify these areas. In particular, as Bulow and Roberts (1989) shows, Weyl's (2010) model is equivalent to Segal's (1999) general model of contracting with externalities with asymmetric information. Thus it seems natural that our model should be closely related to common agency with multiple agents, externalities and asymmetric information. It would therefore be interesting to consider whether IE has a natural analogy to solution concepts invoked in that literature, or whether it offers a potential alternative concept. At a deeper theoretical level, it would be interesting to understand more clearly the dynamic incentives of multi-sided platforms, extending the work of Cabral (2011) to competition and richer heterogeneity.

On the applied side, we believe our paper offers a number of tools that make possible a range of interesting empirical analyses of multi-sided platforms, measuring market power and Spence distortions and predicting counter-factual effects of policy interventions, which we hope will develop in coming years. Making the theory of multi-sided platforms useful to policy makers will also require enriching our model to consider issues that are beyond the scope of this paper such as interconnection, vertical restraints, bundling, predation and regulatory design. We are thus hopeful that the theory and measurement of multi-sided platform industries will be increasingly put to use in helping to clarify an important and often ideologically-driven set of industrial policy debates.

Appendices

A Proofs of Propositions Appearing in Section 6

Proof of Proposition 1: Pricing at the Socially Optimal Allocation. Differentiating (8) with respect to $N^{\mathcal{I},\mathrm{j}}$ yields first-order condition for maximizing social welfare, $\frac{\partial V^{\mathcal{I}}}{\partial N^{\mathcal{I},\mathrm{j}}} + \frac{\partial V^{\mathcal{J}}}{\partial N^{\mathcal{I},\mathrm{j}}} - C^{\mathrm{j}}_{\mathcal{I}} = 0$. By Lemma 2, we have $\frac{\partial V^{\mathcal{I}}}{\partial N^{\mathcal{I},\mathrm{j}}} = P^{\mathcal{I},\mathrm{j}}$, and the numerator in $\overline{v^{\mathcal{I},\mathrm{j}}_{\mathrm{j}}}$ is equal to $\frac{\partial V^{\mathcal{J}}}{\partial N^{\mathcal{I},\mathrm{j}}}$. Thus we have the condition given by (9). □

Proof of Proposition 2: Generalized Armstrong Pricing. Under GAPs, at IE, the common interaction value among side $\mathcal{J}$ consumers is $\widetilde{\beta^{\mathcal{J}}}$. Therefore, part (a) follows from noting that the last term in the general IE pricing formula of Theorem 3,

$$N^{\mathcal{J},\mathrm{j}} \left(\left[-\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}} \right]^{-1} \left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}} \right] \right)_{\mathrm{j},\cdot} \cdot \left[-\mathbf{D}^{\mathcal{I}}_{\cdot,\mathrm{j}} \right], \tag{25}$$

simplifies to

$$N^{\mathcal{J},\mathrm{j}}\left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}\widetilde{\beta^{\mathcal{J}}}\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]\right)_{\mathrm{j},\cdot}\cdot\left[-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathrm{j}}\right]=N^{\mathcal{I},\mathrm{j}}\widetilde{\beta^{\mathcal{J}}},$$

since $\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]^{-1}\widetilde{\beta^{\mathcal{J}}}\left[-\frac{\partial \boldsymbol{N}^{\mathcal{J}}}{\partial \boldsymbol{P}^{\mathcal{J}}}\right]=\widetilde{\beta^{\mathcal{J}}}\boldsymbol{I}$, and $-D^{\mathcal{I}}_{\mathrm{j},\mathrm{j}}=1$. Part (b) follows additionally from the fact that, under GAPs, side $\mathcal{J}$ users' marginal valuations for externalities is constant and thus, for all $\left(\boldsymbol{N}^{\mathcal{I}},\boldsymbol{N}^{\mathcal{J}}\right)\in[0,1]\times(0,1)$, it holds that $\left[\frac{\partial \overline{\boldsymbol{P}^{\mathcal{J}}}(\boldsymbol{N}^{\mathcal{I}};\boldsymbol{N}^{\mathcal{J}})}{\partial \boldsymbol{N}^{\mathcal{I}}}\right]=\widetilde{\beta^{\mathcal{J}}}\boldsymbol{I}$. □

Proof of Proposition 3: Media Pricing. For part (a), given the independence in advertisers' demand for the two platforms, the off-diagonal entries in $\frac{\partial \boldsymbol{N}^{\mathcal{A}}}{\partial \boldsymbol{P}^{\mathcal{A}}}$ and $\frac{\partial \boldsymbol{N}^{\mathcal{A}}}{\partial \boldsymbol{N}^{\mathcal{B}}}$ are zero. Thus, the Jacobian of the Insulating Tariff System is a diagonal matrix with j^{th} entry $\frac{\partial \overline{\boldsymbol{P}^{\mathcal{A},\mathrm{j}}}}{\partial N^{\mathcal{B},\mathrm{j}}}=\frac{\frac{\partial N^{\mathcal{A},\mathrm{j}}}{\partial N^{\mathcal{B},\mathrm{j}}}}{-\frac{\partial N^{\mathcal{A},\mathrm{j}}}{\partial P^{\mathcal{A},\mathrm{j}}}}=\widetilde{\beta^{\mathcal{A}}_{\mathrm{j},\emptyset}}$. This and the fact that $-D^{\mathcal{B}}_{\mathrm{j},\mathrm{j}}=1$ imply that the expression in (25) specializes to $N^{\mathcal{A},\mathrm{j}}\widetilde{\beta^{\mathcal{A}}_{\mathrm{j},\emptyset}}$.

For part (b), also since the off-diagonal entries in $\frac{\partial \boldsymbol{N}^{\mathcal{A}}}{\partial \boldsymbol{P}^{\mathcal{A}}}$ are zero, it holds that $-\boldsymbol{D}^{\mathcal{A}}=\boldsymbol{I}$. Hence, expression (25) becomes

$$\begin{aligned}
& N^{\mathcal{B},\mathrm{j}}\left(\left[-\frac{\partial \boldsymbol{N}^{\mathcal{B}}}{\partial \boldsymbol{P}^{\mathcal{B}}}\right]^{-1}\left[\frac{\partial \boldsymbol{N}^{\mathcal{B}}}{\partial \boldsymbol{N}^{\mathcal{A}}}\right]\right)_{\mathrm{j},\mathrm{j}} \\
= \; & N^{\mathcal{B},\mathrm{j}}\left(\begin{bmatrix} f^{\mathcal{B}}_{\mathrm{j},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & -f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} \\ -f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\end{bmatrix}^{-1}\begin{bmatrix}\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}f^{\mathcal{B}}_{\mathrm{j},\emptyset}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & -\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} \\ -\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & \widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\emptyset}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\end{bmatrix}\right)_{\mathrm{j},\mathrm{j}} \\
= \; & N^{\mathcal{B},\mathrm{j}}\left(\frac{\begin{bmatrix} f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} \\ f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & f^{\mathcal{B}}_{\mathrm{j},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\end{bmatrix}}{f^{\mathcal{B}}_{\mathrm{j},\emptyset}f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}+f^{\mathcal{B}}_{\mathrm{k},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}\begin{bmatrix}\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}f^{\mathcal{B}}_{\mathrm{j},\emptyset}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & -\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} \\ -\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & \widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\emptyset}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\end{bmatrix}\right)_{\mathrm{j},\mathrm{j}} \\
= \; & N^{\mathcal{B},\mathrm{j}}\frac{\begin{bmatrix}\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}f^{\mathcal{B}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\right) & \left(\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\emptyset}}-\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}\right)f^{\mathcal{B}}_{\mathrm{k},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} \\ \left(\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}-\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}\right)f^{\mathcal{B}}_{\mathrm{j},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}} & \widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\mathrm{j}}}f^{\mathcal{B}}_{\mathrm{j},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{k},\emptyset}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}\left(f^{\mathcal{B}}_{\mathrm{j},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\right)\end{bmatrix}_{\mathrm{j},\mathrm{j}}}{f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\right)} \qquad (26) \\
= \; & N^{\mathcal{B},\mathrm{j}}\frac{\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}+\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}f^{\mathcal{B}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\right)}{f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{B}}_{\mathrm{k},\emptyset}+f^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}\right)}=N^{\mathcal{B},\mathrm{j}}\left(\left(1-\omega^{\mathcal{B},\mathrm{j}}\right)\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\emptyset}}+\omega^{\mathcal{B},\mathrm{j}}\widetilde{\beta^{\mathcal{B}}_{\mathrm{j},\mathrm{k}}}\right).
\end{aligned}$$

□

Proof of Proposition 4: Two-Sided Single-Homing. Using the top row of the Jacobian of the

Insulating Tariff System from (26) (replacing $\mathcal{B}$ and $\mathcal{A}$ superscripts with $\mathcal{J}$ and $\mathcal{I}$) gives

$$\begin{aligned} & N^{\mathcal{J},\mathrm{j}}\left(\left[-\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{P}^{\mathcal{J}}}\right]^{-1}\left[\frac{\partial \mathbf{N}^{\mathcal{J}}}{\partial \mathbf{N}^{\mathcal{I}}}\right]\right)_{\mathrm{j},\cdot}\cdot\left[-\boldsymbol{D}^{\mathcal{I}}_{\cdot,\mathrm{j}}\right] \qquad (27) \\ = \; & N^{\mathcal{J},\mathrm{j}}\frac{\left[\widetilde{\beta^{\mathcal{J}}_{\mathrm{j,k}}}f^{\mathcal{J}}_{\mathrm{k},\emptyset}f^{\mathcal{J}}_{\mathrm{j,k}}+\widetilde{\beta^{\mathcal{J}}_{\mathrm{j},\emptyset}}f^{\mathcal{J}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{J}}_{\mathrm{k},\emptyset}+f^{\mathcal{J}}_{\mathrm{j,k}}\right) \quad \left(\widetilde{\beta^{\mathcal{J}}_{\mathrm{k},\emptyset}}-\widetilde{\beta^{\mathcal{J}}_{\mathrm{k,j}}}\right)f^{\mathcal{J}}_{\mathrm{k},\emptyset}f^{\mathcal{J}}_{\mathrm{j,k}}\right]}{f^{\mathcal{J}}_{\mathrm{j,k}}f^{\mathcal{J}}_{\mathrm{k},\emptyset}+f^{\mathcal{J}}_{\mathrm{j},\emptyset}\left(f^{\mathcal{J}}_{\mathrm{k},\emptyset}+f^{\mathcal{J}}_{\mathrm{j,k}}\right)}\begin{bmatrix}1\\ -D^{\mathcal{I}}_{\mathrm{k,j}}\end{bmatrix} \\ = \; & N^{\mathcal{J},\mathrm{j}}\left(\left(1-\omega^{\mathcal{J},\mathrm{j}}\right)\widetilde{\beta^{\mathcal{J}}_{\mathrm{j},\emptyset}}+\omega^{\mathcal{J},\mathrm{j}}\left(\widetilde{\beta^{\mathcal{J}}_{\mathrm{j,k}}}+D^{\mathcal{I}}_{\mathrm{k,j}}\left(\widetilde{\beta^{\mathcal{J}}_{\mathrm{k,j}}}-\widetilde{\beta^{\mathcal{J}}_{\mathrm{k},\emptyset}}\right)\right)\right)\end{aligned}$$

□

Proof of Proposition 5: m Symmetric Platforms. We must derive the factor in parentheses in (15). By the symmetry of platforms, we have that $-\frac{\partial N^{\mathcal{J}}}{\partial P^{\mathcal{J}}}$ is an $m \times m$ diagonal matrix with diagonal entries of $f^{\mathcal{J}}_{\emptyset}+(m-1)f^{\mathcal{J}}_{\mathrm{k}}$ and off-diagonal entries of $-f^{\mathcal{J}}_{\mathrm{k}}$. Inverting $-\frac{\partial N^{\mathcal{J}}}{\partial P^{\mathcal{J}}}$ yields a diagonal matrix with diagonal entries of $\left(f^{\mathcal{J}}_{\emptyset}+f^{\mathcal{J}}_{\mathrm{k}}\right)/\left({f^{\mathcal{J}}_{\emptyset}}^2+mf^{\mathcal{J}}_{\emptyset}f^{\mathcal{J}}_{\mathrm{k}}\right)$ and off-diagonal entries of $f^{\mathcal{J}}_{\mathrm{k}}/\left({f^{\mathcal{J}}_{\emptyset}}^2+mf^{\mathcal{J}}_{\emptyset}f^{\mathcal{J}}_{\mathrm{k}}\right)$. The matrix $\frac{\partial N^{\mathcal{J}}}{\partial N^{\mathcal{I}}}$ has diagonal entries of $\widetilde{\beta^{\mathcal{J}}_{\emptyset}}f^{\mathcal{J}}_{\emptyset}+\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}(m-1)f^{\mathcal{J}}_{\mathrm{k}}$ and off-diagonal entries of $-\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}f^{\mathcal{J}}_{\mathrm{k}}$. Thus, the last two factors in expression 3 are equal to

$$\begin{aligned} & \frac{1}{{f^{\mathcal{J}}_{\emptyset}}^2+mf^{\mathcal{J}}_{\emptyset}f^{\mathcal{J}}_{\mathrm{k}}}\left(\begin{bmatrix} f^{\mathcal{J}}_{\emptyset}+f^{\mathcal{J}}_{\mathrm{k}} & f^{\mathcal{J}}_{\mathrm{k}} & \cdots & f^{\mathcal{J}}_{\mathrm{k}} \\ f^{\mathcal{J}}_{\mathrm{k}} & \ddots & & \\ \vdots & & & \\ f^{\mathcal{J}}_{\mathrm{k}} & & & f^{\mathcal{J}}_{\emptyset}+f^{\mathcal{J}}_{\mathrm{k}}\end{bmatrix}\begin{bmatrix}\widetilde{\beta^{\mathcal{J}}_{\emptyset}}f^{\mathcal{J}}_{\emptyset}+\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}(m-1)f^{\mathcal{J}}_{\mathrm{k}} & -\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}f^{\mathcal{J}}_{\mathrm{k}} & \cdots & -\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}f^{\mathcal{J}}_{\mathrm{k}} \\ -\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}f^{\mathcal{J}}_{\mathrm{k}} & \ddots & & \\ \vdots & & & \\ -\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}f^{\mathcal{J}}_{\mathrm{k}} & & & \widetilde{\beta^{\mathcal{J}}_{\emptyset}}f^{\mathcal{J}}_{\emptyset}+\widetilde{\beta^{\mathcal{J}}_{\mathrm{k}}}(m-1)f^{\mathcal{J}}_{\mathrm{k}}\end{bmatrix}\right)_{\mathrm{j},\cdot}\begin{bmatrix}-D^{\mathcal{I}}\\ \vdots \\ 1 \\ \vdots \\ -D^{\mathcal{I}}\end{bmatrix} \\ = \; & \left(\left(1-\omega^{\mathcal{J}}\right)\widetilde{\beta^{\mathcal{I}}_{\emptyset}}+\omega^{\mathcal{J}}\left(\widetilde{\beta^{\mathcal{I}}_{\mathrm{k}}}+D^{\mathcal{I}}\left(\widetilde{\beta^{\mathcal{I}}_{\mathrm{k}}}-\widetilde{\beta^{\mathcal{I}}_{\emptyset}}\right)\right)\right)\end{aligned}$$

□

References

Ambrus, Attila and Rossella Argenziano (2009), "Asymmetric Networks in Two-Sided Markets." *American Economic Journal: Microeconomics*, 1, 17–52.

Anderson, Robert M., Glenn Ellison, and Drew Fudenberg (2010), "Location Choice in Two-Sided Markets with Indivisible Agents." *Games and Economic Behavior*, 69, 2–23.

Anderson, Simon P. and Stephen Coate (2005), "Market Provision of Broadcasting: A Welfare Analysis." *The Review of Economic Studies*, 72, 947–972.

Anderson, Simon P., André de Palma, and Jacques-François Thisse (1992), *Discrete Choice Theory of Product Differentiation*. MIT Press, Cambridge, Massachusetts.

Armstrong, Mark (2006), "Competition in Two-Sided Markets." *RAND Journal of Economics*, 37, 668–691.

Armstrong, Mark and John Vickers (2001), "Competitive Price Discrimination." *RAND Journal of Economics*, 32, 579–605.

Azevedo, Eduardo, E. Glen Weyl, and Alexander White (2011), "Walrasian Equilibrium without Gross Substitutes.", URL `http://ssrn.com/abstract=1831944`.

Becker, Gary S. (1991), "A Note on Restaurant Pricing and Other Examples of Social Influence on Price." *Journal of Political Economy*, 99, 1109–1116.

Bergemann, Dirk and Stephen Morris (2005), "Robust Mechanism Design." *Econometrica*, 73, 1771–1813.

Berry, Steven, James Levinsohn, and Ariel Pakes (1995), "Automobile Prices in Market Equilibrium." *Econometrica*, 63, 841–890.

Berry, Steven T. (1994), "Estimating Discrete-Choice Models of Product Differentiation." *RAND Journal of Economics*, 25, 242–262.

Borenstein, Severin (1989), "Hubs and High Fares: Dominance and Market Power in the U.S. Airline Industry." *RAND Journal of Economics*, 20, 344–365.

Bulow, Jeremy and John Roberts (1989), "The Simple Economics of Optimal Auctions." *The Journal of Political Economy*, 97, 1060–1090.

Cabral, Luís (2011), "A Dynamic Theory of Two-Sided Markets.", URL `http://csgb.ubc.ca/files/conference/Cabral.pdf`.

Caillaud, Bernard and Bruno Jullien (2003), "Chicken & Egg: Competition among Intermediation Service Providers." *RAND Journal of Economics*, 34, 309–328.

Cass, David and Karl Shell (1983), "Do Sunspots Matter?" *Journal of Political Economy*, 91, 193–227.

Chung, Kim-Sau and J. C. Ely (2007), "Foundations of Dominant-Strategy Mechanisms." *Review of Economic Studies*, 74, 447–476.

Cournot, Augustin (1838), *Researches into the Mathematical Principles of the Theory of Wealth* (N.T. Bacon trans.), 1971 edition. Augustus M. Kelley, New York.

Dybvig, Philip H. and Chester S. Spatt (1983), "Adoption Externalities as Public Goods." *Journal of Public Economics*, 20, 231–247.

Ellison, Glenn and Drew Fudenberg (2003), "Knife-Edge or Plateau: When Do Market Models Tip?" *The Quarterly Journal of Economics*, 118, 1249–1278.

Ellison, Glenn, Drew Fudenberg, and Markus Möbius (2004), "Competing Auctions." *Journal of the European Economic Association*, 2, 30–66.

Farrell, Joseph and Carl Shapiro (2010), "Antitrust Evaluation of Horizontal Mergers: An Economic Alternative to Market Definition." *Berkeley Electronic Journal of Theoretical Economics: Policies and Perspectives*, 10.

Gomes, Renato and Alessandro Pavan (2011), "Price Discrimination in Many-to-Many Matching Markets.", URL `http://faculty.wcas.northwestern.edu/~apa522/`.

Hagiu, Andrei (2006), "Pricing and Commitment by Two-Sided Platforms." *RAND Journal of Economics*, 37, 720–737.

Hotelling, Harold (1929), "Stability in Competition." *The Economic Journal*, 39, 41–57.

Hotelling, Harold (1938), "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates." *Econometrica*, 6, 242–269.

Jackson, Matthew O. (2008), *Social and Economic Networks*. Princeton University Press, Princeton, New Jersey.

Jaffe, Sonia and E. Glen Weyl (2011), "The First-Order Approach to Merger Analysis.", URL `http://www.glenweyl.com/research`.

Jevons, William Stanley (1871), *The Theory of Political Economy*. MacMillan, London.

Katz, Michael L. and Carl Shapiro (1985), "Network Externalities, Competition, and Compatibility." *The American Economic Review*, 75, 424–440.

Klemperer, Paul D. and Margaret A. Meyer (1989), "Supply Function Equilibria in Oligopoly under Uncertainty." *Econometrica*, 57, 1243–1277.

Lee, Robin S. (2010), "Dynamic Demand Estimation in Platform and Two-Sided Markets.", URL `http://pages.stern.nyu.edu/~rslee/papers/DynDemand.pdf`.

Liebowitz, S. J. and Stephen E. Margolis (1994), "Network Externality: An Uncommon Tragedy." *Journal of Economic Perspectives*, 8, 133–150.

Mazzeo, Michael J. (2002), "Product Choice and Oligopoly Market Structure." *RAND Journal of Economics*, 33, 221–242.

Mirrlees, J. A. (1971), "An Exploration in the Theory of Optimal Taxation." *Review of Economic Studies*, 38, 175–208.

Myerson, Roger B. (1981), "Optimal Auction Design." *Mathematics of Operations Research*, 6, 58–73.

Reisinger, Markus (2011), "Unique Equilibrium in Two-Part Tariff Competition between Two-Sided Platforms.", URL `http://www.dynmod.vwl.uni-muenchen.de/publikationen/reisinger_forschung/uniqueness-tsm.pdf`.

Riley, John G. and William F. Samuelson (1981), "Optimal Auctions." *The American Economic Review*, 71, 381–392.

Rochet, Jean-Charles and Jean Tirole (2003), "Platform Competition in Two-Sided Markets." *Journal of the European Economic Association*, 1, 990–1029.

Rochet, Jean-Charles and Jean Tirole (2006), "Two-Sided Markets: A Progress Report." *RAND Journal of Economics*, 37, 645–667.

Rohlfs, Jeffrey (1974), "A Theory of Interdependent Demand for Communications Service." *Bell Journal of Economics and Management Science*, 5, 16–37.

Rosse, James N. (1970), "Estimating Cost Function Parameters Without Using Cost Data: Illustrated Methodology." *Econometrica*, 38, 256–275.

Rysman, Marc (2009), "The Economics of Two-Sided Markets." *Journal of Economic Perspectives*, 23, 125–143.

Segal, Ilya (1999), "Contracting with Externalities." *The Quarterly Journal of Economics*, 114, 337–388.

Shaked, Avner and John Sutton (1982), "Relaxing Price Competition Through Product Differentiation." *The Review of Economic Studies*, 49, 3–13.

Song, Minjae (2011), "Estimating Platform Market Power in Two-sided Markets with an Application to Magazine Advertising.", URL `http://www.simon.rochester.edu/fac/MSONG/papers/Song-twosided.pdf`.

Spence, A. Michael (1973), "Job Market Signaling." *Quarterly Journal of Economics*, 87, 355–374.

Spence, A. Michael (1975), "Monopoly, Quality, and Regulation." *Bell Journal of Economics*, 6, 417–429.

Veiga, Andre and E. Glen Weyl (2011), "Platforms with Heterogeneous Externalities." This work is currently in progress. Contact Weyl at weyl@uchicago.edu for notes.

Weinstein, Jonathan and Muhamet Yildiz (2007), "A Structure Theorem for Rationalizability with Application to Robust Predictions of Refinements." *Econometrica*, 75, 365–400.

Weyl, E. Glen (2010), "A Price Theory of Multi-Sided Platforms." *American Economic Review*, 100, 1642–1672.

Wilson, Robert (1987), "Game Theoretic Analysis of Trading Processes." In *Advances in Economic Theory* (Truman F. Bewley, ed.), Cambridge University Press, Cambridge, UK.