The Disposition Effect and Expectations as Reference Point

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Abstract: This paper shows that aversion to losses relative to a reference point predicts a V-shaped relationship between the optimal position in a stock and current gains from that stock, in contrast to the approximately monotonic relationship implied by the standard theory. Estimates from Odean's (1999) individual trading records show that (i) the predicted V-shape relationship exists for a large majority of investors, and (ii) expectations are the most likely determinant of investors' reference points. The V-shaped relationship and the implication of the initial purchase decision that expectations are mostly positive yield a simple explanation of the disposition effect.

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The disposition effect refers to the observation that stock market investors tend to hold on to their losers for too long and sell their winners too soon, with losers and winners defined by comparing current price to the initial or (when shares were acquired at different times) the average purchase price (Shefrin and Statman (1985), Odean (1998), and Weber and Camerer (1998)).[2] Odean (1998), for instance, analyzes trading records of individual investors at a large discount brokerage house. He finds a strong asymmetry in the sale probabilities of stocks that currently show a gain and those that show a loss relative to the average purchase price.[3]

In Odean's (1998) dataset, most investors do not immediately purchase another stock after selling an old one, so the selling decision is largely a choice between holding a risky stock or safe cash, which mainly reflects attitudes toward risk.[4] The disposition effect thus poses a challenge to explanations based on simple models with expected-utility maximizing investors, in that there is no reason why the sharp changes in risk aversion needed to explain the disposition effect should bunch around the average purchase price, especially when investors have varying wealth levels, different starting portfolios, and distinct purchase prices. Further, Odean explicitly considers expected-utility explanations for the asymmetry in sale probabilities

[2] For consistency with the previous literature, I use this definition of winners and losers below when talking about patterns in the data, even though the true reference level of price may not be the initial or average purchase price. Further, Odean's (1998) analysis suggests that the choice whether to use the initial or the average purchase price makes little difference empirically, so I focus below on average purchase for simplicity.

[3] A similar phenomenon has been documented in the housing market (Genesove and Mayer (2001)). Weber and Camerer (1998) also replicate the disposition effect in the lab.

[4] It is of course always possible that investors purchase other types of risky assets, or they have accounts in other brokerage companies so that the trading records in this sample are not complete. However, given the large number of investors and the long period involved, the time lag between each sale and the next purchase should largely reflect an investment pattern rather than these incidences. For instance, it is not likely that most investors trade in another asset market every time they sell a stock; or they use stock accounts in other brokerage companies to buy a new stock every time they sell an old one.

based on richer specifications of the investor's problem, finding that portfolio rebalancing, transaction costs, taxes, or rationally anticipated mean reversion cannot explain the observed asymmetry.[5]

The most popular informal explanation of the disposition effect has been prospect theory (Kahneman and Tversky (1979), and Odean (1998)). Prospect theory assumes that the carrier of utility is not the level of wealth, but the change in wealth relative to a reference point. The theory has three main elements: Loss aversion (losses relative to the reference point hurt investors more than gains please them), diminishing sensitivity (investors are less sensitive to big gains and losses than small ones), and nonlinear probability weighting (investors systematically overweight small probabilities). [6]

The literature on prospect theory equates the reference point to the status quo (e.g. Shefrin and Statman (1985), Kahneman, Knetsch and Thaler (1990), Benartzi and Thaler (1995), and Genesove and Mayer (2001)). In this setting the status quo reduces to the average purchase price or equivalently, measuring the reference point in terms of gains from investing in a given stock

[5] Weber and Camerer (1998) find that incorrect beliefs of mean reversion cannot explain the disposition effect either. In their experimental study, subjects forced to sell the losers and given a chance to buy them back usually refuse to do so.

[6] Prospect theory generates individual trading behavior that in equilibrium can explain various asset pricing puzzles. For instance, Benartzi and Thaler (1995) use prospect theory to explain the equity premium puzzle. Barberis, Huang and Santos (2001) show that prospect theory preferences, combined with changes in risk attitudes after prior gains and losses can generate high average stock return, high volatility, and cross-sectional predictability. Grinblatt and Han (2005) suggest that the undervaluation of stocks after gains and overvaluation of stocks after losses generated by prospect theory can predict short-run momentum in returns if the distorted prices are corrected by rational investors. In a study on the trading behavior of Chicago Board of Trade proprietary traders, Coval and Tyler (2005) confirm Grinblatt and Han's (2005) predictions. These studies all assume, implicitly or explicitly, that reference point is determined by the status quo.

as I shall do here, zero gains. [7] [8] The literature on the disposition effect often ignores nonlinear probability weighting for simplicity. The informal explanations have so far focused on diminishing sensitivity, which implies that investors in the gains domain are more risk averse, hence more likely to sell a stock (for less risky cash); while investors in the losses domain are risk seeking, hence more willing to hold a stock. However, loss aversion also influences attitudes toward risk in a way that has the potential to explain the disposition effect. When an investor's probability of crossing the reference point is nonnegligible, the kink associated with loss aversion causes first-order risk aversion (Rabin (2000)), potentially decreasing the investor' probability of holding a stock much more than any plausible effect of diminishing sensitivity.[9]

Translating an explanation of the disposition effect based on prospect theory into a formal model has been challenging (e.g. Hens and Vlcek (2005), Gomes (2005), and Barberis and Xiong (2009)). Barberis and Xiong (henceforth "BX") (2009) propose a dynamic model of selling behavior based on both loss aversion and diminishing sensitivity, taking into account that investors' rational expected returns must be positive to justify the initial purchase decision. BX show that for a binomial or lognormal returns distribution with a reasonable range of positive

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[7] In terms of the literature on the disposition effect, Health, Huddart and Lang (1999) is an exception. Because they look at the stock option exercise so there is no natural purchase price to rely on. They show that reaching the highest price of the previous year drives the exercise decision.

[8] A reference point defined by the status quo equates the monetary gains (capital gains and dividends) from investing in a stock to the psychological gains relative to the reference point. However, such equivalence breaks down when the reference point is different from the status quo, in which case I shall use "psychological gains" to describe the latter. Further, reference point is defined on wealth space in prospect theory, but it can be more conveniently referred to in terms of the corresponding reference level of gains in the stock market setting.

[9] Decision makers exhibiting first-order risk aversion are risk averse even for very small gambles; while those with second-order risk aversion, represented by the usual concave utility function, are approximately risk neutral for small gambles.

means, taking the status quo as the reference point, prospect theory in Tversky and Kahneman (1992)'s parameterization actually generates the opposite of the disposition effect in most cases.[10]

While diminishing sensitivity may contribute to the disposition effect in BX's model, the result that generates the opposite of the disposition effect comes from loss aversion. The first-order risk aversion caused by loss aversion suggests that the closer gains are to the reference level, the more risk-averse investors are, hence the more likely they are to sell. Returns distribution with positive mean normally generates large gains and small losses, making gains on average farther away from zero than losses. If an investor's reference level of gains is zero, then she is more likely to sell stocks that currently show a loss because of the proximity to the zero reference level. Loss aversion, even though partially offset by the effect of diminishing sensitivity, therefore generates more sales below than above zero gains.

Although BX carefully investigate the robustness of their results in several directions, they do not consider alternative specifications of the reference point beyond the status quo. However, the empirical literature on prospect theory has taken equivocal positions on what determines reference point. Although the early literature assumes that reference point is the status quo, Kahneman and Tversky (1979) also note that "there are situations in which gains and losses are

[10] BX's original formulation takes the initial wealth invested in a given stock (with interest earnings) as the reference point. In section III of the paper they also sketch a model of realization utility that distinguishes between paper and realized gains and losses. Realization utility is capable of generating the disposition effect. I will discuss this alternative in section IV in light of the empirical facts documented in this paper. My analysis, however, still follows the traditional assumption *not* to distinguish between paper and realized gains and losses.

coded relative to an expectation or aspiration level that differs from the status quo". Kőszegi and Rabin (2006, 2007, 2009) develop this idea in their new reference-dependent model by endogenizing reference points as lagged rational expectations.[11]

This paper reconsiders the possibility of explaining the disposition effect via loss aversion, taking a broader view of the reference point. When the reference point is not closely tied to the status quo, positive expected return and the associated asymmetry of gains and losses around zero play a less important role for the disposition effect, and I show that for a general returns distribution with positive expected return, loss aversion with reference point defined by expectations reliably implies a disposition effect of the kind commonly observed. Diminishing sensitivity reinforces this effect, but it is not essential for an explanation.

Econometric analysis of Odean's (1999) data on individual trading records from a large brokerage house confirms the existence of a large disposition effect. More importantly, a novel and stronger empirical pattern is documented and linked to loss aversion. Being the first attempt to estimate investors' reference points from individual trading data, the econometric analysis also supports expectations as the most reasonable candidate for investors' reference points.

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[11] Several empirical papers have tested Kőszegi and Rabin's assumption that reference point is determined by expectations. In a lab experiment, Abeler, Falk, Goette, and Huffman (forthcoming) manipulate subjects' expectations of earnings and show that their labor supply decision is determined by reference point defined by expectations rather than the status quo. Ericson and Fuster (2009) suggest that reference point defined by expectations plays an important role in driving the classical endowment effect. Gill and Prowse (2009) run a field experiment in a real effort competition setting. Their subjects respond negatively to the rivals' efforts, a prediction from disappointment aversion that treats the certainty equivalent of the lottery—a plausible proxy of expectations—as the reference point. Crawford and Meng (2009) proxy expectations by natural sample averages of the outcomes to show that a reference-dependent model with reference point defined by expectations provides a useful account of New York cabdrivers' labor supply behavior. Card and Dahl (2009)'s empirical analysis suggests that emotional cues generated by unexpected losses by the home team in football increase family violence.

The rest of the paper is organized as follows.

Section I proposes a model of reference-dependent preferences. I model investors' decision problem much as BX (2009) do, but with the following differences: First, I assume loss aversion but not diminishing sensitivity. Since diminishing sensitivity has explanatory power for the disposition effect, ignoring it strengthens my main point, that prospect theory can provide a credible explanation for the disposition effect. Second, instead of equating the reference point to the status quo, I derive the model's implications for any exogenous and deterministic reference point. Third, I generalize returns distribution from BX's binomial or lognormal to any continuous distribution, so that the model can deliver a complete picture of the non-monotonic changes in risk attitudes across all return levels. Finally, although BX's model is dynamic, to keep the matter simple I illustrate the main prediction of loss aversion using a static model.[12]

The major prediction of the model is a V-shaped relationship between the optimal position on a given stock and current gains of that stock, with the bottom point of the V shape closely linked to the reference point. When we change an investor's reference point from the status quo to expected gain, which should be positive due to the initial purchase decision, the bottom of the V shape changes from zero to a positive level. Although most gains are still farther away from zero than most losses, most gains are now generally closer to the positive reference level. Thus the investor is more likely to sell when stock price appreciates from the average purchase price. The V-shaped relationship, combined with the effect of positive expectations on the reference

[12] Section I.C. analyzes a dynamic model and a model of stochastic reference point as robustness check.

point, therefore generates the disposition effect. However, the V shape is a stronger testable prediction. For instance, a threshold strategy of selling the stocks once certain positive threshold is reached is also capable of generating the disposition effect, but will not yield a V-shaped relationship.

Section II analyzes individual trading records from a large brokerage house used by Odean (1999) and Barber and Odean (2000, 2001). I pool observations across investors to estimate the aggregate trading pattern. Since investors in the dataset hold either all or none of their positions on a given stock most of the times, the probability of holding a stock becomes a good proxy for the normalized stock position.[13] I indeed document a novel and largely V-shaped relationship between the probability of holding a stock and current gains of that stock. Several papers have partially characterized the implications of this relationship using various datasets, but to my knowledge this paper is the first to document and analyze the complete quantitative pattern.[14]

My theoretical characterization of the V-shaped relationship also suggests a useful empirical strategy to identify the reference point from the bottom of the V. Using a multi-threshold model and treating investors as if they had homogenous reference level of gains, I

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[13] Each investor's position on a given stock in the portfolio at different times need to be normalized relative to the initial position to facilitate the analysis across stocks and investors.

[14] Using a dataset of individual trading records different from the one used in this paper, Odean (1998) shows that investors are more likely to hold big winners and losers than small ones. Grinblatt and Keloharju (2001) find the same tendency with losers using trading records of Finnish investors. Working with the same data set as this paper, Ivković, Poterba and Weisbenner (2005) find that the relationship between the probability of holding a stock and positive capital gains is negative within six months and positive after twelve months since purchase, a natural implication of the V-shaped relationship with positive expectations as the reference point, because in the domain of positive gains, small (large) gains are located to the left (right) of the reference level, leading to a negative (positive) relationship.

estimate the bottom of the V shape to be around a gain of 5.5%, which is significantly different from zero, suggesting that the status quo cannot be a reasonable candidate of the reference point. The estimate is, however, closely tied to investors' expectations. Qualitatively, investors' expected gains should be strictly positive to justify their initial purchase decision. Quantitatively, investors' expected gains should be reasonably related to market returns. For the average holding period (230 days) in the sample, the market return is 4.8% five years prior to the sample period and 5.9% during the sample period, both very close to the 5.5% estimate.

Section III extends the empirical analysis to allow heterogeneous reference points, particularly between (i) frequent traders and infrequent traders, and (ii) stocks with good price history and bad price history. A reference point determined by the status quo does not predict systematic differences along these dimensions but the one determined by expectations does: First, compared to infrequent traders, frequent traders should have a lower bottom of the V, and their relationship between the probability of holding a stock and gains from the stock should be closer to a perfect V shape. These predictions follow because frequent traders hold their stocks for only a short period, which makes their expected gains lower (given positive expected returns) and closely bunching together. Second, controlling for changes in beliefs about future stock returns, stocks with mostly appreciating prices after purchase should demonstrate a higher bottom of the V, simply because past returns are part of the total gains expected from investing in a stock, even in the absence of learning. These predictions are confirmed by the data.

Section IV concludes the paper.

I. Theoretical Model

A. Set-up

The section looks at a static wealth allocation problem in which an investor has to decide on how to split her wealth between a risky asset (stock) and cash. For simplicity I assume no short selling, no time discounting, and no return and inflation risk on cash. The net return of the stock is r_t, $t = 1,2$, which is independently and identically distributed with a continuous distribution $f(r_t)$ on the support $(-1,+\infty)$. The investor starts with a given initial wealth W_0 out of which $x_0 P_0$ is allocated to the stock, and the rest to cash, where x_0 is the number of shares and P_0 is the purchase price. $P_t = P_{t-1}(1+r_t)$ and $g_t = (P_t - \overline{P}_{t-1})/\overline{P}_{t-1}$ denote price and gain from the stock in period t respectively, where $\overline{P}_{t-1}$ is the average purchase price at the end of period $t-1$. To keep the model static, I treat the initial position as given here, but as Barberis and Xiong (2009) and Hens and Vlcek (2005) suggest, I impose positive expected return $E(r_t) > 0$ to reflect the restriction on beliefs about returns implied by the initial purchase decision. Section I.C discusses the dynamic problem and the implications of including the initial decision into the analysis.

In period one, $P_1 = P_0(1+r_1)$ is realized. The investor chooses a stock position x_1 to maximize the gain-loss utility in period two given the reference point W_1^{RP} and subject to the budget constraint $0 \le x_1 P_1 \le W_0 + x_0 P_0 r_1$. In period one, $g_1 = r_1$, because the average purchase price at the end of period zero is simply the initial purchase price $\overline{P}_0 = P_0$.

In period two, $P_2 = P_1(1+r_2)$ is realized. The investor incurs the gain-loss utility over changes in wealth relative to the reference point W_1^{RP}. Equation (1) specifies the expected gain-loss utility in period two.[15] [16]

$$E(U(W_2 \mid W_1^{RP})) = E(1_{\{W_2 - W_1^{RP} > 0\}}(W_2 - W_1^{RP}) + \lambda 1_{\{W_2 - W_1^{RP} \leq 0\}}(W_2 - W_1^{RP})) \tag{1}$$

$$W_2 = W_1 + x_1 P_1 r_2 = W_1 + x_1 P_0 (1+g_1) r_2 \tag{2}$$

$$W_1^{RP} = W_0 + x_0 \overline{P}_0 g_1^{RP} = W_0 + x_0 P_0 g_1^{RP} \tag{3}$$

W_2 is the wealth level in period two. W_1^{RP} is the deterministic reference point relevant for period-one decision and period-two utility. The reference point is lagged in the sense that it adjusts to neither the price realization P_1 nor the position x_1 in period one, reflecting the possibility that the investor cannot make peace with the current situation immediately. It is perhaps realistic and certainly convenient to focus on g_1^{RP}, the reference level of gains in period

[15] Following BX (2009), this paper defines utility directly over wealth. Equation (1) can be understood as an implicit function that reflects utility from an optimal consumption plan in the future given certain wealth level today. The fact that wealth fluctuations compared to a reference point generate utility today can be motivated by the implied changes in future consumption plan relative to a reference point, a concept that Kőszegi and Rabin (2009) term as "prospective gain-loss utility".

[16] Kőszegi and Rabin (2006) develop a more general version of the reference-dependent model in which the total utility is a weighted average of the consumption utility and gain-loss utility, and the unit for gain-loss comparison is also the consumption utility:

$$E(U(W_2 \mid W_1^{RP})) = E(u(W_2) + \eta(1_{\{W_2 - W_1^{RP} > 0\}}(u(W_2) - u(W_1^{RP})) + \lambda 1_{\{W_2 - W_1^{RP} \leq 0\}}(u(W_2) - u(W_1^{RP})))).$$

$u(.)$ is the traditional consumption utility and η is the weight attached to the gain-loss utility. This more general version keeps the essentials of loss aversion while incorporating the effect of standard consumption utility. Equation (1) can be viewed as a simplified version in which the consumption utility is linear and it has a negligible weight. Having a concave consumption utility function does not change the V shape, but it shifts the bottom of the V shape away from the reference point. Having a non-zero weight on the consumption utility does not change the V shape either, since it only affects the gain-loss utility quantitatively but not the qualitatively.

two from period one's perspective, also the level of gains in period one at which the investor can simply sell the entire stock position and reach the reference point in period two. $g_1^{RP} = 0$ corresponds to the status quo assumption while $g_1^{RP} = g_1^{E} = E((1+r_1)(1+r_2)) - 1$ treats the lagged expected gain as the reference level.[17] Positive expected return $E(r_t) > 0$ ensures positive expected gain $E(g_1^{E}) > 0$. The loss indicator $1_{\{W_2 - W_1^{RP} \le 0\}}$ takes the value "one" if there is a loss relative to the reference point ($W_2 - W_1^{RP} \le 0$), otherwise "zero". If wealth in period two falls below the reference point, their difference is multiplied by $\lambda > 1$, representing loss aversion. Without assuming diminishing sensitivity, the utility function is piece-wise linear. Thus the kink at the reference point characteristic of loss aversion is the only source of risk aversion.

Following BX, my model makes a non-trivial assumption called "narrow framing" or "mental accounting" (Thaler 1990). First of all, I assume that the investor opens a mental account for each stock after purchase and closes the account once the stock is sold. Thus she incurs gain-loss utility at the individual stock level rather than the portfolio level. Correspondingly, W_0 can be viewed as the maximum amount of money that she is willing to lose on a particular stock (Barberis, Huang and Thaler (2006), and Barberis and Xiong (2009)).

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[17] This paper's specification of expectations as the reference point departs from Kőszegi and Rabin's (2006) model in the following aspects. First of all, reference point here is specified as the mean expectation rather than the whole stochastic distribution. It turns out that depending on the returns distribution, stochastic reference point may or may not affect the quasiconvex relationship between the optimal position in a stock and gains from that stock (see section 1.C and appendix C). Second, the reference point is exogenous to g_1 and x_1. Endogenous reference point as in Kőszegi and Rabin's (2006) "personal equilibrium" does not generally lead to the V shape observed in the data. For the purposes of explaining the disposition effect and distinguishing between the status quo and expectations, it is therefore a better choice to start with a deterministic and exogenous reference point.

Barberis and Huang (2001) show that treating trading decision as if investors were considering each stock separately fits the empirical facts better than including portfolio choice. Odean (1998) also shows that the disposition effect cannot be explained by portfolio concern. Second, I assume that the investor evaluates her investment outcomes and incurs gain-loss utility over a certain narrow period. Benartzi and Thaler (1995) call it "myopic loss aversion". According to their estimation from the aggregate stock returns, the average evaluation period is about one year. My static model can thus be viewed as describing the optimal decision within one such evaluation period.

I do not take a position on what the reference point is when solving the model. Instead I derive the model's predictions for any exogenous and deterministic reference point, in preparation for section II's econometric analysis, where the model will be used to infer its location from the patterns in the data.

B. Solution

Loss aversion introduces a cut-off point that divides future return r_2 into those generating gains and those generating losses relative to the reference level, which are assigned weight 1 and $\lambda > 1$ respectively. The cut-off point $K(x_1) = \frac{x_0}{x_1}(\frac{1+g_1^{RP}}{1+g_1} - 1)$ is a function of current position x_1, given the reference level g_1^{RP} and current gain g_1. $K(x_1)$ is a specific level of r_2 that makes period two wealth equal to the reference point $(W_2 = W_1^{RP})$. Equation (4) gives the expected marginal utility of holding an additional share.[18]

$$E(MU(K(x_1))) = P_1(\int_{K(x_1)}^{+\infty} r_2 f(r_2)dr_2 + \int_{-1}^{K(x_1)} \lambda r_2 f(r_2)dr_2) \tag{4}$$

The optimal interior solution x_1^* satisfies the first-order condition $E(MU(K(x_1^*))) = 0$. Under the piece-wise linear assumption, the choice variable x_1 enters the first-order condition only through $K(x_1)$, which is the sufficient statistic for the optimal solution. Proposition 1 gives the optimal interior position. The restrictions imposed on the returns distribution indicate that returns should be good enough to induce purchase in the first place ($E(r_t) > 0$), but they should not be too lucrative to make even a loss-averse investor never want to sell the stock ($E(MU(K(x_1) = 0)) < 0$).

Proposition 1. (See appendix A for proof) For any returns distribution $f(r_t)$ satisfying $E(r_t) > 0$ and $E(MU(K(x_1) = 0)) < 0$, there exist two deterministic return levels $K_1 < 0$ and $K_2 > 0$ that satisfy $E(MU(K_1)) = E(MU(K_2)) = 0$. The optimal interior position is given by

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[18] Equation (4) should also include the effect of the change in x_1 on the cut-off point $K(x_1)$. But this term is zero by the fact that at the return level $K(x_1)$, future wealth equals the reference point $W_2 = W_1^{RP}$. Marginal change in $K(x_1)$ thus brings almost no change to the expected gain-loss utility.

$$x_1^* = \begin{cases} \frac{x_0}{K_2}(\frac{1+g_1^{RP}}{1+g_1}-1), for\ g_1 < g_1^{RP} \\ \frac{x_0}{K_1}(\frac{1+g_1^{RP}}{1+g_1}-1), for\ g_1 > g_1^{RP} \\ 0, \qquad\qquad\qquad for\ g_1 = g_1^{RP} \end{cases} \tag{5}$$

The budget constraint $x_1^* P_1 \le (W_0 + x_0 P_0 r_1)$ binds for extremely high and low g_1.

With a binomial returns distribution, BX (2009) show that it is optimal for the investor to gamble until the highest wealth leaves her at or near the reference point if she is currently in the losses domain, and vise versa. Proposition 1 confirms and generalizes this conclusion with any continuous returns distribution, by showing that the investor will gamble until the wealth generated by a fixed return level $K_2 > 0$ ($K_1 < 0$) reaches the reference point when she is currently in the losses (gains) domain. Given such optimal strategy, it is easy to see how the optimal position changes with different levels of gains.

Corollary 1. The optimal interior position x_1^*

(i) decreases in g_1 when $g_1 < g_1^{RP}$ and increases in g_1 when $g_1 > g_1^{RP}$.

(ii) is concave in g_1 when $g_1 < g_1^{RP}$ and convex in g_1 when $g_1 > g_1^{RP}$.

Different levels of current gains generate different distances from the reference level g_1^{RP}. The closer g_1 is g_1^{RP}, the more risk averse the investor is, so she takes few risks by demanding few shares of the stock. Meanwhile she takes more risks by enlarging the position in the stock as g_1 deviates farther from g_1^{RP}. These facts bring a V-shaped relationship between the optimal position and gains from the stock, with the bottom of the V shape reached at the reference level $g_1 = g_1^{RP}$. Such relationship comes from both loss aversion and the monotonic probability of crossing the reference point as the position becomes larger. The optimal position x_1^* is concave when $g_1 < g_1^{RP}$ and convex when $g_1 > g_1^{RP}$ because it is more expensive to purchase more shares when stock price is higher. Consequently, the tendency to enlarge the optimal position is mitigated (exacerbated) as gains become larger (smaller). [19]

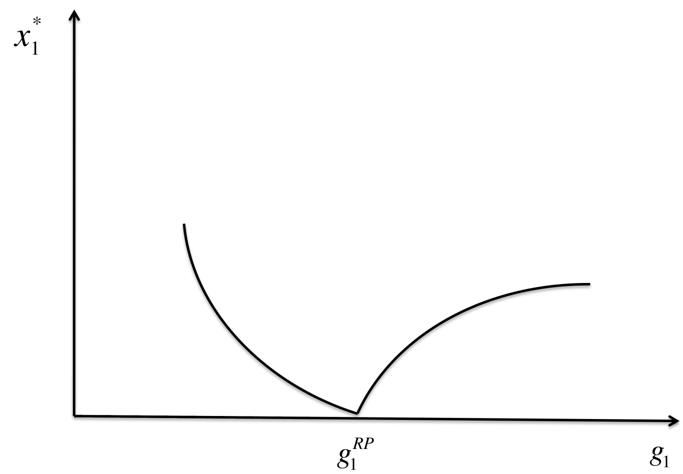
Figure 1 illustrates the V-shaped relationship in the region where the budget constraint does not bind. The optimal position reaches its minimum with a kink at the reference level $g_1 = g_1^{RP}$. This kink comes from different cut-off return levels K_1 and K_2 used below and above the reference level g_1^{RP}. It is clear that the investor is most likely to sell the stock around g_1^{RP}, given a fixed x_0. For gains located symmetrically around g_1^{RP}, which one leads to a larger optimal position x_1^* is ambiguous.[20] However, since the disposition effect describes asymmetric behavior around zero gains, the asymmetry around the reference level g_1^{RP}, if any, is less relevant in explaining the disposition effect once g_1^{RP} is different from zero.

[19] Comparative statics analysis shows that higher loss aversion coefficient λ leads to smaller x_1^* at every level of current gains. Loss aversion, like conventional risk aversion, reduces investment demand for the risky asset. Similarly, any move of probability mass from positive returns to negative returns decreases the optimal position at all levels of current gains, but the V-shaped relationship remains unchanged.

[20] The answer depends on where K_1 and K_2 stand relative to zero, which in turn depends on the returns distribution. For example, any returns distribution with an increasing $f(r_t)$ in the region $r_t \in [K_1, K_2]$ implies $|K_1| > |K_2|$, leading to a relatively small x_1^* at the high gains level in the symmetric pair.

Figure 1: The V-Shaped Relationship

This figure shows the relationship between the optimal position x_1^* from a given stock and gains g_1 from that stock for any exogenous and deterministic reference level g_1^{RP}.



With the V-shaped relationship, it is convenient to illustrate both BX's argument for why loss aversion fails to predict the disposition effect when zero gains is treated as the reference level (figure 2) and how assuming positive expected gain as the reference level generates the disposition effect (figure 3).

In figure 2 BX's argument relies crucially on $g_1^{RP} = 0$. Under returns distribution with positive mean, a typical realization of gains (g_1^G) will be relatively farther away from zero compared to that of losses (g_1^L). It therefore takes a larger position for future wealth generated by the cut-off return K_1 to reach the reference point at g_1^G. Diminishing sensitivity in BX's model mitigates this tendency in favor of the disposition effect but is not enough to totally offset it.

Figure 2: The V-Shaped Relationship when the Reference Point is the Status Quo

This figure shows the relationship between the optimal position x_1^* in a given stock and gains g_1 from that stock when the reference level is determined by the status quo ($g_1^{RP} = 0$). g_1^G and g_1^L are the typical realization of gains and losses respectively.

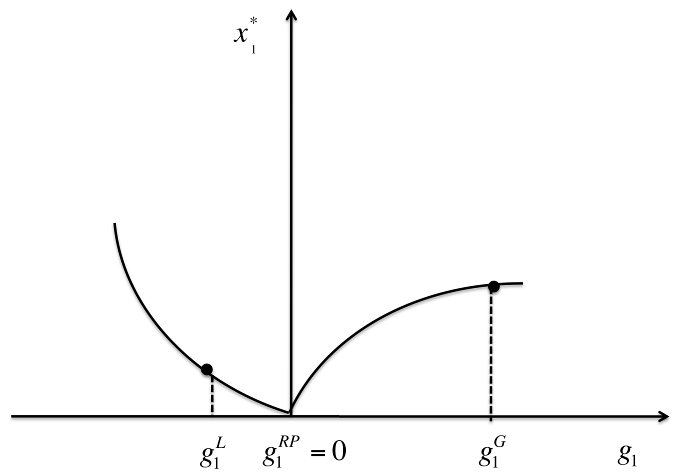
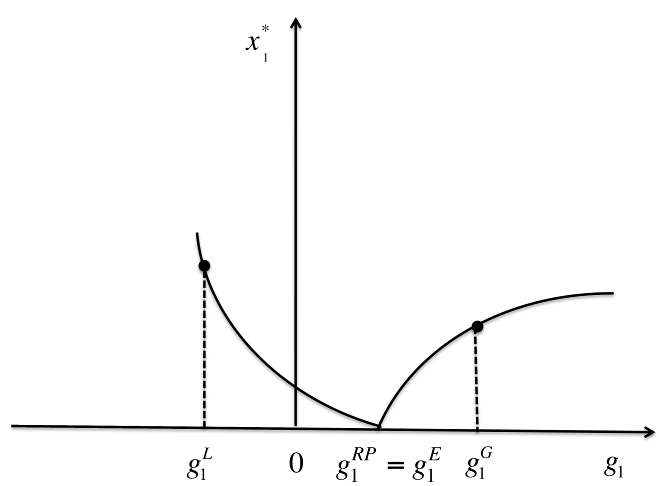


Figure 3: The V-Shaped Relationship when the Reference Point is Expectation

This figure shows the relationship between the optimal position x_1^* from a given stock and gains g_1 from that stock when the reference level is determined by expected gain ($g_1^{RP} = g_1^E$). g_1^G and g_1^L are the typical realization of gains and losses respectively.



As figure 3 illustrates, this paper generates the disposition effect without relying on diminishing sensitivity because of the shift of the reference level of gains from zero to the positive expected gain, making g_1^G closer to g_1^{RP} than g_1^L. Because the investor is most likely to sell around the reference level due to loss aversion, she is on average more likely to sell the stock when it has a gain than when it has a loss.

C. Robustness

My simple static model predicts a V-shaped relationship between the optimal stock position in a given stock and gains from that stock. This relationship is sufficient but not necessary to generate the disposition effect, because the disposition effect does not require an increasing tendency to hold the stock as gains become large. The model also implies that the bottom of the V shape is the reference level. I discuss the robustness of these results by relaxing one assumption at a time. It turns out that the V-shaped relationship is relatively robust, but not to the endogenous reference point, or the stochastic reference point with certain distributions. Under more general conditions, however, the bottom of the V shape may not be reached exactly at the reference level. But the bottom point is still largely driven by and therefore provides valuable information about the reference point.

• **Multiple periods:** In a reasonable dynamic model the investor has many periods to make choices and incur gain-loss utilities. In the beginning of period t, she learns about P_t (hence g_t) and incurs gain-loss utility relative to the reference point W_{t-1}^{RP}. Then she forms a new (but still lagged) reference point W_t^{RP} for utility in period $t+1$ and chooses x_t accordingly. The investor understands that her choice this period will affect wealth levels and reference points in the following periods. Formally, the investor's decision problem is

$$\max_{\{x_0, x_1, \ldots, x_T\}} \sum_{t=0}^{T} E_0(\beta^t U(W_{t+1} \mid W_t^{RP})) \tag{6}$$

Where

$$U(W_{t+1} \mid W_t^{RP})) = 1_{\{W_{t+1} - W_t^{RP} > 0\}}(W_{t+1} - W_t^{RP}) + \lambda 1_{\{W_{t+1} - W_t^{RP} \leq 0\}}(W_{t+1} - W_t^{RP})) \tag{7}$$

$$W_{t+1} = W_t + x_t P_t r_{t+1} = W_t + x_t \overline{P}_{t-1}(1+g_t) r_{t+1} \tag{8}$$

$$W_t^{RP} = W_{t-1} + x_{t-1} \overline{P}_{t-1} g_t^{RP} \tag{9}$$

The investor chooses the optimal positions $\{x_0^*, x_1^*, \ldots, x_T^*\}$ to maximize the summation of the gain-loss utilities from period 0 up to a final period $T+1$, given the reference points $\{W_0^{RP}, W_1^{RP}, \ldots, W_T^{RP}\}$. By definition $x_{T+1} = 0$ and the investor sells the entire position. W_0^{RP} is taken as given before making initial decision, but $\{W_1^{RP}, W_2^{RP}, \ldots, W_T^{RP}\}$ are partially determined by $\{x_0^*, x_1^*, \ldots, x_{T-1}^*\}$. I again focus on the sequence of deterministic reference level of gains $\{g_0^{RP}, g_1^{RP}, \ldots, g_T^{RP}\}$.

Proposition 2. (See appendix B for proof) In the dynamic model defined above, the optimal interior position x_t^* is decreasing in g_t when $g_t < g_t^{RP}$ and increasing in g_t when $g_t > g_t^{RP}$.

According to proposition 2, the general V shape is robust to the dynamic consideration. The bottom point of the V shape is still determined by the reference point. The intuition is the following: Both future wealth levels and reference points adjust to gains g_t in the same direction so their differences (the gains and losses) are not affected by g_t. The current position x_t has a non-zero effect on future gain-loss utilities but such effect is orthogonal to g_t. Therefore the dynamic consideration affects the level of x_t^* but not its relative relationship to g_t.

The dynamic model nonetheless sheds light on the impact of including the initial purchase decision into consideration, which is potentially important because it restricts the range of expected returns we should focus when discussing the subsequent selling decision.[21] As BX correctly suggest, the investor should purchase stocks with only strictly positive expected returns, especially when she is loss averse with respect to the status quo. If her sequence of reference points is determined by positive expectations, however, she is willing to accept stocks with lower (but still positive) expected returns. Because only by taking more risks can she have the opportunity to obtain the desired positive gains. On one hand, since the disposition effect in my model is driven by treating the positive expectations as the reference points, a lower but still

[21] For a consistent investor who believes that the returns distribution is independently and identically distributed, if the reference level of gains is zero and remains so over time, once she purchases the stock she will never want to sell. This theoretical reasoning makes it unlikely for the status quo to be the reference point in my simple static model. The problem of never wanting to sell does not exist under the additional assumptions such as dynamic adjustment of the reference point, diminishing sensitivity and stochastic reference points. My empirical analysis, however, does not impose the theoretical structure on estimation, so the econometric model is free to pick up zero as the reference level, if the data suggests so. If I indeed estimate a bottom point of the V shape at zero, then I need to modify my static model to address the problem of never wanting to sell. However, given that this is not happening in the data, this issue seems to be minor.

positive expected return makes the disposition effect weaker but still present. On the other hand, a lower expected return reduces the asymmetry of gains around zero, which according to BX's argument, favors the disposition effect.

- **Stochastic reference point:** If the investor's reference point is stochastic in nature, the overall utility is a probability-weighted average of the gain-loss utilities relative to different realization of the reference point.

Proposition 3. (See appendix C for proof) For a stochastic reference level of gains g_1^{RP} with the density $h(g_1^{RP})$, there exists a lower bound $\underline{g}_1$ and a higher bound $\overline{g}_1$ such that: the optimal position x_1^* is decreasing in g_1 for $g_1 < \underline{g}_1$ and increasing in g_1 for $g_1 > \overline{g}_1$. The relationship is ambiguous when $\underline{g}_1 \leq g_1 \leq \overline{g}_1$.

Although the V shape does not literally exist given stochastic reference point, its essential nature is still preserved. However, there is hardly any bottom point, global or local, that can be identified with the reference point. A highly relevant case under the stochastic reference point is the possibility of comparing outcomes to both the status quo and expectations. Depending on the returns distribution $f(r_2)$ and the probability attached to each reference point, there are two possible patterns. Under the "single-trough" pattern the optimal position is still V-shaped but the bottom of the V is located between the status quo and expected gain. Under the "twin-trough" pattern the optimal position is W-shaped, with a local minimum position at each reference level.

- **Endogenous reference point:** An endogenous reference point that is affected by the decision variable x_1 does *not* create the kind of variations in psychological gains and losses as x_1 and P_1 vary that is necessary to keep the V shape.[22] It is interesting to test the predictions of endogenous reference point, but given a strong V-shaped pattern in the data, it seems more natural to explore the possibilities of models that explain this pattern with exogenous reference point first.

- **Diminishing sensitivity:** Making the gain-loss utility function concavity above and convexity below the reference point as Tversky and Kahneman (1992) estimate them keeps the V-shaped relationship, because diminishing sensitivity itself generates monotonically decreasing position in a given stock as gains from that stock increase. The bottom of the V shape, however, is not exactly at the reference level. Instead it is pushed slightly to the right of its reference level (see Gome (2005), and Barberis and Xiong (2009)). Since the gain-loss utility function with diminishing sensitivity is not globally concave, there is a discontinuous decline in the optimal position at the bottom of the V shape (Gomes (2005)), partially reflecting the power of diminishing sensitivity to explain the disposition effect.

[22] Gill and Prowse (2009) provide the first lab evidence for the existence of endogenous reference point

II. Empirical Analysis: Overall Sample

A. Data Summary

The dataset is from a large discount brokerage house on the investments of 78,000 households from January 1991 through December 1996. It was used by Odean (1999) and Barber and Odean (2000,2001) but is different from the one that Odean (1998) used in his pioneering analysis of the disposition effect. The original dataset includes end-of-month position statements, trading records (trade date, trade quantity and trade price) for each stock held by each account, together with some background information about the account owners (e.g. gender, age, income and net wealth). Odean (1999) gives more detailed description of this dataset. I also have data for the daily stock price, daily trading volume, shares outstanding, an adjustment factor for splits and dividends and market returns (S&P) from CRSP. Appendix D reports my data cleaning process.

In this study I focus on the trading of common stocks at the individual stock level, ignoring portfolio concern. My empirical analysis relies on constructing an investor's trading history for each stock she holds. The history includes dates of purchase, hold and sale. Following Odean (1998) and Grinblatt and Keloharju (2001), I generate observations of hold dates in the following way: Any time that at least one sale takes place in the portfolio, I count the untraded stocks in the portfolio as holds and obtain price information from CRSP for them. These are stocks that investors could have sold but did not. In other words, I select holding dates conditional on having at least one sale taking place in the portfolio on that day. This procedure is standard in the empirical literature of the disposition effect. It ensures that any holding decision in the sample comes from deliberation rather than inattention.[23]

Table I reports the summary statistics of the dataset. All prices are appropriately adjusted for commission, dividend and splits so the calculated gains from a stock include both capital gains and other forms of income.[24]

Table I: Summary Statistics

	Mean	Standard Deviations	Min	Max	Observations
Holding Days	230	290	1	2126	279,968
Paper Gains	0.005	0.353	-1	24.01	848,756
Realized Gains	0.051	0.371	-1	21.55	279,968
Portfolio Size	4.044	5.677	1	309	1,128,724
Principal at the Initial Purchase	10,622	28,302	6	6,011,361	394,637
Commission per Share	.448	1.319	0	55	394,637
Income	95,851	3,728,442	500	588,671,000	36,174
Age	55	14	1	80	18,724

For each investor, I construct the variable "trading frequency" as the inverse of the average day between two trades using her entire trading records. Investors with trading frequency higher

[23] For the purpose of studying the relationship between the optimal position in a stock and gains from that stock, it does not necessarily imply sample selection problem. However, there is some risk of bias if portfolio size and trading frequency are correlated to gains. I control for these potential confounding factors in the regression.

[24] Whether to adjust for commissions doesn't generate a big difference in the estimates.

(lower) than the mean are treated as frequent (infrequent) traders. Frequent traders on average trade every month and infrequent traders trade every five months. To measure the desirability of price history during the holding period, I calculate the proportion of increasing prices (compared to the price of the previous observation) between the initial purchase date and the date in question, and assign observations with this ratio higher (lower) than the mean as having good (bad) history, in the sense that the stock price keeps going up (down) on average after the initial purchase.[25] Stocks with good price history yield an average gain of 22.4% and those with bad price history yield an average loss of 16.3%. For both sample-splitting criterions, ties are randomly assigned to each group.

I follow Odean's (1998) method of measuring the disposition effect to analyze his (1999) dataset and report the results in table II. Odean (1998) count winners and losers relative to the average purchase price.[26] He defines the proportion of gains realized (hence PGR) as the number of realized gains divided by the number of realized and paper gains. Similarly the proportion of losses realized (hence PLR) is defined as the number of realized losses divided by the number of realized and paper losses.

[25] I have tried other ways to measure the desirability of price history, including constructing the proportion of positive gains and the proportion of positive market adjusted gains between the initial purchase date and the date in question. To check whether investors judge good or bad history by more recent history, I also calculate these ratios for the past week or past two months. The qualitative results of this paper are robust to these alternative specifications.

[26] A hold observation is counted as a gain (loss) if the lowest (highest) price of that day is higher (lower) than the average purchase price.

Table II: Proportion of Losses Realized (PLR) and Proportion of Gains Realized (PGR)

This table calculates the proportion of gains realized (PGR) and the proportion of losses realized (PLR) following the strategy of Odean (1998 Table I). PGR is calculated as the ratio between numbers of realized gains and total (realized and paper) gains; PLR is calculated as ratio between numbers of realized losses and total (realized and paper) losses. RG, PG, RL, PL represent numbers of realized gains, paper gains, realized losses and paper losses. The standard error for the t-statistic is constructed by $\sqrt{\frac{PGR(1-PGR)}{RG+PG}+\frac{PLR(1-PLR)}{RL+PL}}$.

	Overall Sample	Dec.	Jan.-Nov.	Frequent Traders	Infrequent Traders	Good History	Bad History
PGR	0.360	0.309	0.365	0.227	0.424	0.380	0.218
PLR	0.245	0.313	0.238	0.152	0.304	0.248	0.244
PLR-PGR	-0.115	0.004	-0.127	-0.076	-0.120	-0.132	0.026
t-statistic	-130.054	1.326	-137.925	-58.724	-104.358	-69.649	14.496
RG	184,802	11,872	172,930	37,475	147,327	171,456	13,346
RG+PG	512,647	38, 434	474, 213	164,974	347,673	451,519	61,128
RL	136,041	15,572	120,469	32,780	103,261	15,051	120,990
RL+PL	556,375	49,741	506,634	216, 325	340,050	60,746	495,629

My dataset does exhibit substantial disposition effect. In the overall sample, investors realize significantly more gains than losses. The difference is large (PLR-PGR=-0.115).[27] In December the difference is not significant, probably due to tax incentive to realize more losses than gains.[28] The difference between frequent traders and infrequent traders is also consistent with Odean's finding. Although both types demonstrate the disposition effect, infrequent traders

[27] The difference between PLR and PGR for the overall sample is -0.05 in Odean (1998 table I), smaller than the one reported in table II. The qualitative nature is nonetheless consistent across the two samples.

[28] Because of tax on positive capital gains, it is often more beneficial for investors to realize losses than gains at the end of the year. Ivković, Poterba and Weisbenner (2005) identify the existence of tax-driven selling using the dataset in this paper.

are especially vulnerable to it.[29] The distinction between good history and bad history, which has not been investigated before, also reveals a surprising asymmetry. Investors with good price history are more likely to sell stocks with gains and hold on to those with losses. The pattern is reversed in the case of bad history.

B. The Probability of Holding a Stock: Illustration.

While my model predicts a relationship at the individual level, this section's empirical analysis pools the observations across investors. For comparison purpose, investors' positions on stocks are normalized relative to the initial purchasing positions. In the individual trading records constructed from my dataset, only 4% of the observations are partial sales or repurchases. In other words, investors sell either all or none of their positions most of the times. This fact makes the normalized stock positions essentially binary, and so I can further proxy the normalized optimal position of account i, stock j and time t using a binary "hold" variable h_{ijt} ($h_{ijt} = 0$ if sell and $h_{ijt} = 1$ otherwise).[30] Also let $g_{ijt} = P_{ijt} / \overline{P}_{ijt} - 1$ denote account i's gain from investing in stock j at time t, where $\overline{P}_{ijt}$ is the average purchase price. Because of the almost

[29] BX (2009) explain this fact by noting that infrequent traders have long average holding period, so future independent risks can cancel each other and make infrequent traders more willing to accept stocks with lower expected returns today. Gains and losses are then distributed more symmetrically around zero, making their model more likely to generate the disposition effect. I develop an alternative explanation in section III based on the effect of the reference points defined by expectations.

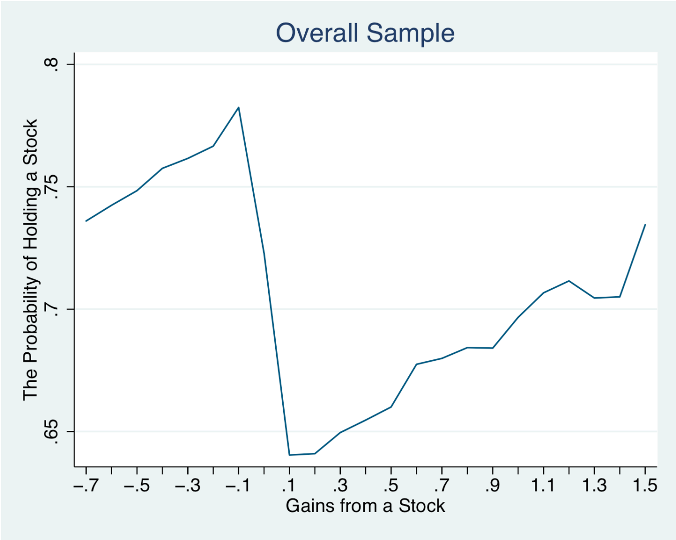
[30] For comparison purpose h_{ijt} is purposefully constructed to be exactly one minus the "sell" dummy variable normally used in the literature.

binary nature of the decision in the sample, $\overline{P}_{ijt}$ takes constant values at the initial purchase price most of the times.

Figure 4 illustrates the probability of holding a stock calculated as the average of the dummy variable h_{ijt} within each 10% gains interval. The patterns in this and the upcoming figures are shown to be robust to the influences of trading frequency, portfolio size, and the stock's own returns and market returns in the past two months, etc. (see the full set of control variables in table III and appendix F.)

Figure 4: the Probability of Holding a Stock (Overall Sample)

This figure shows the probability of holding a stock across different levels of gains for the overall sample. The probability of holding a stock is calculated as the average of a dummy variable h_{ijt} ($h_{ijt}=0$ if sell and $h_{ijt}=1$ otherwise)) within each 10% gains interval.



The relationship is clearly non-monotonic. Starting from a loss of 10% it is also V-shaped: The probability of holding a stock starts to decline from a loss of 10% to a gain of 10% and rises after a gain of 10%. The overall pattern implies that investors are most likely to sell stocks with small positive gains and hold stocks with large gains and losses. Such pattern implies the disposition effect, but it is a stronger empirical regularity. The left-tail drop in the probability of holding a stock is not directly predicted by loss aversion, and I will discuss possible explanations in section III.

Figure 4 suggests that investors' risk attitudes change non-monotonically as gains increase. Such sharp and non-monotonic change in risk attitudes is not readily explained by expected utility model. The commonly used utility functions (e.g. CRRA) normally imply a monotonic change in risk attitudes as wealth increases, hence a monotonic relationship in figure 4.[31] This largely V-shaped pattern is nonetheless consistent with a model of loss aversion. Further, it is visually clear that the bottom of the V shape is reached at a strictly positive level of gains. This is strong evidence that investors' reference points are affected by something higher than the status quo.[32]

Figure 4(S) is a "zoom-in" version of figure 4 on the interval [-10%, 10%], with h_{ijt} averaged for each 0.1% gains interval to calculate the probability of holding a stock.

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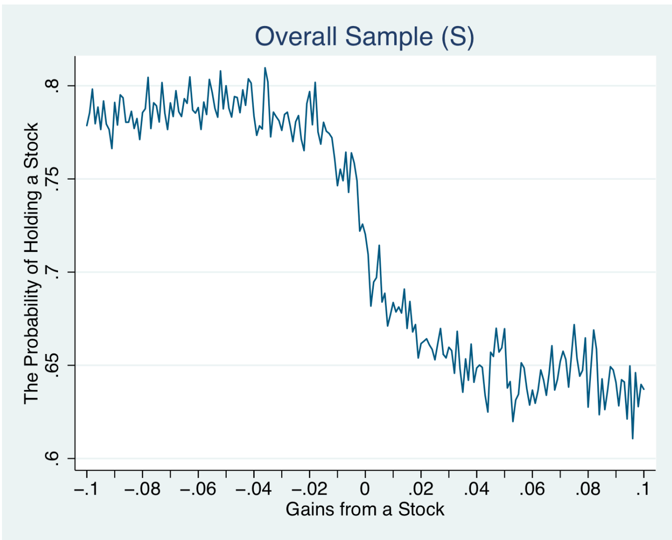
[31] One may argue that because of diversification of risks, gains from one stock in the portfolio may be offset by losses from other stocks so that the effect of gains from individual stock on total wealth is ambiguous, making figure 4 an inaccurate reflection of change in risk attitudes across different wealth levels. This point is well taken. But it should be noted that to rely on this particular point to explain the non-monotonic relationship observed, the expected utility model requires a complicated and specific pattern of correlation among prices of stocks in the portfolios, which may be true but needs more careful study.

[32] In two cases a reference point determined by the status quo can generate a bottom of the V shape located at a positive level of gains. The first one is loss aversion combined with diminishing sensitivity (Gomes 2005), but it cannot be an empirically plausible explanation because it fails to predict the disposition effect (Barberis and Xiong 2009). The second one is Kőszegi and Rabin's (2006) more general reference-dependent model with a concave consumption utility to serve as the unit of gain-loss comparison. However, the magnitude of such shift is likely to be too small to offset the asymmetric gains and losses realization around zero. So it may fail to generate the disposition effect according to BX's argument. Further, both alternatives have a hard time accounting for heterogeneity in the bottom point of the V shape across trading frequency and price history (section III).

Interestingly, although the bottom of the V shape is not reached at zero gains, there is a steep decline in the probability of holding there. This observation suggests the role of the status quo is not completely negligible. My econometric model includes a dummy variable for positive gains to capture the effect of the status quo.[33]

Figure 4(S): the Probability of Holding a Stock (Small Region)

This figure shows the probability of holding a stock across different levels of gains for the gains interval [-10%, 10%]. The probability of holding a stock is calculated as the average of a dummy variable h_{ijt} ($h_{ijt} = 0$ if sell and $h_{ijt} = 1$ otherwise)) within each 0.1% gains interval.



[33] Appendix C analyzes the possibility of treating both the status quo and expectations as reference points. Depending on the parameters, such model may generate prediction indistinguishable from the model with a single reference point determined by expectations. It is also possible that there are heterogeneous investor types, where one type has zero gains while the other type has positive gains as the reference levels. However, in the subsamples generated by trading frequency and price history, reaching zero gains almost always has significant influences over trading behavior. Even if there are different types it seems hard to find an intuitive criterion to sort them out.

C. The Probability of Holding a Stock: Estimation

This section estimates the non-monotonic relationship shown in figure 4 using a multi-threshold model with unknown thresholds (Andrews (1993), Bai (1996), and Hansen (2000a, 2000b)). A formal estimation is intended to check the robustness of the V shape to the inclusion of more control variables and to obtain accurate estimate of the bottom point of the V shape.

In this section I impose homogeneity in the reference point by treating the location of the bottom of the V shape as a single fixed number. The resulting estimate reflects the average reference level across investors and stocks in the sample.[34] However, this homogeneity assumption is obviously too restrictive, especially when expectations is a candidate for the reference point. Section III partially relaxes this assumption by allowing heterogeneity in trading frequency and price history.

The underlying specification is a linear probability model regressing the binary holding decision h_{ijt} on gains from the stocks g_{ijt} and other variables represented by the vector X_{ijt}.[35] I

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[34] If there is heterogeneity in the threshold, and if it is correlated with the regressors in the equation, then the current specification causes biases to the slope parameters. However, whether it leads to systematic bias on the estimated location of the threshold is unknown. Section III adds some important heterogeneity to attenuate this problem. To remedy this problem, I have also tried maximum likelihood estimation where I specify the threshold as a linear function of holding period, trading frequency, price history, and a random error. The estimation failed to converge possibly because of the multiple changes in slope. Specifying the threshold as a linear function of these variables without a random component requires developing new tests beyond the scope of Hansen (2000). I leave this to future research.

[35] Another reasonable alternative is to use the Cox proportion hazard model, which has the advantage of controlling for the effect of holding period non-parametrically. However, as far as I know, there is no appropriate procedure to test the location of the threshold in the Cox model. What's more, in my sample holding period has an almost linear effect on the probability of holding a stock, thus a linear model may not cause severe bias. To double check, I include dummy variables for different lengths of holding period to allow nonlinear effect, and the estimation results are essentially unchanged. Although my model implies that the probability of holding is concave below and convex above the bottom of the V shape, I choose to estimate the simple linear model as the first step. More complicated models such as a polynomial, or nonparametric estimation can be used to obtain more accurate estimates.

use linear rather than nonlinear specification for the probability mainly because the multi-threshold model I use (Hansen 2000) applies to the linear case. Also significant bias due to probability boundary effects is unlikely because my sample selection of holding observations is conditional on having at least one sale taking place on that day. As the estimation results show, the linear model fits the relationship reasonably well.[36]

Equation (10) represents the specification with one threshold. Later I introduce a sequential procedure to estimate multiple thresholds based on this specification. The reason to allow for multiple thresholds is the observation from figure 4 that besides the slope change at the bottom of the V shape there is also an obvious slope change at a slight negative level of gains. We need to control for such structural change to allow for a better estimate of the bottom point.

$$h_{ijt} = \alpha_0 + \alpha_1 I(g_{ijt} > 0) + \beta_0 g_{ijt} + \beta_1 \max(g_{ijt} - g^s, 0) + X_{ijt}\gamma + \varepsilon_{ijt} \tag{10}$$

Equation (10) allows the slope of gains to change at an unknown threshold g^s while keeps the probability of holding continuous.[37] I am interested in both the magnitude of the change β_1 and the location of the threshold g^s. $I(g_{ijt} > 0)$ is introduced to capture any influence generated

[36] I include 5th and 95th percentiles of the predicted probability of holding in related tables in appendix E to show that most of the predicted values are within the range (0,1).

[37] Equation (10) imposes two restrictions on parameters. First, the slopes of control variables (γ) do not change at the threshold. There are no particular reasons that I can think of for variables such as holding period, portfolio size and past returns to have the threshold effect. So this restriction is made to achieve high efficiency. Second, the regression function is assumed to be continuous at the threshold, because the purpose of the estimation is to find the bottom point of the V shape, which is reflected by a slope change rather than a discrete jump. To check the robustness of the results to these two restrictions, I estimate a model that allows all the coefficients to change at the threshold, including the intercept. The locations of the thresholds are not much different. It also turns out that most control variables do not experience significant changes at the thresholds, except for some variables indicating past returns. There is also no significant discrete jump in the probability of holding at the threshold.

by the distinction between gains and losses. This term controls for the effect of the status quo, and so allows the model to concentrate on finding the reference level via estimation of the bottom point. Further, since my focus is to find the point where slope changes, controlling for the intercept change at zero gains still permits the model to estimate a threshold there, if any. X_{ijt} contains control variables.

A common problem with estimating a threshold model with unknown threshold is the existence of nuisance parameter (see the discussion in Andrews (1993), and Hansen (1996)). Under the null-hypothesis of no change in slope ($\beta_1 = 0$), the threshold g^s does not even exist. The test of any change in parameter value is therefore non-standard and normally requires simulation. Card, Mas and Rothstein (forthcoming) propose a simple solution to this problem: They randomly split the data into an estimating sample and a testing sample, using the former to estimate the thresholds and the latter to test the magnitude of parameter changes taking the estimated thresholds as given. This procedure allows for a standard hypothesis testing. I follow their estimation strategy.

I am also interested in testing the hypothesis that $g^s = 0$. Hansen (2000a) constructs a confidence interval for the estimated threshold based on the likelihood ratio test.[38] His test statistic is non-standard but free of nuisance parameter problem. I follow his econometric technique.

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[38] He makes an assumption from the change-point literature, which states that as sample size increases to infinity, the change in the parameter value converges to zero. It implies that the statistical test and confidence interval are asymptotically correct if the change in the parameter value is small.

Bai (1997) and Bai and Perron (1998) propose a sequential procedure to estimate multiple thresholds with efficiency.[39] The first step is to perform a parameter constancy test to the entire sample and estimate a threshold if the test is rejected. The second step is to split the sample into two subsamples using the estimated threshold from the first step and estimate a threshold in each subsample, if any. Continue this process until no further threshold is detected on each subsample. The third step is to go back and re-estimate the thresholds that are previously estimated using samples containing other thresholds. The third step ensures efficiency. I follow this sequential procedure in estimating multiple thresholds.

Table III reports the estimates of the key parameters. Column 1 regresses the binary holding decision on gains from the stock and a dummy variable that indicates positive gains. Column 2 additionally controls for December effect, holding period, trading frequency, portfolio size, tax rate, income, net wealth, daily trading volume and total shares out. Column 3 further adds market returns and the stock's own returns dating back as far as two months to control for beliefs.[40] β_1 and β_2 are the changes in the slope of gains at threshold I and II respectively. By definition, these changes are all significantly different from zero at 1% significance level. Instead of reporting β_1 and β_2 separately, table III reports $\beta_0 + \beta_1$ (the slope between threshold I and threshold II) and $\beta_0 + \beta_1 + \beta_2$ (the slope after threshold II).

[39] Bai and Perron (1998) construct a model that can estimate and test multiple change points simultaneously. They show that the sequential procedure introduced here is consistent with the simultaneous estimation.

[40] These include the market returns and the stock's own returns in the past 0~1 days, 1~2 days, 2~3 days, 3~4 days, 4~5 days, 6~20 days, 21~40 days, 41~60 days.

Table III: A Multi-threshold Model of the Probability of Holding a Stock (Overall Sample)

This table reports the estimation results of a multi-threshold model that regresses a binary decision to hold a stock or not on gains from that stock and other control variables. The slope of gains is allowed to change at multiple unknown thresholds. I randomly split observations into an estimating sample (to identify the thresholds) and a testing sample (to test the magnitude of the changes). In the estimating sample I use the procedure developed by Hansen (2000) to construct the heteroskedastic-consistent 99% confidence interval for the location of the threshold based on a likelihood ratio test. In the testing sample, because of the sample splitting the test for the slope change is standard. standard errors clustered by account number are reported in brackets. Estimates of control variables are reported in table III (continue) of appendix E.

	(1)	(2)	(3)
Estimating Sample			
Threshold I [99% confidence interval]	-0.034*** [-0.043,-0.025]	-0.039*** [-0.047,-0.029]	-0.039*** [-0.048,-0.030]
Threshold II [99% confidence interval]	0.051*** [0.034,.0068]	0.053*** [0.033,0.085]	0.055*** [0.038,0.078]
Observations	566,599	566,437	562,081
Testing Sample			
β_0 (Slope before threshold I)	0.084*** (0.005)	0.125*** (0.005)	0.157*** (0.010)
$\beta_0 + \beta_1$ (Slope between threshold I and II)	-1.149*** (0.092)	-0.960*** (0.078)	-0.812*** (0.075)
$\beta_0 + \beta_1 + \beta_2$ (Slope after threshold II)	0.055*** (0.007)	0.013* (0.007)	0.018*** (0.006)
α_1 (Discontinuity at zero)	-0.052*** (0.005)	-0.051*** (0.004)	-0.055*** (0.006)
Market and stock's own returns in the past two months	-	-	Yes
Other control variables	-	Yes	Yes
Adjusted R^2	0.018	0.088	0.094
Observations	562,125	561,944	557,604

The two estimated thresholds are very robust to the inclusion of more control variables, with one estimated at negative levels (-3.4%, -3.9%, -3.9%) and the other at positive levels (5.1%, 5.3%, 5.5%), both significantly different from zero at 1% significance level. Since the estimation results are very similar across columns, I focus on discussing column 3. In the region immediately before threshold I (g_{ijt}<-3.9%) an one unit increase in gains makes investors 15.7% more likely to hold a given stock. After threshold I, the estimated relationship is V-shaped: At threshold II the slope changes from a significantly negative level (-0.812) to a significantly positive level (0.018), making threshold II at 5.5% the bottom point of the V shape. Investors are also 5.5% less likely to hold a given stock once there are positive gains, reflecting the influence of the status quo.

The estimated bottom point of the V shape (5.5%) is closely related to investors' expectations. Qualitatively, investors' expected gains should be mostly positive to justify the initial purchase decision, and the estimated bottom point of the V shape is significantly different from zero. Quantitatively, for comparison purpose, I calculate the average market returns (S&P) as a proxy for expectations. Over the average holding day in the sample (230 days) the market return is 4.8% in the five years prior to the sample period and 5.9% within the sample period, both very close to an estimated gain of 5.5%. The bottom point of the V shape is also higher than the interest rate. Between the year 1991 and 1996, the return of 1-year Treasury Bill ranges from 3.33% to 5.69%, lower than 5.5% over an average of 230 days.[41] Treating expectations as the reference point therefore generates predictions consistent with the empirical estimates from the overall sample.

[41] See http://www.federalreserve.gov/releases/h15/data/Annual/discontinued_AH_Y1.txt.

Most control variables have significant effects as well. The effect of trading frequency is strongly positive, which may sound surprising at first because intuitively frequent traders should be less likely to hold a stock. However, it is a reflection of the sample selection procedure and the trading pattern of frequent traders: Conditional on at least one sale taking place on any given day, frequent traders actually sell only a small proportion of stocks in their portfolios, making the probability of holding a stock on a given day higher. Portfolio size and holding period also have slightly positive and significant effects. Investors are also less likely to hold a given stock in December. High trading volume of the stock on the day makes investors very likely to sell. Income and net wealth do not have very significant effects. Market returns in the past two months mostly affect the probability of holding positively, whereas the stock's own returns in the same period affect the probability of holding negatively.

III. Empirical Analysis: Heterogeneity

A. The Probability of Holding a Stock: Illustration.

This section investigates heterogeneity by splitting the sample first by frequent traders versus infrequent traders, and then by stocks with good history versus bad history. As the initial step of the analysis, I still keep homogeneity assumption within each subsample by treating the bottom of the V shape as a fixed number, but allow the estimate to vary across subsamples.

To demonstrate the predictions of treating expectations as the reference points, I make the following assumption based on the dynamic model in section I.C:

$$g_t^{RP} = g_t^E = \prod_{m=1}^{m=t-1}(1+r_m)E(\prod_{n=t}^{n=T+1}(1+r_n))-1 \quad (11)$$

The lagged expected gain g_t^E incorporates the effect of past returns realization up to period $t-1$ and takes expectation over returns from period t to $T+1$. The simple specification is convenient to explain the predicted heterogeneity, but the basic intuition should be robust to a range of more complicated specifications of expectations.

The first heterogeneity is between frequent traders and infrequent traders. I model their difference by different lengths of evaluation period. Frequent traders are assumed to have a relatively short time interval to take an action and evaluate gain-loss utilities.[42] (To reiterate, in the sample frequent traders on average trade every month while infrequent traders every five months). Due to the short evaluation period, frequent traders naturally expect low gains, given $E(r_t) > 0$. If investors' reference points are affected by expectations, in aggregate the average bottom point of the V shape of frequent traders should be lower than that of infrequent traders.[43]

There is a more subtle prediction. Frequent traders are less likely to adjust their expected gains too much from the initial levels due to the short holding period, while infrequent traders' expected gains may deviate a lot from the initial levels because they experience realization of

[42] The implicit assumption here is that the decision period and action period have the same length. This assumption can be easily relaxed to the case where investors have multiple periods to adjust their stock positions before evaluating gains and losses. Other things equal, future independent risks in returns would cancel each other, making investors more likely to take larger positions. This channel brings monotonic effect to the levels of the optimal positions for every level of gains. The V shape still remains.

[43] It could also be that for some exogenous reasons not related to time horizon some investors expect to earn low gains from holding a stock, and such reference levels make them sell the stock quickly. The prediction of a low bottom point for the sample of frequent traders, however, only requires a correlation (rather than a causality) between time horizon and expected gains.

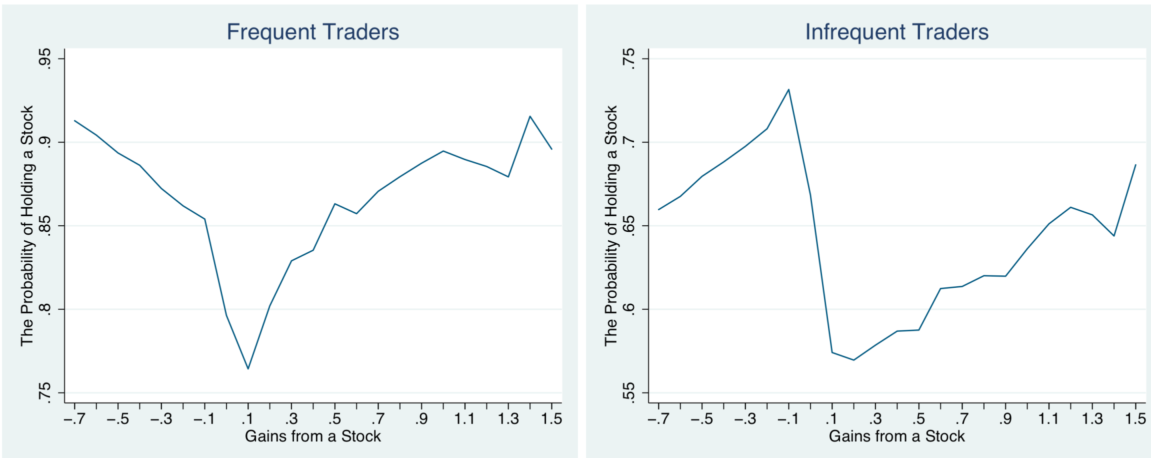
gains and losses over many periods. The relative bunching of frequent traders' expected gains reinforces the V-shaped relationship, because the V shape is derived from assuming a single reference point. On the contrary, the large variation of infrequent traders' expected gains may not generate a perfect V shape.

The second heterogeneity is between stocks with good history and bad history. Other things equal, investors' expected gains on stocks with rising prices between period 0 and period t should be higher, simply because good price histories generate higher cumulative returns $\prod_{m=1}^{m=t-1}(1+r_m)$, which is part of the expected gains. Admittedly, past returns may change beliefs about the returns distribution $f(r_t)$ if there is learning. Learning either reinforces (the momentum belief) or mitigates (the mean reversion belief) the direct effect of past price history on expected gains. The regression includes the stock's own returns and market returns in the past two months to control for learning.

These predictions are confirmed by the data.

Figure 5 illustrates the probability of holding a stock in the samples of frequent traders as well as infrequent traders. Consistent with the predications, the bottom point of the V shape of frequent traders is indeed lower than that of infrequent traders. Further, frequent traders have a V-shaped relationship that almost perfectly matches the prediction of loss aversion, whereas the pattern of infrequent traders is not strictly V-shaped, mainly because there exists a left-tail drop in the probability of holding a stock.

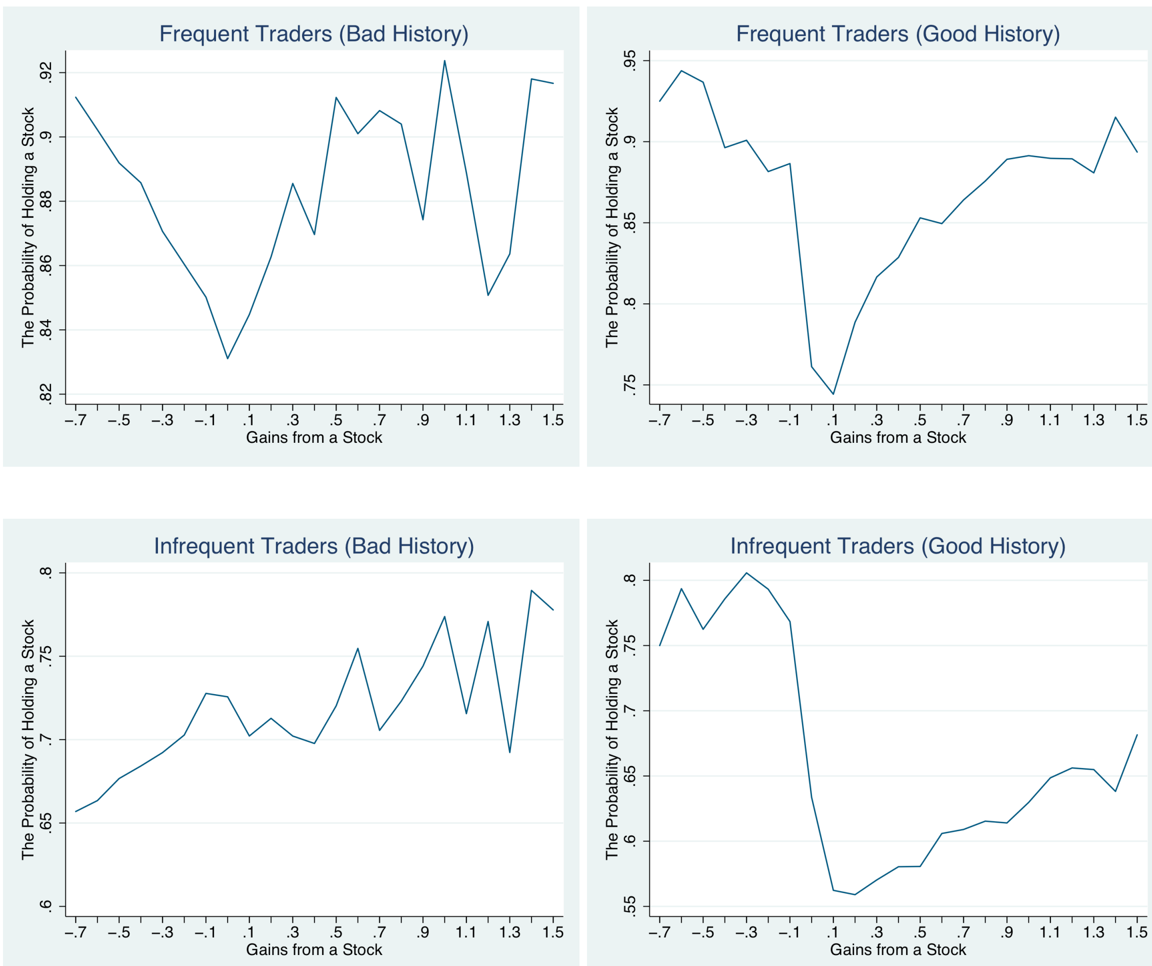
Figure 5: The Probability of Holding a Stock (Frequent Traders and Infrequent Traders)
This figure shows the probability of holding a stock across different levels of gains for both frequent and infrequent traders. The probability of holding a stock is calculated as the average of a dummy variable h_{ijt} ($h_{ijt} = 0$ if sell and $h_{ijt} = 1$ otherwise)) within each 10% gains interval.



I further split the observations of each investor type by whether the stocks have good price history or bad price history. Figure 6 illustrates the probability of holding a stock in the resulting four subsamples. The two subsamples of frequent traders still keep the basic V shape, except for some noise at tails, because there is a positive correlation between current gains and price history that leads to relatively more observations at the right (left) tail of stocks with good (bad) price history. Interestingly, the bottom point of the V shape in the sample of stocks with bad price history is around zero, while that in the sample of stocks with good price history is positive. The two subsamples of infrequent traders demonstrate large differences. The probability of holding a stock in the sample of stocks with bad price history is almost monotonically increasing as gains increase, with a slight drop when gains become positive. The pattern of stocks with good price history is largely consistent with the V shape prediction, with no obvious left-tail drop in the probability of holding a stock.

Figure 6: The Probability of Holding a Stock (Stocks with Bad History and Good History)

This figure shows the probability of holding a stock across different levels of gains for stocks with good history and bad history for each trader type. The probability of holding a stock is calculated as the average of a dummy variable h_{ijt} ($h_{ijt}=0$ if sell and $h_{ijt}=1$ otherwise)) within each 10% gains interval.



It is also informative to compare these patterns across columns. For stocks with bad price history, frequent traders do not increase the tendency to sell as losses become large, but infrequent traders do so. For stocks with good price history, frequent traders have a steep increase in the tendency of holding after reaching the bottom of the V shape while infrequent traders keep the probability of holding almost flat. Recall that the aggregate pattern in figure 4 is largely V-shaped except for the left-tail drop in the probability of holding. It is clear now that such a pattern mainly comes from infrequent traders holding stocks with bad price history.

B. The Probability of Holding a Stock: Estimation

This section estimates the same multi-threshold linear probability model for each subsample. Table IV and table V report the estimates of key parameters in the samples of frequent traders and infrequent traders respectively.

Table IV suggests that frequent traders as a whole (column 1) have three thresholds estimated at -2% (threshold I), 1.8% (threshold II) and 26.2% (threshold III) respectively, all significantly different from zero at 1% significance level. The slope of gains changes from a significantly negative level (-1.954) to a significantly positive level (0.234) at threshold II, making it the bottom point of the V shape. Before threshold I and after threshold III the slope is not significantly different from zero. Such pattern is very close to the V shape prediction of the model assuming a fixed reference point. Frequent traders as a whole are also 2.7% less likely to hold a given stock once gains become positive.

Table IV: A Multi-threshold Model of the Probability of Holding a Stock (Frequent Traders)

This table reports the estimation results of a multi-threshold model that regresses a binary decision to hold a stock or not on gains from that stock and other control variables. The slope of gains is allowed to change at multiple unknown thresholds. I randomly split observations into an estimating sample (to identify the thresholds) and a testing sample (to test the magnitude of the changes). In the estimating sample I use the procedure developed by Hansen (2000) to construct the heteroskedastic-consistent 99% confidence interval for the location of the threshold based on a likelihood ratio test. In the testing sample, because of the sample splitting the test for the slope change is standard. Standard errors clustered by account number are reported in brackets. Control variables include December dummy, holding period, trading frequency, portfolio size, tax rate, income, net wealth, daily trading volume, total shares out, market and the stock's own returns dating back as far as two months. Estimates of control variables are reported in table IV (continue) of appendix E.

	(1) Frequent traders (Overall)	(2) Frequent traders (Bad History)	(3) Frequent traders (Good History)
Estimating Sample			
Threshold I [99% confidence interval]	-0.020*** [-0.045,-0.011]	-0.043*** [-0.059,-0.024]	-0.014*** [-0.037,-0.004]
Threshold II [99% confidence interval]	0.018*** [0.008, 0.041]	0.007 [-0.043,0.038]	0.018*** [0.005,0.047]
Threshold III [99% confidence interval]	0.262*** [.157, .360]	-	0.264*** [0.163,0.370]
Observations	203987	118941	85046
Testing Sample			
β_0 (Slope before threshold I)	0.024* (0.013)	0.003 (0.013)	0.051 (0.035)
$\beta_0+\beta_1$ (Slope between threshold I and II)	-1.954*** (0.341)	-0.850*** (0.184)	-2.582*** (0.620)
$\beta_0+\beta_1+\beta_2$ (Slope between threshold II and III)	0.234*** (0.039)	-0.019 (0.024)	0.271*** (0.039)
$\beta_0+\beta_1+\beta_2+\beta_3$ (Slope after threshold III)	-0.011 (0.014)	-	-0.011 (0.015)
α_1 (Discontinuity at zero)	-0.027*** (0.010)	0.007 (0.010)	-0.040** (0.017)
Adjusted R^2	0.061	0.044	0.079
Observations	202,323	117,909	84,414

Frequent traders holding stocks with bad price history (column 2) have two thresholds estimated at -4.3% (threshold I) and 0.7% (threshold II). For the first time threshold II is not significantly different from zero. At a gain of 0.7% the slope of gains changes from a significant negative level (-0.850) to a level indistinguishable from zero (0.019), perhaps because there is more noise in the region of positive gains. Although the slope is not strictly increasing at the right tail, for consistency reason I still call 0.7% the bottom point of the V shape because the slope of gains experiences a sharp positive change here. The decline of the probability of holding at zero gains is not significant.

Frequent traders holding stocks with good price history (column 3) again have three thresholds estimated at -1.4% (threshold I), 1.8% (threshold II) and 26.4% (threshold III) respectively, all significantly different from zero at 1% significance level. The overall pattern is shown to be V-shaped, very similar to the case of frequent traders as a whole. The bottom point of the V shape (threshold II) is also estimated at 1.8%. Investors in this case are 4% less likely to hold a given stock once zero gains is reached, reflecting the effect of the status quo.

In summary, among frequent traders the bottom point of the V shape is higher in the sample of stocks with good price (1.8%) than with bad history (0.7%), but the distance between them is very small. Frequent traders adjust their reference levels of gains in the same direction of average price realization during the holding period, but not too much. The aggregate pattern of frequent traders closely matches the V-shaped prediction of loss aversion with a fixed reference point.

Table V: A Multi-threshold Model of the Probability of Holding a Stock (Infrequent Traders)

This table reports the estimation results of a multi-threshold model that regresses a binary decision to hold a stock or not on gains from that stock and other control variables. The slope of gains is allowed to change at multiple unknown thresholds. I randomly split observations into an estimating sample (to identify the thresholds) and a testing sample (to test the magnitude of the changes). In the estimating sample I use the procedure developed by Hansen (2000) to construct the heteroskedastic-consistent 99% confidence interval for the location of the threshold based on a likelihood ratio test. In the testing sample, because of the sample splitting the test for the slope change is standard. Standard errors clustered by account number are reported in brackets. Control variables include December dummy, holding period, trading frequency, portfolio size, tax rate, income, net wealth, daily trading volume, total shares out, market and the stock's own returns dating back as far as two months. Estimates of control variables are reported in table V (continue) of appendix E.

	(1) Infrequent traders (Overall)	(2) Infrequent traders (Bad History)	(3) Infrequent traders (Good History)
Estimating Sample			
Threshold I	-0.045***	-0.043***	-0.019
[99% confidence interval]	[-0.057,-0.034]	[-0.058,-0.025]	[-0.061,0.012]
Threshold II	0.103***	0.009	0.119***
[99% confidence interval]	[0.073,0.147]	[-0.027,0.073]	[0.078,0.160]
Observations	358094	170384	187710
Testing Sample			
β_0	0.186***	0.155***	0.092**
(Slope before threshold I)	(0.011)	(0.011)	(0.039)
$\beta_0+\beta_1$	-0.586***	-1.324***	-0.487***
(Slope between threshold I and II)	(0.043)	(0.171)	(0.044)
$\beta_0+\beta_1+\beta_2$	0.005	-0.038	-0.001
(Slope after threshold II)	(0.007)	(0.025)	(0.008)
α_1	-0.070***	-0.029***	-0.081***
(Discontinuity at zero)	(0.005)	(0.010)	(0.006)
Adjusted R^2	0.156	0.110	0.183
Observations	355,281	168,379	186, 902

In table V infrequent traders as a whole (column 1) have two thresholds estimated at -4.5% (threshold I) and 10.3% (threshold II), all significantly different from zero at 1% significance level. Before threshold I, one unit increase in gains makes infrequent traders 18.6% more likely to hold a given stock. Between threshold I and threshold II, the slope of gains is significantly negative (-0.586). The slope changes to a level indistinguishable from zero (0.005) after threshold II. Again I call threshold II (10.3%) the bottom point of the V shape even though the slope is not strictly positive at the right tail. Infrequent traders are 7% less likely to hold a stock at zero gains.

Infrequent traders who own stocks with bad price history (column 2) have two thresholds estimated at a gain of -4.3% (threshold I) and 0.9% (threshold II). Similar to the sample of stocks with bad history owned by frequent traders, threshold II is not significantly different from zero. At a gain of 0.9% the slope of gains changes from a significant negative level (-1.324) to a level indistinguishable from zero (-0.038). There is also significant decline in the probability of holding once gains become positive.

Infrequent traders who own stocks with good price history (column 3) have two thresholds estimated at -1.9% (threshold I) and 11.9% (threshold II), both significantly different from zero. The overall pattern is similar to the case of infrequent traders as a whole, with the bottom point of the V shape (threshold II) estimated at a gain of 11.9%. Investors in this case are 8.1% less likely to hold a stock at zero gains.

In summary, among infrequent traders the bottom point of the V shape is much higher in the sample of stocks with good price history (11.9%) than stocks with bad history (0.9%). This could result from infrequent traders adjusting their reference points substantially in the same direction of average returns realization in the past. The relationship is not globally V-shaped both because of a positive slope at the left tail and an almost flat relationship at the right tail.

Loss aversion with the reference points defined by expectations alone has a hard time explaining the last observation about patterns at the tails. One possibility comes from the literature on changing risk attitudes in sequential gambles. Thaler and Johnson (1990) find that people generally become more risk averse after prior losses and take more risks after prior gains, mainly because they use a heuristic editing rule to integrate or segregate prior gains and losses. Barberis, Huang, and Santos (2001) incorporate this idea into an asset allocation model with changing risk attitudes. This theory can easily explain the left-tail drop in the probability of holding a stock, but it somehow fails to capture the high level of risk aversion at the right tail. It is also possible that returns in the past two months cannot control for beliefs adequately so there exists momentum beliefs. Such beliefs may explain the observed left-tail pattern, but the implication is again incompatible with the almost flat relationship at the right tail. What's more, learning alone should generate smoother transition rather than the sharp changes observed in the data. Momentum beliefs combined with more extreme movements in the reference points may provide a better account of the patterns at tails. For instance, the adjustment of expectations to both large gains and losses leads to low probability of holding a given stock in these regions. Loss aversion reinforces the effect of momentum beliefs at the left tail but counter-balances it at the right tail. Thus the decreasing in the probability of holding a stock as losses become large is very salient, but at the same time there is almost no change in the probability of holding a stock as gains becomes large.

The left-tail and right-tail changes observed on infrequent traders remain a puzzle. My conjecture is that changes in expectations hence the reference points should play an important role in a satisfactory explanation.

IV. Discussion and Conclusion

BX (2008, 2009 section III) develop an alternative model of realization utility based on the distinction between paper and realized gains and losses. The optimal solution is characterized by a threshold strategy that makes investors sell the stocks once certain threshold (higher than the purchase price if with transaction cost) is reached. Combined with positive time discounting, realization utility predicts more sales above the purchase price, among a wide range of other predictions. The threshold selling strategy explains better the behavior of infrequent traders than that of frequent traders in this sample. In particular, this strategy has some difficulties in explaining why the probability of holding a stock rises significantly after passing the bottom point of the V shape in the sample of frequent traders. It is also not easy to reconcile the threshold selling strategy with the fact that the aggregate probability of holding a stock begins to decline significantly after gains become larger than -3.9% (estimated threshold I in table III column 3) rather than after a slight positive level of gains as a model of realization utility would predict.

I believe that realization utility is an important psychological factor in trading that is complementary to loss aversion in explaining the disposition effect. Incorporating it into any model of reference-dependent preferences should substantially improve the quantitative accuracy of the predictions.

To conclude, this paper relies on aversion to losses relative to a reference point defined by expectations to explain the disposition effect. Loss aversion predicts a V-shaped relationship between the optimal position in a given stock and current gains from that stock, which implies the disposition effect when the reference point is defined expectations but is itself a stronger and novel empirical regularity. Empirical analysis using individual trading records has indeed discovered such V-shaped relationship. The theoretical prediction of loss aversion allows a reasonable econometric identification of the reference point, and the estimates from both the overall sample and heterogeneous subsamples strongly support expectations as the most reasonable candidate. The common assumption in the literature that treats the status quo as the reference point renders prospect theory, the most popular informal explanation, incapable of generating the disposition effect. The theoretical and empirical results from this paper resolve this puzzle by emphasizing the fact that investors' reference points are defined by positive expectations. More careful studies regarding the nature of such reference point and the implications to trading behavior are needed.

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Appendix A: Proof of Proposition 1

Since the investor assigns any return below the cut-off return $K(x_1)$ a higher weight λ, and any negative return brings negative marginal utility, the expected marginal utility is decreasing in $K(x_1)$ when $K(x_1) < 0$. When current gain is higher than the reference level $g_1 > g_1^{RP}$, only negative future returns can make future wealth equal to the reference point so $K(x_1) < 0$. If $K(x_1)$ is zero, by assumption the expected marginal utility is negative $E(MU(K(x_1) = 0)) < 0$. As $K(x_1)$ approach negative infinity, all the returns are in the gains domain hence weighted equally, so the expected marginal utility is positive. Therefore there exists a cut-off point $K_1 < 0$ that satisfies the first-order condition that the expected marginal utility is zero. The optimal position is then determined by $K(x_1^*) = K_1$.

The case when $g_1 < g_1^{RP}$ or $g_1 = g_1^{RP}$ follows similar analysis.

Appendix B: Dynamic Model (Proof of Proposition 2)

The dynamic problem can be written as the following recursive structure:

$$V(W_t, W_t^{RP}) = \max_{x_t} E_t(U(W_{t+1} \mid W_t^{RP}) + \beta V(W_{t+1}, W_{t+1}^{RP})) \tag{12}$$

where

$$W_{t+1} = W_t + x_t P_t r_{t+1} \tag{13}$$

$$W_t^{RP} = W_{t-1} + x_{t-1} P_{t-1}((1 + r_t^{RP})(1 + r_{t+1}^{RP}) - 1) \tag{14}$$

$$\overline{P}_{t-1} g_t^{RP} = P_{t-1}((1 + r_t^{RP})(1 + r_{t+1}^{RP}) - 1) \tag{15}$$

I express the reference point in terms of the reference level of returns in period t (r_t^{RP}) and $t+1$ (r_{t+1}^{RP}) rather than gains from the stock g_t^{RP} for computational convenience. But they are related by equation (15).

Let $U_{W_{t+1}} = \dfrac{\partial U(W_{t+1} \mid W_t^{RP})}{\partial W_{t+1}}$, $U_{W_t^{RP}} = \dfrac{\partial U(W_{t+1} \mid W_t^{RP})}{\partial W_t^{RP}}$, $V_{W_{t+1}} = \dfrac{\partial V(W_{t+1}, W_{t+1}^{RP})}{\partial W_{t+1}}$ and $V_{W_{t+1}^{RP}} = \dfrac{\partial V(W_{t+1}, W_{t+1}^{RP})}{\partial W_{t+1}^{RP}}$. The F.O.C is given by equation (16).

$$E_t(U_{W_{t+1}} r_{t+1} + \beta V_{W_{t+1}} r_{t+1} + \beta V_{W_{t+1}^{RP}}((1 + r_{t+1}^{RP})(1 + r_{t+2}^{RP}) - 1) = 0 \tag{16}$$

Through simple algebra, we know that

$$V_{W_t^{RP}} = E_t(U_{W_t^{RP}}) \quad (17)$$

$$V_{W_t} = E_t(U_{W_{t+1}} + \beta V_{W_{t+1}} + \beta V_{W_{t+1}^{RP}}) \quad (18)$$

$$E_t(U_{W_{t+1}} + U_{W_t^{RP}}) = 0 \quad (19)$$

According to the law of iterated expectation and equation (19),

$$V_{W_t} = E_t(U_{W_{t+1}} + \beta^{T-t}(V_{W_T} + V_{W_T^{RP}})) = E_t(U_{W_{t+1}}) \quad (20)$$

In the final decision period T, $V(W_T, W_T^{RP}) = \max_{x_T} E_T(U(W_{T+1} \mid W_T^{RP})$. So $V_{W_T} = E_T(U_{W_{T+1}})$ and $V_{W_T^{RP}} = E_T(U_{W_T^{RP}})$, which cancel out. We now have

$$V_{W_t} = E_t(U_{W_{t+1}}) \quad (21)$$

Plug in equation (17) and (21) into the F.O. C (16). After simplifying the equation, we get

$$E_t(U_{W_{t+1}} r_{t+1} + \beta U_{W_{t+1}^{RP}}((1+r_{t+1}^{RP})(1+r_{t+2}^{RP}) - (1+r_{t+1}))) = 0 \quad (22)$$

To have an understanding of the relationship between the optimal position in the stock and stock return in period t, I take total derivative with respect to equation (22).

Let $K_t = \frac{W_t^{RP} - W_t}{x_t P_t} = \frac{x_{t-1}}{x_t}(\frac{(1+r_t^{RP})(1+r_{t+1}^{RP})}{(1+r_t)} - 1)$. Instead of looking at $\frac{dx_t^*}{dr_t}$, it is computationally more convenient to look at

$$\frac{dr_t}{dx_t^*} = \frac{(1+r_t)}{x_t^*}\left(\frac{(1+r_t)}{(1+r_t^{RP})(1+r_{t+1}^{RP})} - 1\right) - \frac{A}{f(K_t^*)\frac{x_{t-1}^{*2}}{x_t^{*2}}\frac{(1+r_t^{RP})(1+r_{t+1}^{RP})((1+r_t^{RP})(1+r_{t+1}^{RP}) - (1+r_t))}{(1+r_t)^3}} \quad (23)$$

where

$$A = \int_{-1}^{+\infty} f(K_{t+1}) \frac{((1+r_{t+1}^{RP})(1+r_{t+2}^{RP}) - (1+r_{t+1}))^2}{(1+r_{t+1})x_{t+1}^*} f(r_{t+1}) dr_{t+1} > 0 \quad (24)$$

From equation (23) and (24) it is easy to see that when $r_t < (1+r_t^{RP})(1+r_{t+1}^{RP}) - 1$, $\frac{dx_t^*}{dr_t} < 0$; when $r_t \geq (1+r_t^{RP})(1+r_{t+1}^{RP}) - 1$, $\frac{dx_t^*}{dr_t} \geq 0$.

Expressing the result in term of gains from the stock g_t, first recall that $P_{t-1}(1+r_t) = \overline{P}_{t-1}(1+g_t)$ and $P_{t-1}(1+r_t^{RP})(1+r_{t+1}^{RP}) = \overline{P}_{t-1}(1+g_t^{RP})$, where $\overline{P}_{t-1}$ is the average purchase price at the end of period $t-1$, g_t is the gains in period t, and g_t^{RP} is the reference level of gains in period t. We also have the same relationship between the optimal position x_t^* and gains from the stock g_t: When $g_t < g_t^{RP}$, the optimal position is decreasing in g_t; when $g_t > g_t^{RP}$, the optimal position is increasing in g_t.

Appendix C: Stochastic Reference Point (Proof of Proposition 3)

Assume that g_1^{RP} follows a distribution $h(g_1^{RP})$. The expected marginal utility of adding an extra share is given by equation (25).

$$E(MU(K(x_1))) = P_1 \int_{-1}^{+\infty} (\int_{K(x_1)}^{+\infty} r_2 f(r_2) dr_2 + \int_{-1}^{K(x_1)} r_2 f(r_2) dr_2) h(g_1^{RP}) dg_1^{RP} \tag{25}$$

According to the F.O.C that $E(MU(K(x_1^*))) = 0$, we have

$$\frac{dx_1^*}{dg_1} = -\frac{\int_{-1}^{+\infty} (\frac{1+g_1^{RP}}{1+g_1})(g_1^{RP} - g_1) f(K(x_1^*)) h(g_1^{RP}) dg_1^{RP}}{\int_{-1}^{+\infty} \frac{1}{x_1^*}(g_1^{RP} - g_1)^2 f(K(x_1^*)) h(g_1^{RP}) dg_1^{RP}} \tag{26}$$

The denominator is always positive. Thus the relationship between the optimal position and gains depends on the sign of the numerator, which in turn depends on the sign of $(g_1^{RP} - g_1)$ and the properties of the densities $f(.)$ and $h(.)$. When $g_1 = -1$, it follows that $g_1^{RP} - g_1 \geq 0$ for all levels of g_1^{RP}, so $\frac{dx_1^*}{dg_1} \leq 0$. Because $\frac{dx_1^*}{dg_1}$ is continuous in g_1, there exists a lower bound $\underline{g}_1$ such that for $g_1 < \underline{g}_1$ the relationship between optimal position and gains is negative. Similarly, when $g_1 \to +\infty$, it follows that $g_1^{RP} - g_1 < 0$ for all levels of g_1^{RP}, so $\frac{dx_1^*}{dg_1} > 0$. There exists a higher bound $\overline{g}_1$ such that for $g_1 > \overline{g}_1$ the derivative is positive. For $\underline{g}_1 \leq g_1 \leq \overline{g}_1$, the relationship is ambiguous. It depends on the characteristics of $f(.)$ and $h(.)$.

As a relevant example, Let us look at the case of two reference points ($W_1^{RP,L}$, p ; $W_1^{RP,H}$, 1- p) with the application in mind that the low reference point is the status quo and the high one is the mean expectation. Let $g_1^{RP,L}$ and $g_1^{RP,R}$ be the corresponding reference levels of gains for the two reference points, respectively. Through some tedious algebra, we have the following observations:

(i) when $g_1 \leq g_1^{RP,L}$, the optimal position is decreasing in g_1; when $g_1 > g_1^{RP,H}$, the optimal position is increasing in g_1.

(ii) when $g_1^{RP,L} < g_1 < g_1^{RP,H}$ the relationship is ambiguous:

a. Single-trough pattern: if $f(r_2)$ is relatively constant around $r_2 = 0$, there exists a level of gains $\tilde{g}_1$ such that $g_1^{RP,L} < \tilde{g}_1 < g_1^{RP,H}$. The optimal position x_1^* decreases in g_1 in the region $g_1^{RP,L} < g_1 < \tilde{g}_1$, and increases in g_1 in the region $\tilde{g}_1 < g_1 < g_1^{RP,H}$, with the minimum position reached at $g_1 = \tilde{g}_1$.

b. Twin-trough pattern: if $f(r_2)$ is strongly increasing in the region of small negative returns, and decreasing in the region of small positive returns, there exists a level of gains $\hat{g}_1$ such that $g_1^{RP,L} < \hat{g}_1 < g_1^{RP,H}$. The optimal position x_1^* increases in g_1 in the region $g_1^{RP,L} < g_1 < \hat{g}_1$, and decreases in g_1 in the region $\hat{g}_1 < g_1 < g_1^{RP,H}$, with the two local minimum positions reached at the two reference levels of gains $g_1^{RP,L}$ and $g_1^{RP,H}$ respectively.

Appendix D: Data Cleaning Process

1. Construct Trading Records.

In this study I focus on common stocks only. Trading records with short selling are excluded from the sample for simplicity. This is about 2% of the observations. Intraday trades are netted with price aggregated to be the volume weighted average price.

2. Add Position Records.

For each stock held by each account owner, I connect trading data to the end-of-the-month position data in chronological order. I match the first trading record of a stock in the sample with the most recent end-of-the-month position record before the first trading date. Such end-of-the-month position record serves as the initial position to construct the complete trading records for each stock based on the trading data.

Any trading history not starting with an initial position of zero is dropped out of the sample, so I end up with consistent trading histories of stocks whose initial purchase prices are known.

3. Fill in Dates of Hold.

I expand the trading records of each stock by adding dates in which at least one sale is made in the portfolio this stock belongs to. I then obtain daily stock price (either closing price of the date or the average of ask and bid prices if closing price is not available), market returns (S&P), trading volume, number of shares out, and adjustment factor for dividend and split from CRSP. Prices are adjusted for commission, dividend and splits. Commissions for potential sales are assumed to be equal to the average commission incurred when purchasing this stock. To avoid being driven by outliers I winsorize 0.5% of the extreme gains and losses at tails, which restricts gains from each stock to be within (-.788,2.046).

Appendix E: The Estimation Results: Control Variables

Table III (continue): A Multi-threshold Model of the Probability of Holding a Stock (Overall Sample)

This table continues to report the estimates of the parameters associated with control variables from the testing sample in the overall sample. Standard errors clustered by account number are reported in brackets.

	(1)	(2)	(3)
December	-	-0.015*** (0.003)	-0.012*** (0.003)
Holding Days/100	-	0.014*** (0.000)	0.011*** (0.000)
Trading Frequency	-	0.856*** (0.064)	0.850*** (0.066)
Portfolio Size	-	0.002 (0.001)	0.002 (0.001)
Tax	-	0.001*** (0.000)	0.001*** (0.000)
Income/ 10^7	-	0.001 (0.003)	0.002 (0.003)
Net worth/ 10^7	-	0.002* (0.001)	0.001 (0.001)
Total Daily Trading Volume of the Holding Stock/ 10^8	-	-2.323*** (0.110)	-2.401*** (0.124)
Total Outstanding Shares of the Holding Stock/ 10^7	-	0.101*** (0.004)	0.088*** (0.004)
Market Returns 1	-	-	0.749*** (0.112)
Market Returns 2	-	-	0.755*** (0.113)
Market Returns 3	-	-	0.257** (0.109)
Market Returns 4	-	-	0.110 (0.110)
Market Returns 5	-	-	0.110 (0.115)
Market Returns 6~20	-	-	0.170*** (0.043)

Market Returns 21~40	-	-	0.044 (0.035)
Market Returns 41~60	-	-	-0.044 (0.034)
S&P index	-	-	0.000*** (0.000)
Own Returns 1	-	-	-0.286*** (0.023)
Own Returns 2	-	-	-0.155*** (0.032)
Own Returns 3	-	-	-0.118*** (0.036)
Own Returns 4	-	-	-0.172*** (0.022)
Own Returns 5	-	-	-0.178*** (0.019)
Own Returns 6~20	-	-	-0.100*** (0.010)
Own Returns 21~40	-	-	-0.059*** (0.006)
Own Returns 41~60	-	-	-0.047*** (0.005)
Constant	0.788*** (0.001)	0.651*** (0.009)	0.526*** (0.010)
Adjusted R^2	0.018	0.088	0.094
5% and 95% Percentiles of the fitted probability of holding	[0.637,0.783]	[0.545,.932]	[0.530,0.942]
Observations	562,125	561,944	557,604

Table IV (continue): A Multi-threshold Model of the Probability of Holding a Stock (Frequent Traders)

This table continues to report the estimates of the parameters associated with control variables from the testing sample in the sample of frequent traders. Standard errors clustered by account number are reported in brackets.

	(1) Frequent traders (Overall)	(2) Frequent traders (Bad History)	(3) Frequent traders (Good History)
December	-0.008* (0.004)	-0.028*** (0.005)	0.020*** (0.007)
Holding Days/100	0.013*** (0.001)	0.011*** (0.001)	0.012*** (0.001)
Trading Frequency	0.132** (0.056)	0.188*** (0.059)	0.022 (0.075)
Portfolio Size	0.001** (0.000)	0.001** (0.000)	0.001*** (0.001)
Tax	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
Income/ 10^7	-0.063*** (0.019)	-0.060*** (0.020)	-0.071*** (0.024)
Net worth/ 10^7	0.093*** (0.024)	0.064** (0.026)	0.130*** (0.031)
Total Daily Trading Volume of the Holding Stock/ 10^8	-2.791*** (0.157)	-2.320*** (0.174)	-3.031*** (0.181)
Total Outstanding Shares of the Holding Stock/ 10^7	0.081*** (0.006)	0.071*** (0.007)	0.083*** (0.009)
Market Returns 1	0.753*** (0.159)	0.542*** (0.202)	1.099*** (0.243)
Market Returns 2	0.737*** (0.166)	0.860*** (0.195)	0.507** (0.258)
Market Returns 3	0.474*** (0.152)	0.739*** (0.175)	0.074 (0.260)
Market Returns 4	0.093 (0.158)	0.212 (0.190)	0.040 (0.247)
Market Returns 5	0.133 (0.166)	0.289 (0.183)	0.023 (0.279)
Market Returns 6~20	0.137** (0.068)	0.153* (0.078)	0.142 (0.088)
Market Returns 21~40	0.034 (0.056)	0.020 (0.066)	0.055 (0.082)

Market Returns 41~60	-0.043 (0.050)	0.008 (0.061)	-0.106 (0.072)
S&P index	0.000* (0.000)	-0.000 (0.000)	0.000*** (0.000)
Own Returns 1	-0.091** (0.046)	0.208*** (0.048)	-0.416*** (0.069)
Own Returns 2	-0.174*** (0.044)	0.232*** (0.047)	-0.568*** (0.065)
Own Returns 3	-0.121*** (0.038)	0.062* (0.038)	-0.316*** (0.070)
Own Returns 4	-0.077** (0.037)	0.009 (0.037)	-0.206*** (0.043)
Own Returns 5	-0.115*** (0.029)	-0.036 (0.034)	-0.204*** (0.053)
Own Returns 6~20	-0.045*** (0.012)	-0.050*** (0.013)	-0.021 (0.016)
Own Returns 21~40	-0.033*** (0.009)	-0.048*** (0.010)	0.006 (0.015)
Own Returns 41~60	-0.022*** (0.009)	-0.044*** (0.010)	0.028* (0.015)
Constant	0.759*** (0.021)	0.797*** (0.021)	0.726*** (0.027)
Adjusted R^2	0.061	0.044	0.079
5% and 95% Percentiles of the fitted probability of holding	[0.705,0.990]	[0.766, 0.990]	[0.646,0.987]
Observations	202,323	117,909	84,414

Table V (continue): A Multi-threshold Model of the Probability of Holding a Stock (Infrequent Traders)

This table continues to report the estimates of the parameters associated with control variables from the testing sample in the sample of infrequent traders. Standard errors clustered by account number are reported in brackets.

	(1) Infrequent traders (Overall)	(2) Infrequent traders (Bad History)	(3) Infrequent traders (Good History)
December	-0.011*** (0.003)	-0.078*** (0.004)	0.062*** (0.004)
Holding Days/100	0.010*** (0.001)	0.006*** (0.001)	0.011*** (0.001)
Trading Frequency	2.655*** (0.271)	2.621*** (0.216)	2.621*** (0.343)
Portfolio Size	0.019*** (0.003)	0.016*** (0.003)	0.020*** (0.004)
Tax	0.000* (0.000)	0.000 (0.000)	0.001*** (0.000)
Income/ 10^7	-0.000 (0.002)	0.004 (0.006)	-0.002 (0.001)
Net worth/ 10^7	0.001 (0.001)	-0.000 (0.002)	0.002** (0.001)
Total Daily Trading Volume of the Holding Stock/ 10^8	-1.903*** (0.124)	-1.872*** (0.099)	-1.513*** (0.209)
Total Outstanding Shares of the Holding Stock/ 10^7	0.094*** (0.005)	0.080*** (0.005)	0.088*** (0.008)
Market Returns 1	0.820*** (0.125)	0.919*** (0.175)	0.766*** (0.182)
Market Returns 2	0.853*** (0.133)	1.264*** (0.181)	0.513*** (0.190)
Market Returns 3	0.298* (0.162)	0.500** (0.200)	0.098 (0.221)
Market Returns 4	0.516*** (0.144)	0.573*** (0.187)	0.518*** (0.197)
Market Returns 5	0.179 (0.146)	0.339* (0.186)	0.120 (0.205)
Market Returns 6~20	0.141** (0.064)	0.054 (0.070)	0.243*** (0.080)

Market Returns 21~40	0.063* (0.038)	0.032 (0.052)	0.126** (0.051)
Market Returns 41~60	0.027 (0.035)	-0.004 (0.051)	0.078* (0.046)
S&P index	0.000*** (0.000)	0.000 (0.000)	0.000*** (0.000)
Own Returns 1	-0.310*** (0.026)	-0.001 (0.026)	-0.583*** (0.052)
Own Returns 2	-0.117*** (0.033)	0.297*** (0.032)	-0.429*** (0.061)
Own Returns 3	-0.096** (0.039)	0.096*** (0.033)	-0.238*** (0.080)
Own Returns 4	-0.184*** (0.025)	-0.027 (0.026)	-0.364*** (0.046)
Own Returns 5	-0.167*** (0.021)	-0.056*** (0.021)	-0.280*** (0.048)
Own Returns 6~20	-0.105*** (0.014)	-0.067*** (0.010)	-0.118*** (0.027)
Own Returns 21~40	-0.061*** (0.007)	-0.060*** (0.008)	-0.050*** (0.009)
Own Returns 41~60	-0.049*** (0.006)	-0.068*** (0.008)	-0.019** (0.009)
Constant	0.430*** (0.010)	0.508*** (0.011)	0.363*** (0.012)
Adjusted R^2	0.156	0.110	0.183
5% and 95% Percentiles of the fitted probability of holding	[0.401, 0.959]	[0.518, 0.956]	[0.348,0.956]
Observations	355,281	168,379	186,902

Appendix F: The Residuals of the Probability of Holding a Stock

The residuals of the probability of holding a given stock comes from a linear probability regression where the binary decision to hold a given stock is regressed on the full set of control variables in column 3 of table III. In general, the residuals look very similar to the raw relationships except that the left-drop in the probability of holding is more severe in the residuals graph.