Optimal Screening by Risk-Averse Principals*

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Abstract

This paper studies the effects of principal's risk aversion on principal-agent relationship under hidden information. It finds that the agent's equilibrium effort increases and approaches the efficient level as the principal's risk aversion increases and tends to infinity. Allowing for random participation by the agent, his effort is likely to be efficient even the principal's risk aversion is finite. For the case of common agency with random participation, it is optimal for the principals to make the agent the residual claimant on profits and the principals' net profits monotonically decrease to zero when their risk aversion tends infinite.

Keywords: principal-agent model, risk aversion, random participation, common agency

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1 Introduction

This paper studies systematically the effects of the risk aversion of principals on the optimal screening contracts. Existing literature in screening by nonlinear payment schedules, originating in papers of Rothschild and Stiglitz (1976), Mussa and Rosen (1978), and Maskin and Riley (1984), typically considers a certainty environment or assumes that principals are risk neutral and maximize expected payoffs. Though several papers and textbook treatments use a formulation of the screening problem that allows for risk aversion on the principal's side,[1] most of them do not study systematically the effects of principal's risk aversion on the equilibrium allocation. A notable exception is Gence-Creux (2000), who studies a joint problem of adverse selection and moral hazard in a regulation contract with a risk-averse principal and two different types of agents.

The reason for this omission in the theory development may be due to the recognition that the population of agents is sufficiently large and the effects of uncertainty are negligible, although the risk aversion of principals is well accepted as a more plausible and more realistic assumption than risk neutrality. Uncertainty and risk attitude, however, become crucial when a principal faces a small finite number of agents with hidden information. For example, at the time when a firm hires a CEO the firm may not know exactly or is not certain about the CEO's ability of profit-making and work ethics. Consequently, the firm's profits under the CEO's management are uncertain. Such uncertainty to principals is also quite obvious when a principal, such as a small

[1] See, for example, Fudenberg and Tirole (1992) for a textbook treatment, and papers by Anglin and Arnott (1991), Spier (1992), Page (1997), Gence-Creux (2000), and Celik (2003).

municipality, signs a regulatory contract with a private "public service" operator, such as a large multinational firm (Gence-Creux (2000), Laffont and Rochet (1998)), because the transfer from the principal to the operator depends on the unknown characteristics of the agent such as operating costs. In financial market, when a liquidity provider (principal) posts a limit order on an electronic trading system, she does not know the size of next market order (agent) she will meet although she can impose a limit on the maximum amount of buying or selling (Biais, Martimort and Rochet (2000)). Under these circumstances, the payoff to a principal from the use of a particular contracting mechanism is uncertain and therefore the optimal mechanism will be affected by the degree of risk aversion of the principal. This paper focuses on the role played by the risk aversion of principal(s) and its effect on the optimal screening and on the equilibrium effort exerted by the agent.

The hidden information under the consideration, which is summarized by the *type* of an agent, may include his valuation for services he can exchange with the principal(s) as well as his valuation of outside opportunities. For concreteness, we investigate the case, where the principal hires the agent to perform a task for her. To focus on screening, we abstract from the moral hazard issue by assuming that work effort is contractible by observing the output of the agent through a deterministic function of effort, but the cost of effort is private information to the agent. In Section 2, we consider a scenario where the set of possible types (i.e., marginal costs of effort) forms an interval on the Euclidean line but the value of the outside option is constant and known to the principal. Thus, in the spirit of Mussa and Rosen (1978), we develop a model of bilateral contracting, which can accommodate the risk aversion on the principal side. It is found that risk aversion makes pooling at equilibrium less likely. More importantly, the equilibrium effort provision increases in the degree of the principal's risk aversion under certain relatively non-restrictive conditions. When the principal becomes sufficiently risk averse, the effort reaches the efficient level. Total payment and marginal payment to the agent increase as well when the principal becomes more risk averse. What are the driving forces behind these results? If the principal is risk neutral, her goal is to maximize the expected profits, net of compensation to the agent. However, if she is risk averse, the volatility of net profits, including the risk of breaking down the principal-agent relationship, matters too. Thus, she is willing to pay the agent more in exchange for a lower volatility, which also stimulates the agent to work harder as he faces a higher and steeper pay. In the extreme, the principal's degree of risk aversion is infinitely high and she wants a non-random net profit. Then, she can optimally designs a contract to let the agent to be residual claimant. In turn, the agent will exert an efficient level of work effort. This section ends with a closed-form optimal wage designed by a risk-averse principal in a special case of the model.

Section 3 extends the model in Section 2 by allowing the agent's private value of the outside option to be random but independent of his marginal cost of effort, paralleling the Rochet and Stole's (2002) extension of the Mussa-Rosen model (1978). Therefore, the type is actually two-dimensional. However, since the value of outside option does not affect the marginal cost of effort, screening is possible only on one dimension: the cost of effort. If there is only one principal, we find once again that the equilibrium effort exerted by the agent approaches the efficient level when the principal is sufficiently risk averse. Moreover, there is a distribution of participation costs for which making the agent residual claimant on the profits is the optimal solution to the principal's problem and the principal's profits decline in her coefficient of absolute risk aversion. In the case of common agency, making the agent residual claimant on the profits is optimal in a full-participation symmetric equilibrium, for any distribution of participation costs. Similar to the case of single principal, the net profits of the principals monotonically decrease and converge to zero as their coefficient of absolute risk aversion tends to infinity. However, there is an important difference between the single principal and common agency cases. In the common agency case, the profits left to the principals affect only the distribution of the surplus because the agent definitely works for a principal in equilibrium. But the agent may fail to be hired in the single principal equilibrium so that a decrease in the principal's net profits can improve the probability of the agent to be hired and in turn the efficiency of the economy.

Our analysis shows that the risk aversion on the principal(s)' side generally plays a positive role in terms of improving efficiency and the agents' welfare. This finding is similar to Rochet and Stole's (2002) results on the positive effects of random participation on nonlinear pricing, although the economic mechanisms behind them are different.

2 Agent with a known outside option

Consider a principal-agent relationship, where a risk-averse principal hires an agent to perform a task for her. The task involves exerting effort, e, which produces profits $\pi(e)$ for the principal,[2] where $\pi(\cdot)$ is strictly increasing, strictly concave function. Although she may not be able to directly observe the agent's effort she can infer it uniquely from the profit generated by the agent. The agent's preferences are summarized by:

$$u(w, e) = w - et, \tag{1}$$

where type, t, can be interpret as the marginal cost of the agent's effort. Although t is the private information of the agent, the principal knows that t is drawn from a distribution with a strictly positive probability density function, $f(t)$, on (t_1, t_2), where $0 \leq t_1 < t_2 \leq \infty$. By the Taxation Principle (Rochet (1985)) one can without loss of generality restrict mechanisms used by the principal to be nonlinear wage schedules, $w(e)$.

For a given wage schedule $w(\cdot) : R \to R$, define the agent's surplus by:

$$s(t) \equiv \max_{e}(w(e) - et). \tag{2}$$

Denote $e(t)$ the utility-maximizing effort of a type-t agent. By the envelope theorem, we have

$$s'(t) = -e(t). \tag{3}$$

[2] We abstract from moral hazard problem by assuming that $\pi(\cdot)$ is deterministic.

The principal's expected utility is given by:

$$\int_{t_1}^{t_2} V(\pi(e(t)) - w(e(t)))f(t)dt,$$

where utility function $V(\cdot)$ is strictly increasing, concave, and twice differentiable. Thus, the principal's problem is to choose a wage schedule, $w(\cdot)$, to maximize her expected utility, subject to (3) and the implementability constraint, which states that $e(\cdot)$ is non-increasing.

Let us first concentrate on the relaxed problem, i.e. drop the implementability constraint. Then the monopolist solves

$$\max \int_{t_1}^{t_2} V(\pi(e(t)) - w(e(t)))f(t)d(t) \quad (4)$$

$$\text{s.t. } s'(t) = -e(t), \ s(t_2) = 0. \quad (5)$$

Note, an agent is characterized by his marginal cost of effort so that the principal can always properly select the wage schedule making the surplus of the most inefficient agent, agent t_2, equal to his outside option. In (5), the outside option has been normalized to zero. The Hamiltonian for this problem is:

$$H \equiv V(\pi(e) - s(t) - e(t)t)f(t) - \lambda(t)e(t). \quad (6)$$

The Pontryagin maximum principle states that if $\{e(\cdot), s(\cdot)\}$ is the solution to the principal's problem then there exists $\lambda(t) \in C^1(\Omega)$ such that

$$\begin{cases} s'(t) = -e(t) \\ \lambda'(t) = V'(\pi(e) - s(t) - e(t)t)f(t) \\ \lambda = V'(\pi(e) - s(t) - e(t)t)f(t)(\pi'(e) - t) \\ s(t_2) = 0,\ \lambda(t_1) = 0 \end{cases} . \tag{7}$$

In particular, the above system implies that

$$\pi'(e(t_1)) = t_1, \tag{8}$$

i.e. the marginal profit of the principal is equal to the marginal cost of effort exerted by agent t_1. In other words, we obtain the well-known result of no efficiency distortion at the top.[3] On the other hand, the second equation of system (7) implies that $\lambda'(t) > 0$ for all $t \in (t_1, t_2)$. Since $\lambda(t_1) = 0$, it is obvious that $\lambda(t) > 0$ for all $t \in (t_1, t_2]$. Recalling the third equation of system (7), the strict positivity of $\lambda(t)$ means

$$\pi'(e(t)) > t \tag{9}$$

for all $t \in (t_1, t_2]$. Thus, all agents, except for agent t_1, underprovide effort, no matter in pooling or separating equilibrium.

Substituting the third equation of (7) into the second and after some rearrangement one obtains:

$$\frac{d}{dt}((\pi' - t)f - F) = R(\pi' - t)^2 f e', \tag{10}$$

[3] Note, t_1 type (i.e. the type for whom the effort is least costly) generates the highest total surplus. Therefore, we call it the top type.

where $R \equiv -V''/V'$ is the Arrow-Pratt's coefficient of absolute risk aversion and $F(\cdot)$ is the cumulative density function corresponding to f. Note, we have dropped the variables in the functions to simplify notations and $e' \equiv de(t)/dt$. The initial condition for (10) is (8). In the analysis below, we assume that R is constant, i.e. $V(\cdot)$ is a CARA utility function,

$$V(x) = \frac{1 - \exp(-Rx)}{R}, \tag{11}$$

with $V(0) = 0$ and $\lim_{R \to +0} V(x; R) = x$. If the principal is risk neutral, then $R = 0$ and (10) implies

$$\pi'(e(t)) = v(t) \equiv t + \frac{F(t)}{f(t)}. \tag{12}$$

Expression (12) is similar to the well-known characterization of the efficiency distortion by a risk-neutral principal (see Varian, 1989). It leads to the solution for the complete problem provided that the virtual type, $v(\cdot)$, is non-decreasing. If the virtual type is decreasing in some range of types the optimal solution will entail some pooling. Note that if $\pi(\cdot)$ satisfies Inada condition at zero all agents will participate in the contract. However, exclusion region will be non-empty if $\pi'(0) < \inf_t v(t)$. Our first proposition establishes that pooling, if it happens, will occur for $R \in [0, R^*)$ for some $R^* \geq 0$. In particular, if equilibrium is fully separating under risk neutrality it will remain fully separating for any degree of risk aversion. More precisely, the following proposition holds:

Proposition 1 *Assume that the pooling region is non-empty for $R = R_0$. Then, it is also is non-empty for any $R < R_0$.*

Proof. With routine calculus, (10) can be written as

$$e'(t) = \frac{2f(t) - (\pi'(e) - t)f'(t)}{[\pi''(e) - R(\pi'(e) - t)^2]f(t)}. \tag{13}$$

Let $e^r(t, R)$ denote the solution of the relaxed problem (4)-(5),[4] it thus satisfies (13). Then (8) and (9) imply that

$$\pi'(e^r(t, R)) \geq t, \tag{14}$$

where equality holds only when $t = t_1$. By the assumption of non-empty pooling region for $R = R_0$, there exists $t = t^*$ such that

$$e'(t^*, R_0) > 0.$$

Therefore, (13) yields

$$2f(t^*) - (\pi'(e^r(t^*, R_0)) - t)f'(t^*) < 0,$$

which requires $f'(t^*) > 0$ because of (14) and $f(t^*) > 0$. It is straightforward to show that Hamiltonian (6) is supermodular in (e, R) and in turn $e^r(\cdot, R)$ is increasing in R. Since $e^r(\cdot, R)$ is increasing in R and $\pi'(\cdot)$ is decreasing, we have, for any $R < R_0$, that

$$2f(t^*) - (\pi'(e^r(t^*, R)) - t)f'(t^*) < 0,$$

[4] Equilibrium effort $e(t)$ is implicitly determined by R. So we explicitly express the effect of R by $e(t, R)$ when it is relevant. The same treatment is also applied to equilibrium wage, etc. For simplify, we slightly abuse notations and $e'(t, R)$ below should be considered as $\partial e(t, R)/\partial t$.

which implies from (13) that

$$e'(t^*, R) > 0,$$

i.e., there is pooling equilibrium at t^* for all $R < R_0$. Q.E.D.

Proposition 1 indicates that risk aversion on the principal side reduces the chance of pooling equilibrium. Therefore, the condition ensuring separating equilibrium under risk neutral also guarantees separating equilibrium under risk aversion as stated by the following corollary.

Corollary. 1 *If*

$$f'(t) \leq \frac{2f^2(t)}{F(t)}, \tag{15}$$

then for all $R \geq 0$ the equilibrium is separating.

Proof. Condition (15) is equivalent to the requirement that $v'(t) \geq 0$, i.e. that the virtual type is increasing. It guarantees that there is no pooling for $R = 0$ and therefore, by Proposition 1, for any $R > 0$. Q.E.D.

It is worth to notice that condition (15) in the corollary does not impose a significant restriction because the log-concavity of $F(t)$ or the monotonicity of hazard rate $\frac{f(t)}{F(t)}$, a widely adopted assumption in the literature (e.g. Biais, Martimort and Rochet (2000)), requires $f'(t) \leq \frac{f^2(t)}{F(t)}$.

Proposition 2 *Assume that $f(t)$ satisfies* (15) *or the equilibrium is fully separating* *for all R. Then, $e(t,R)$ increases in R.*

Proof. By Corollary 1, pooling has been ruled out, solution to (10) or equivalently to (13) provides the solution to the complete problem. Now observe that $e'(t)$ increases in R for given t and e, and the value of e at $t = t_1$ does not depend on R. The Gronwall's lemma (see, for example, Bellman, 1943) implies that $e(t,R)$ is increasing in R. Q.E.D.

Proposition 3 *Under the assumptions of Proposition 2, the equilibrium supply of effort is efficient when R tends to infinity, i.e.*

$$\lim_{R \to +\infty} e(t,R) = e^{eff}(t),$$

where the efficient effort, $e^{eff}(t)$, is determined by $\pi'(e^{eff}) = t$.

Proof. Rewrite equation (13) as

$$e'(t)(\frac{\pi''(e)}{R} - (\pi'(e) - t)^2)f(t) = \frac{2f(t) - (\pi'(e) - t)f'(t)}{R}.$$

Define function $G(t,e) = 2f(t) - (\pi'(e) - t)f'(t)$. Since $(t,e) \in [t_1, t_2] \times [0, e(t_1)] = C$, where $C \subset R^2$ is a compact set, there exists $K > 0$ such that $0 < G(t,e) < K$ for all $(t,e) \in C$. Taking into account that under the assumptions of Proposition 2, $e'(t) < 0$, one obtains

$$\lim_{R \to +\infty} \pi'(e(t,R)) = t,$$

which completes the proof. Q.E.D.

The intuition behind Propositions 2 and 3 are straightforward. If the principal is risk neutral, what she concerns is the expected profits, net of wage expenditure. However, if she is risk averse, the volatility of net profit, $\pi(e) - w(e)$, concerns her too. Thus, she is willing to sacrifice net profits and pay the agent more to reduce the volatility. The increased wage payment will stimulate the agent to work harder and in turn the efficiency is improved. The more risk averse is the principal, the higher wage is she willing to pay, and in turn the greater effort is exerted by the agent. When $R \to +\infty$, the principal tries to avoid all profit uncertainty. This requires choosing a wage schedule, which ensures that the net profit yielded from meeting any agent is constant; that is, $\pi(e) - w(e) = \pi_0$. Clearly, this strategy makes the agent the residual claimant on the firm's profits. Taking its derivative with respect to e and recalling the first order condition for agent utility maximization $w'(e) = t$, we immediately obtain $\pi'(e(t)) = t$, i.e., the efficient effort provision. Now we formalize our claim on the effects of risk aversion on the wage schedule.

Proposition 4 *Under the assumptions of Proposition 2, both total wage $w(e)$ and marginal wage $w'(e)$ increase in R; i.e. $w_R(e, R) > 0$ and $w_{eR}(e, R) \geq 0$.*

Proof. Given the agent's surplus, the total wage can be written as $w(e) = \inf_t(s(t) + et)$ (see, for example, Basov 2005). Now

$$w_R(e, R) = s_R = \int_t^{t_2} e_R(\tau, R) d\tau > 0, \tag{16}$$

where the first equality follows from the envelope theorem, the second equality is derived from (3) using initial condition (5), and the last inequality follows from Proposition 2.

For marginal wage, the first- and second-order conditions for the type-t agent who faces wage schedule $w(e, R)$ imply

$$\begin{cases} w_e(e, R) = t \\ w_{ee}(e, R) \leq 0 \end{cases} . \tag{17}$$

Totally differentiating the first equation in (17) with respect to R and using the second inequality and Proposition 2 one obtains $w_{eR} = -w_{ee}(e, R)e_R \geq 0$. Q.E.D.

It is clear that as the principal becomes more risk averse the agent is paid more for the same effort level, and therefore his welfare increases (see (16)). In other words, risk aversion on the principal side actually makes the agent better off. Next, we provide an example for the special case of the model. The explicit solution to the example will illustrate the main results of our previous analysis.

Example. Let us assume that the production function is $\pi(e) = e - \frac{e^2}{2}$ and the type is distributed uniformly on $(0, 1)$. Then system (10) becomes:

$$e'(t) = -\frac{2}{1 + R(1 - e - t)^2} \tag{18}$$

with initial condition $e(0) = 1$. Apparently, there is $e'(t) < 0$, i.e. any solution of this differential equation is decreasing in t. Therefore, there is no pooling at positive effort levels and (18) completely determines the solution of the problem. Let us introduce $q(t)$ by $q(t) \equiv 1 - e(t)$, then

$$q'(t) = \frac{2}{1 + R(t-q)^2} \tag{19}$$

and $q(0) = 0$. Define implicit function $q^*(t)$ by:

$$q^*(t) \equiv \frac{1}{\sqrt{R}} \ln \frac{1 - \sqrt{R}(t - q^*(t))}{1 + \sqrt{R}(t - q^*(t))}. \tag{20}$$

It is easy to check that (20) satisfies system (19). The corresponding effort solves:

$$e(t) = 1 - \frac{1}{\sqrt{R}} \ln \frac{1 - \sqrt{R}(t + e(t) - 1)}{1 + \sqrt{R}(t + e(t) - 1)}.$$

Agents who choose the outside option are those whose type t falling in $(0, t^*(R))$, where

$$t^*(R) = \frac{(\exp(\sqrt{R}) - 1)}{\sqrt{R}(1 + \exp(\sqrt{R}))}.$$

Note that $t^*(R)$ is decreasing in R with $t^*(0) = 0.5$ and $\lim_{R\to\infty} t^*(R) = 0$; i.e. as the principal becomes more risk averse the chance that the agent takes the principal's offer approaches one, which agrees with our previous observation that the effort provision approaches the efficient level.

To solve for the optimal wage, let us introduce variable $m \equiv \sqrt{R}(t + e - 1)$. Then equation (20) implies:

$$m = \tanh \frac{\sqrt{R}(e-1)}{2}, \tag{21}$$

where the hyperbolic tangent $\tanh(\cdot)$ is defined by $\tanh y = \frac{\exp(y)-\exp(-y)}{\exp(y)+\exp(-y)}$. Substituting the first order condition for the agent's optimization $w'(e) = t$ into (21), one obtains:

$$w'(e) = 1 - e + \frac{\tanh \frac{\sqrt{R}(e-1)}{2}}{\sqrt{R}}.$$

Finally, carrying out integration and taking into account the participation constraint $w(0) = 0$, one obtains:

$$w(e) = \pi(e) + \frac{2}{R} \ln \frac{\cosh \frac{\sqrt{R}(e-1)}{2}}{\cosh \frac{\sqrt{R}}{2}},$$

where the hyperbolic cosine, $\cosh(\cdot)$, is defined by $\cosh y = \frac{\exp(y)+\exp(-y)}{2}$.

3 Agents with random outside option

Now we are going to enrich the model of the previous section by assuming that the value of the outside opportunity is unknown to the principal, i.e. it is a part of the private information of the agent. Formally, the type of an agent is characterized by $(t, x) \in (t_1, t_2) \times (x_1, x_2) \subset R^2$. His utility, when he provides effort e and receives wage w, is:

$$U(t, e, w, x) = w - et - kx. \tag{22}$$

We further assume that random variable x is independent of t and has a cumulative density function $N(\cdot)$. To simplify the notation we will assume that $k = 1$ in the single principal case of Subsection 3.1 but will restore it in the common agency case of Subsection 3.2.

3.1 A single risk-averse principal

We first consider the case of a single risk-averse principal. The agent's surplus is still defined by (2). Then the probability that an agent accepts the contract and provides a positive effort is $N(s)$. Therefore, the principal solves

$$\max \int_{t_1}^{t_2} V(\pi(e) - s(t) - e(t)t)N(s(t))f(t)dt \tag{23}$$

subject to (3). Note, the boundary condition $s(t_2) = 0$ in (5) no longer holds because the value of the outside option is uncertain to the principal. Therefore, it is an optimal control problem with both ends being free. The Hamiltonian for this problem is:

$$H = V(\pi - s - et)N(s)f(t) - \lambda(t)e(t) \tag{24}$$

and the Pontryagin maximum principle implies:

$$\begin{cases} \lambda'(t) = V'(\pi - s - et)N(s)f(t) - V(\pi - s - et)N'(s)f(t) \\ V'(\pi - s - et)(\pi'(e) - t)N(s)f(t) - \lambda(t) = 0 \\ \lambda(t_1) = \lambda(t_2) = 0 \end{cases} \tag{25}$$

Proceeding in the same way as in the previous section, one obtains:

$$\frac{1}{N}\frac{d}{dt}(N(\pi' - t)f) - f = R(\pi' - t)^2 e' f + \frac{N'V}{NV'}f. \tag{26}$$

If x takes only one value x_0 with certainty, then $N(s) = 1$ for $s \geq x_0$ and equation (26) is reduced to (10). The boundary conditions in (25) implies

$$N(s(t_i))(\pi'(e(t_i)) - t_i) = 0 \quad \text{for } i = 1, 2. \tag{27}$$

It is reasonable to only consider the case where agents participate in equilibrium with a positive probability. Then, $N(s(t_1)) > 0$ and $\pi'(e(t_1)) = t_1$, which is the conventional "no distortion at the top" property. On the other hand, the boundary condition at t_2 implies either a type-t_2 agent participates with zero probability or the provision of his effort is also efficient. This result of either no-service or no-distortion at the bottom does not depend on the risk aversion of the principal and has been observed by Rochet and Stole (2002) too.

To investigate the effects of risk aversion, let us assume a Bernoulli utility function for the agent again. It is obvious that the first term on the right hand side of (26) becomes dominant when R is sufficiently large. Therefore, it leads to an efficient effort when R tends to infinity. The formal proof of this conclusion is similar to Proposition 3 and is omitted. So we only state it as a corollary below.

Corollary 2 *The results in Proposition 3 hold with random participation.*

The possibility of both type-t_1 and type-t_2 agents exerting an efficient effort leads us to ask under what condition *all* agents will exert an efficient effort when they face a wage schedule proposed by a principal with finite risk aversion. The efficiency requires

$$w(e) = \pi(e) - A_M, \tag{28}$$

where A_M is a constant, which can be interpreted as the net profits to the principal. Suppose $N(x)$ is an exponential function that $N(x) = B\exp(\alpha x)$ with $\alpha > 0$. Because $N(x)$ is a cdf on some interval (x_1, x_2), there are $N(x_1) = 0$ and $N(x_2) = 1$, which require $x_1 = -\infty$ and $B = \exp(-\alpha x_2)$, respectively. In words, these conditions say that the value of the outside option must be distributed on $(-\infty, x_2)$ according to cdf $N(x) = \exp(\alpha(x - x_2))$ for the equilibrium to be efficient, conditional on participation. Recalling $w'(e) = t$, substituting (28) into (26) yields:

$$\frac{V'}{V} = \frac{R}{\exp(RA_M) - 1} = \alpha. \tag{29}$$

Thus, we arrive at the following Proposition.

Proposition 5 *A risk-averse principal imposes a two-part wage schedule (28) with*

$$A_M = \frac{1}{R}\ln(1 + \frac{R}{\alpha}). \tag{30}$$

if and only if the outside option is distributed according to the following cumulative distribution function

$$N(x) = \min\{1, \exp(\alpha(x - x_2))\}. \tag{31}$$

Proof. We have already proved that if the optimal wage has two parts as (28) the distribution of the outside option should satisfy (31). The reverse follows from log-concavity of distribution (31) (see, Rochet and Stole, (2002)). Finally, (30) follows from (29). Q.E.D.

Note that while wage schedule (28) implies that the agent provides efficient effort subject to participation, the participation decision is not efficient as long as A_M is positive. However, A_M in (30) decreases monotonically from $1/\alpha$ to zero as R increases from zero to infinity, indicating that increase in the principal's risk aversion leads to efficiency gains and, as R tends to infinity, the optimal effort converges to the efficient one.

It might be interesting to compare the results in Corollary 2 and Proposition 5 to the findings of Rochet and Stole (2002). They discovered that the introduction of random participation can improve the allocative efficiency in the sense that random participation lifts the quality supplied to the consumer of each type and moves it closer to the efficient level. The findings in Corollary 2 and Proposition 5 take us one step further and indicate that effort provision actually is likely to be efficient, conditional on participation, when random participation combines with risk aversion on the principal side.

3.2 Common agency

This subsection extends the analysis in the previous subsection by allowing the agent to be hired by either of two principals, i.e. we have a common agency problem. Both principals are assumed to have the same production function, π, and the same constant coefficient of absolute risk aversion, R, as we have specified in the single principal case. Let us consider a model in the spirit of Hotelling model, i.e. the two principals are located at the ends of a linear city with a length equal to one, while the agent is randomly located along the Hotelling line. Thus, we can interpret x^i in (22) as the distance of the agent from principal i, where superscript $i \in \{L, R\}$ indexes the principals, and k as the marginal transportation cost. The principal's profit and the agent's effort cost satisfy the following condition,

$$\max_e(\pi(e) - te)) > \frac{k}{2} + \kappa, \tag{32}$$

where $\kappa \equiv 2kN(\frac{1}{2})/N'(\frac{1}{2})$. Assumption (32) simply says that the total surplus, ignoring transportation cost, is sufficiently big for positive gains of trade to exist, irrespective to the location of the agent on the Hotelling line. This guarantees the existence of equilibrium at which the agent provides positive effort with probability one.

Following (2), the agent's surplus function from the relationship with principal i is $s^i(t) = \max_e(w^i(e) - te)$. Given the surplus functions, the probability that the type-t agent takes the contract from the left end principal is:

$$Q^L(s^L, s^R) = N(\min\{\frac{s^L(t)}{k}, \frac{1}{2} + \frac{s^L(t) - s^R(t)}{2k}\}).$$

Therefore, given the right principal's wage schedule, $w^R(e)$, and the surplus function, $s^R(t)$, the left principal determines her wage strategy by maximizing her expected utility:

$$\max \int_{t_1}^{t_2} V(\pi(e) - s^L(t) - te))Q^L(s^L, s^R)f(t)dt.$$

Clearly, the only difference between this program and (23) is $N(s)$ in (23) has been replaced by $Q^L(s^L, s^R)$. So, with given s^R the first-order condition for the optimal solution to the program is the same as (25) with proper notation changes. We would like to concentrate on symmetric equilibrium.

Proposition 6 *Let condition (32) hold. Then the necessary and sufficient conditions for the existence of symmetric equilibrium in which the agent works for one of the principals and supplies an efficient effort are that each principal adopts the following wage schedule:*

$$w(e) = \pi(e) - A_D, \tag{33}$$

where fixed fee A_D is given by:

$$A_D = \frac{1}{R} \ln(1 + \kappa R). \tag{34}$$

Proof. *Necessity*: Without loss of generality we assume that the agent works for the left principal. Since the provision of effort is efficient, there is $\lambda(t) = 0$ for all $t \in [t_1, t_2]$ from the corresponding second equation of (25). Thus, $\lambda'(t) = 0$ and the first equation in (25) implies

$$V \frac{\partial Q^L(s^L, s^R)}{\partial s^L} = V' Q^L(s^L, s^R). \tag{35}$$

On the other hand, the first-order condition of agent's optimization requires $w'(e) = t$ and efficiency implies $\pi'(e) = t$. Therefore, the wage schedule must be (33). To determine A_D, we notice that

$$Q^L(s^L, s^R) = N(\frac{1}{2} + \frac{s^L(t) - s^R(t)}{2k}). \tag{36}$$

Then, substituting (33) into (35) yields

$$\frac{1}{\kappa} = \frac{R}{\exp(RA_D) - 1},$$

and in turn (34).

Sufficiency: There is $\pi'(e) = t$ if both principals impose a wage schedule (33), which means that the first-order conditions for both principals are satisfied and the equilibrium is efficient. It remains to check that given (33) the agent will always accept the contract from one of the principals. But, note that (34) implies that A_D decreases monotonically from κ to zero as R increases from zero to infinity, i.e. $A_D \leq \kappa$. Hence, (32) ensures that the agent will will definitely participate, irrespective to the principals' risk attitude. Q.E.D.

Proposition 6 states that efficient wage schedule prevails with a random but full participating common agent. This echoes the finding that duopolists impose a cost-based two-part tariff in nonlinear price competition by Armstrong and Vickers (2001), and Rochet and Stole (2002). Although the risk aversion of the principals does not affect the efficiency of the economy, it does change the income distribution. As (34) shows, the profit of the principals, A_D, declines in R and reach zero when R is infinite. Therefore, as the principals become more risk averse, the agent becomes better off.

4 Conclusions

This paper has developed a model that focuses on the effects of principals' risk aversion on the optimal work effort supplied by an agent with an unobservable cost of effort. Although it features principals facing a single agent, extension to any finite set of agents is straightforward. While the model builds on the scenario of principals hiring an agent, the theory and its results are directly applicable to other screening problems of hidden information. We started by revisiting the standard Mussa-Rosen (1978) model and showed that an increase in risk aversion on the principal's side monotonically pushes the provision of effort toward the efficient level. Intuitively, this happens because risk aversion makes the principal put a greater weight on avoiding profit uncertainty, particularly the worst possible scenario — a failure to make a hiring. Risk aversion also reduces the chance that the contracting results in a pooling equilibrium. We then extended the base model to allow for random participation, paralleling Rochet and Stole's (2002) extension to the Mussa-Rosen (1978) model. We have established conditions for the optimal wage schedule leading the agent to be the residual claimant on the profits under both single principle and common agency regimes. While under the common agency the conditions are quite general and simply require the unit transportation cost is sufficient low to allow for full participation, the efficient wage schedule in the case of single principle require a quite specific distribution of participation costs. In both cases the net profits of the principal(s) decrease in their coefficient of absolute risk aversion. Such a decrease improves efficiency through more efficient participation decision in the case of a single principal, however, it only affects the distribution of rents between the principals and the agent in the case of common agency. Generally, our analysis indicates that risk aversion on the principal's side plays a positive role in improving either productive efficiency or agent's welfare.

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