# Two-sided Search in International Markets

**(preliminary draft)**

J. Eaton[a,b], D. Jinkins[c], J. Tybout[a,b], D. Xu[d,b][1]

[a]Penn State U., [b]NBER, [c]Copenhagen Business School, [d]Duke U.

April 2017

[1]This research was supported by the National Science Foundation (Grant No. SES-1426645). All results have been reviewed by the U.S. Census Bureau to ensure that no confidential information is disclosed. Any opinions, findings, conclusions and recommendations expressed herein are those of the authors and do not necessarily represent the views of the NSF or the U.S. Census Bureau.We thank Costas Arkolakis, Andrew Bernard, and participants in many seminars for many useful comments on earlier drafts. We also wish to thank Pietra Rivoli (Georgetown U., McDonough School of Business) and Julia Hughes (President, U.S. Fashion Industry Association) for very helpful discussions.

# 1 Introduction

To break into global markets, either as an exporter or an importer, firms must first identify foreign business partners. And since most international partnerships are short-lived, trading firms must continually seek new connections if they wish to maintain or expand their foreign market presence. The resulting patterns of international buyer-seller connections are surprisingly fluid, and they largely determine the dynamics of firm-level trade flows.

Herein we develop a new empirical model of these search and matching processes, quantify the associated costs, and explore their implications for trade dynamics and welfare. Specifically, we develop a dynamic model of trade in consumer goods with three types of agents: foreign exporters, domestic retailers, and domestic consumers. Heterogeneous exporters and retailers engage in costly search for one another, taking stock of their current situation and the structure of the buyer-seller network. The resulting matching patterns determine which retailers carry which varieties of goods. Consumers then choose how to allocate their expenditures across retailers and the individual goods that they offer. When a retailer and an exporter form a new business relationship, they divide the associated rents in a forward-looking Nash bargaining game, thereby determining the wholesale prices at which trade occurs. The retailer then passes the goods on to domestic consumers after adding an optimal mark-up.

Fit to customs records on U.S. apparel imports, our model speaks to a variety of empirical issues[1] First, it provides estimates of the value of international business connections for different types of agents with different portfolios of business partners. Second, it allows us to decompose trade and welfare changes into two basic driving forces: market entry by

[1] An application to U.S. apparel importers is in progress.

Chinese firms, and reductions in search costs. Similarly, it quantifies the capital gains and losses induced by these two types of shocks for different types of firms. Third, it characterizes the effects of search costs and foreign competition on firm dynamics. Finally, since firms with more clients find it less expensive to meet additional business partners, and since the rate at which firms acquire connections is partly due to luck, it quantifies the extent to which large firms owe their success to fortuitous events early in their life cycles.

Our model is related to a wide variety of earlier contributions. First, speaking broadly, it follows in the tradition of papers that analyze firm-to-firm matching in international markets, beginning with work by Rauch (2001), Rauch and Trindade (2002), and Rauch and Watson (2003). Some of these studies explores the implications of firms' uncertainty regarding the appeal of their products to foreign buyers (Rauch and Watson, 2003; Albornoz *et al.*, 2012; Eaton *et al.*, 2014), the prices that prevail in a foreign location (Allen, 2014; Steinwender, 2014; Bernard *et al.*, 2014a), or the characteristics and coordinates of potential foreign clients (Albornoz *et al.*, 2012; Rauch and Watson, 2003; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Antras and Costinot, 2011; Fernandez-Blanco, 2012). Other firm-to-firm trade models presume full information, and focus instead on the question of who matches with whom (Sugita *et al.*, 2014; Bernard *et al.*, 2014b; Lim, 2016).[2] Our model draws features from both strands of the literature. It incorporates uncertainty inasmuch as agents on each side of the market are unable to observe the characteristics and coordinates of potential business partners before they meet each other. And it generates assortative matching in a way similar to Bernard *et al.* (2014b): high-quality agents choose to search more intensively, and thus end up matching

[2] There is also a large literature on value chains that treats firm-to-firm matching patterns (e.g., Antras and Shor (2013)). It is less related to our work in that it focuses on agency issues and sequences of upstream-downstream relationships.

with a broader spectrum of partner types.

Second, our model resembles a number of recent trade papers in its emphasis on customer accumulation as a driver for firm dynamics (Albornoz *et al.*, 2012; Drozd and Nosal, 2012; Eaton *et al.*, 2014; Chaney, 2014; Piveteau, 2015; Fitzgerald *et al.*, 2016).[3] We depart from these papers by treating both exporters and importers as choosing their search intensity optimally. This formulation better conforms to actual practices. It also allows us to generate richer exporter-importer network structures than would have been possible with a one-sided search model.

By making retailers a central feature of our model, we also contribute to the literature on intermediated trade. This includes papers that predict which kinds of exporters will use intermediaries (Blum *et al.*, 2009; Ahn *et al.*, 2011), and more relevant to our work, papers on the effects of trade on welfare under different types of intermediation and bargaining (Rauch and Watson, 2004; Antras and Costinot, 2011; Fernandez-Blanco, 2012; Bernard and Dhingra, 2015). Among these latter papers, the one most closely related to ours is Bernard and Dhingra (2015). Therein, exporters bargain with retailers abroad in order to avoid double marginalization and (in some cases) the price-depressing effects of competition among retailers. We too invoke a Nash bargaining game between retailers and exporters, but our focus is not on the endogenous choice of contract form.

Finally, we contribute to the literature on the life-cycle of exporters and importers. As with the earlier literature on firm dynamics, our model is partly motivated by the "fat" tails that typically characterize firm-size distributions. One way earlier studies have generated

[3]Interest in this approach to firm dynamics is not confined to the trade literature. Recent contributions that focus on the accumulation of domestic customers include Foster *et al.* (2015) and Gourio and Rudanko (2014).

these tails is through stochastic shocks to firm productivity or demand (Luttmer, 2007, 2011; Arkolakis, 2016). Another possibility for generating fat tails is to use a matching model and a convenient search cost function (Eaton *et al.*, 2014). We follow the latter modeling strategy. In particular, as in Eaton *et al.* (2014), we allow a firm's cost of finding new business partners to vary as the number of its current clients increases.

# 2 Data and Stylized Facts

The framework we develop could be applied to a variety of buyer-seller networks, including many that do not involve international trade. But for several reasons, we choose to study the network of U.S. apparel importers (buyers) and their foreign suppliers (sellers). First, by choosing a sector in which most of the buyers are wholesale/retail firms, we are able to keep the buyer side of the market relatively tractable. That is, within each wholesale or retail firm, revenue functions are nearly separable across categories of consumer goods, and firms' payoff functions can be reasonably approximated with relatively simple expressions. Second, cross-border transactions between firms can be observed in customs records, but data on domestic firm-to-firm transactions are difficult to come by.[4] Accordingly, we wish to study an industry where domestic suppliers play a minor role relative to foreign suppliers. Finally, the U.S. apparel market has changed dramatically over the past 30 years, as major new sources of merchandise have emerged abroad and quantitative restrictions on imports have been phased out. These changes have induced transition dynamics in the network structure that we can compare with our model's predictions.

[4] In the U.S., the main source of such data is the Commodity Flow Survey. It is of little use to us because it does not provide the precise identification of the buyer and it is far from comprehensive. Countries that collect value-added data could in principle supply the type of information we need, but it is very difficult to gain access to these records.

Before describing the details of our model, we review some aggregate patterns in U.S. apparel trade over the past 25 years and some micro features of the associated buyer-seller network. Some of these network features have been documented for other markets in the emerging literature on firm-to-firm trade, which includes studies on the United States and Colombia (Eaton *et al.*, 2008, 2014; Bernard *et al.*, 2014b), Chile (Blum *et al.*, 2010), Mexico (Sugita *et al.*, 2014), Norway (Bernard *et al.*, 2014b), and Ireland (Fitzgerald *et al.*, 2016). Other network features have received less attention, particularly those concerning transition dynamics.

## 2.1 Data Sources

Our empirical analysis is largely based on customs records obtained from the U.S. Census Bureau. These data describe all merchandise shipments into the United States during the period 1996 to 2011.[5] Each record includes a ten-digit Harmonized Schedule (HS) product code, shipment value, shipment quantity, entry or exit port, date of transaction, mode of transportation (land, sea, air), and the domestic firm's identification code. Critically for our study, each record also includes a string identifier based on the name and address of the foreign firm that is party to the transaction. This allows us to track buyer-seller pairs through time.[6]

## 2.2 Aggregate trends in apparel trade

Consider the trade aggregates first. Figure 1 shows that after 2000, imports rapidly displaced domestic production as the primary source of apparel for U.S. consumers, going from an

[5]We end our sample period in 2011 because this is the most recent year for which data are available at the shipment level from the U.S. Census Bureau.

[6]Unavoidably, there is some noise in the string identifiers, since the name and addresss for a given exporter may be recorded differently for different shipments. This leads to some overstatement in the number of exporters and business relationships. The problem of fuzzy matching is further analyzed in Tybout *et al.* (2016).

import penetration rate of approximately 30 percent in 1992 to around 80 percent in 2007 and thereafter.[7] Partly, this reflected the emergence of China and other transition economies as attractive locations for low-wage apparel production. It also reflected the phasing out of the Multifibre Arrangement, which placed quantative limits on these countries' exports to the U.S. until 2005. [**JT: more on the MFA here**]

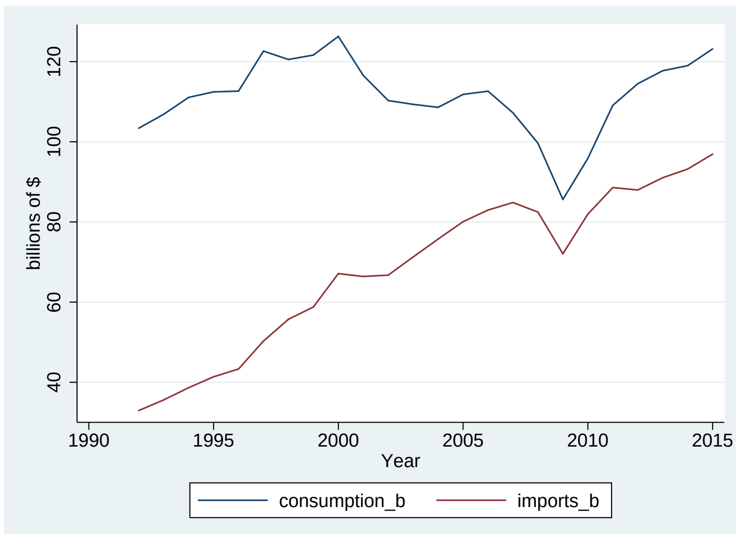


**Figure 1: U.S. apparel consumption and imports**

As imports have come to dominate the domestic market, the number of firms exporting to the United States has steadily grown. This can be seen in Figure **??**, which shows the number of foreign suppliers ("sellers") making shipments to the U.S.

Figures 2 and 3 show aggregate imports and number of suppliers for each of the 10 largest source countries by value as of 2011. Clearly, the primary source of growth has been China. India gained market share aftter 2005 as well, while other countries held stable or lost ground.[8]

[7]The dip in both series around 2009 reflects the financial crisis. Domestic consumption is calculated as the gross value of domestic apparel production plus apparel imports, less apparel exports. The value of domestic production is downloaded from the Bureau of Economic Analysis and trade aggregates are taken from the WTO. [**JT: We need to regenerate this graph using the same raw data that supports the micro analysis.**]

[8]There was subtantial turnover in the set of 10 leading sources of apparel exports between 2000 and 2011. Accordingly, a substantial fraction of the growth in China's exports came at the expense of countries that are

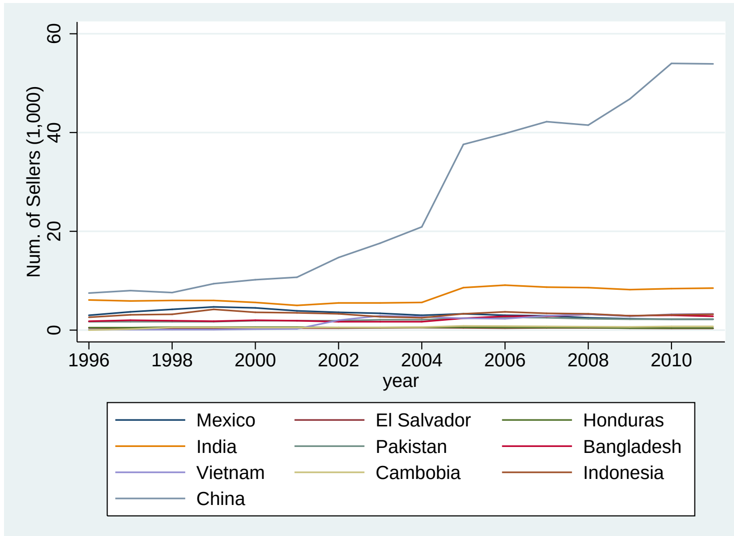


**Figure 2: Number of sellers by country, 1996-2011**

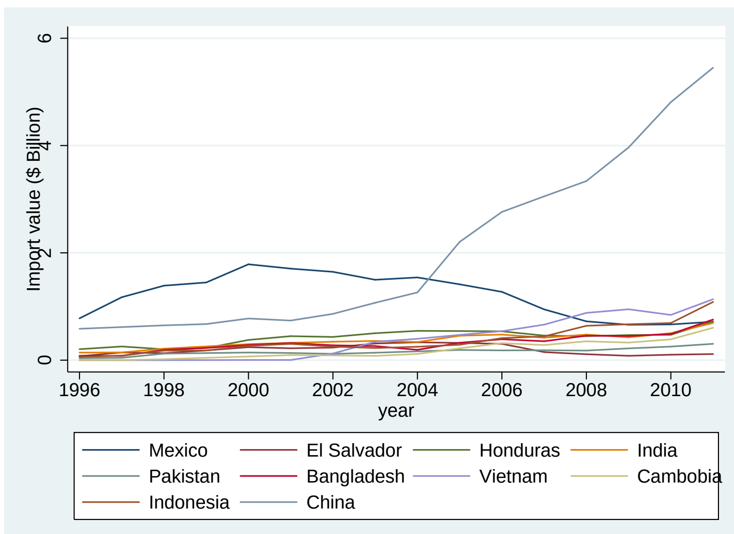


**Figure 3: Value of imports by country, 1996-2011**

## 2.3 Micro features of the apparel market

### 2.3.1 Types of players

We turn next to some patterns that characterize the buyer-seller (importer-exporter) networks behind the aggregates. For this discussion, it is useful to distinguish four different types of agents in the international apparel market: manufacturers, sourcing firms (match-makers),

not depicted in Figures 3 and 4.

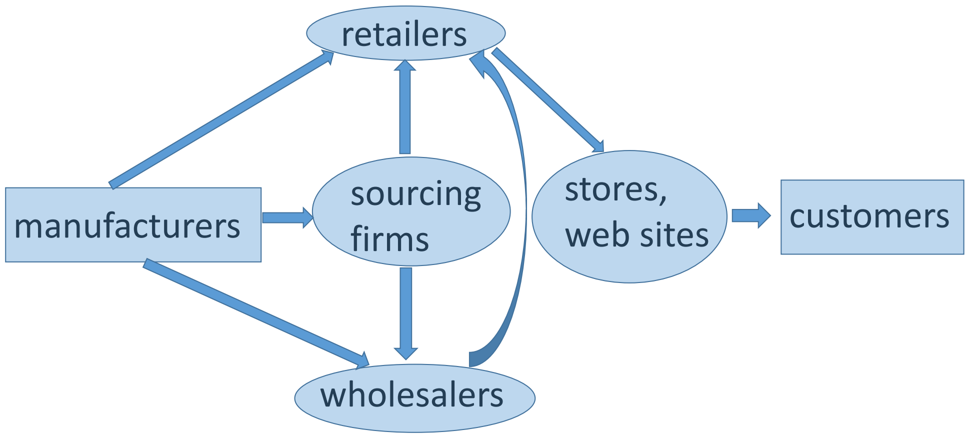


**Figure 4: Industry structure**

wholesalers/branded importers, and general merchandise retailers (refer to figure 4).[9]

Manufacturers use fabric and other intermediate inputs to produce the goods that are eventually offered in the retail market.They then sell their output to general merchandise retailers or wholesalers/branded importers. The former include big box stores like Walmart and Target, as well as department stores like Macy's and Nordstrom's. The latter include a multitude of small-scale designer/importers as well as large apparel firms like Ralph Lauren, Gap, Land's End, VF Corp, and Hanes.[10] Finally, some of these connections between foreign manuacturers and U.S. importers are brokered by sourcing firms that provide buyer-seller match-making, design, and other services. Examples include Gulati Group, Apparel Sourcing Group, Inc., Li & Fung, and W. E. Connor.

The lines between the different types of agents are fuzzy, as it is not unusual for a firm to engage in more than one activity. For example, The Gulati Group also does some clothing

[9]Related clasifications can be found in (Analytics, 2015) and (Gereffi and Memedovic, 2003).

[10]The reader may know VF Corp. by the brands it owns, which include JanSport, The North Face, Timberland, Lee, WRangler, and Nautica.

manufacturing, and Hanes owns some of its own manufacturing facilities. Also, in addition to selling their merchandise to department store chains and/or big box stores, some branded importers like Ralph Lauren and VF Corp maintain their own web sites and/or brick-and-mortar stores for retail sales.

Nonetheless, to keep our analysis tractable, we will approximate the structure of the global apparel market by treating it as populated by three mutually unaffiliated types of agents: manufacturers, wholesale/retailers, and consumers. Accordingly, we will not model firms that own their production facilities. Also, we will gloss over the distinction between importers that use in-house sourcing staff to locate possible suppliers and those that use a sourcing firm to do so. And when wholesalers/branded importers sell their merchandise through unaffiliated retail outlets, we will assume they set retail prices in the same way that the retailer would have set them if it had imported the merchandise itself. That is, we will abstract from bargaining between wholesalers and retailers and we will assume double marginalization does not occur.

How much distance do these abstractions create between our model and the actual market structure? First, while a few branded importers like Hanes own some production facilities, the vast majority do not (Analytics, 2015). The lack of vertical integration is apparent in the fact that only a small fraction of apparel imports are classified as affiliated trade (figure 5), and virtually all of the growth in matches has been due to these arm's-length relationships.

Second, it does not appear that apparel importers rely heavily on sourcing firms to match them with foreign manufacturers. Small-scale operations often get started by attending trade fairs like "Apparel Sourcing USA" or "Sourcing at Magic," which bring them face to face with foreign manufacturers[11] And McFarlan *et al.* (2012) report that in 2004, 9 of the 10

[11]"Apparel Sourcing USA . . . offers apparel brands, retailers, wholesalers and indepen-

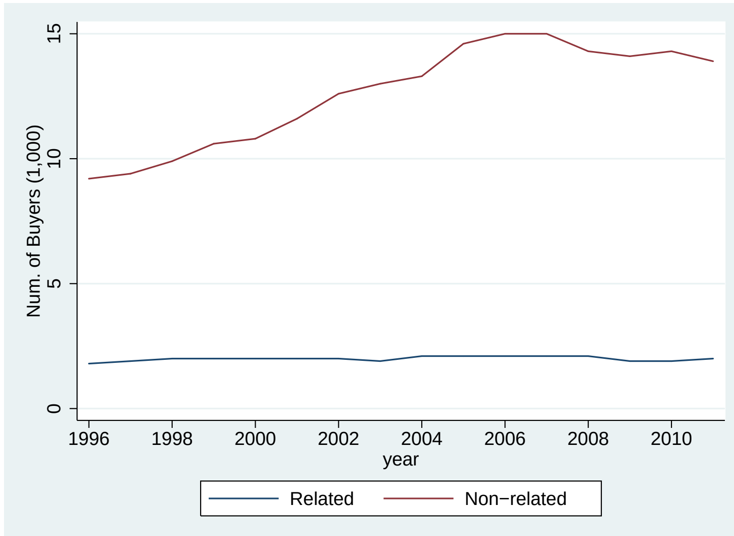


**Figure 5: Value of imports, related party versus arm's length trade**

largest apparel retailers in the U.S. ran their own sourcing offices.[12] In their latest report, the U.S. Fashion Industry Assocation surveyed 30 executives representing various segments of the apparel importing market (Lu, 2016).[13] Among this group, 78 pecent indicated that they "direct source from a selected supplier and mill matrix using [their] own designs and selecting fabric from the mill resource." In contrast, only 41 percent indicated that they engaged a third party to source production.

Third, interviews with industry experts suggest that the branded importers maintain similar retail prices for their products, whether they market them directly or through general

dent design firms a dedicated sourcing marketplace for finding the best international apparel manufacturers." http://www.apparelsourcingshow.com/newyork/en/for-attendees/about-International-Apparel-Sourcing-Show.html?nc.

"Sourcing at Magic" advertises on Facebook as "The largest fashion sourcing event in North America offering one-stop shopping for the entire apparel, footwear and accessories supply chain." The event website (http://10times.com/sourcing-at-magic) provides a partial list of attendees, which includes many representatives of apparel manufacturers located in South Asia.

[12]Kohl's was an exception. More recently, some of these retailers have been to augment the efforts of their own sourcing offices with services purchased from sourcing companies. For example, WalMart signed a 6-year deal with Li & Fung in 2010 (McFarlan *et al.*, 2012).

[13]Almost all of these executives represented large firms. Among them, 77 percent self-identified as retailers, 69 percent identified as branded importers, 69 percent identifed as importer/wholesalers, and 27 percent identified as manfucturer/suppliers. (Percentages do not sum to 100 because most firms engage in more than one activity.)

retail outlets. This is consistent with the broader finding that 72 percent of on-line prices are identical to the prices charged by brick-and-mortar stores for the same products (Cavallo, 2016). Apparently, therefore, branded importers do not frequently price discriminate across outlets, and they might reasonably be viewed as simply outsourcing their brick-and-mortar sales operations while retaining control of pricing.[14] Further, although apparel exporters are more likely to deal with wholesalers than retailers (figure 6), most imports by volume go to firms that are directly involved in retailing (Figure 7).[15].

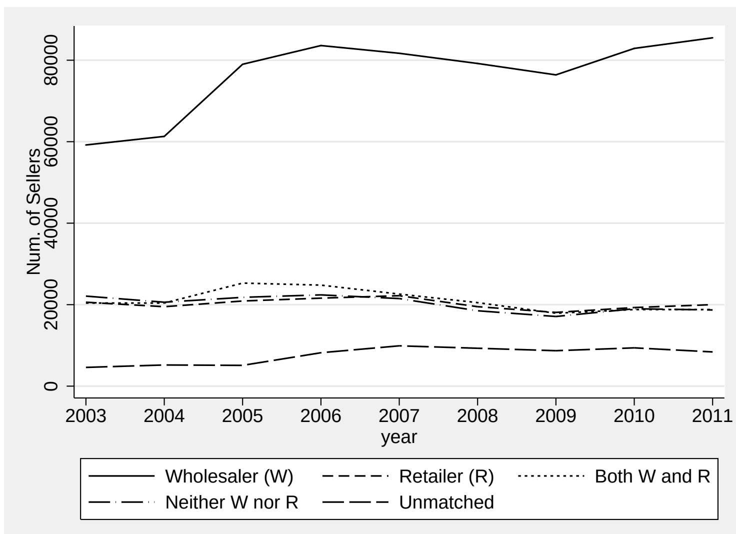


**Figure 6: Number of sellers by buyer type, 1996-2011**

### 2.3.2 Network dynamics

Because fashions rapidly change, retailers and branded importers frequently need to source new products. This has become increasingly true as real-time scanner data on sales flows have

[14]On line apparel sales accounted for roughly 9 percent of total apparel sales in 2014, up from 4 percent in 2006. (Global market review of online apparel retailing, chapter 5. Need to get complete citation.)

[15]Many of the apparel importers that do neither have names that include the words "apparel" or "clothing." This is consistent with the findings of Ha-Brookshire and Dyer (2009), who note that "nearly half of the [firms in their national sample of apparel import intermediaries] misclassified themselves as apparel manufacturers or other business types." They find that these misclassifed firms are typically former manufacturers that survived by shutting down their production opertations, retaining their design departments, and becoming branded importers. The "unmatched" category is comprised of firms that could not be clasified because of missing data problems.

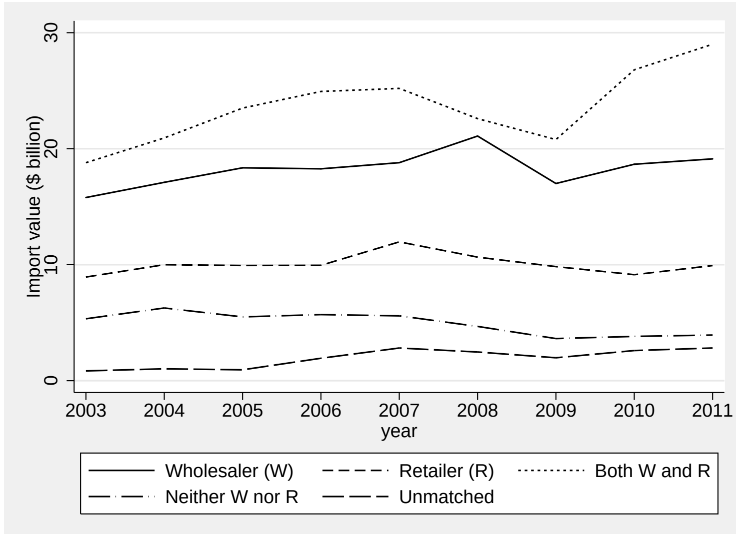


**Figure 7: Value of imports by buyer type, 1996-2011**

allowed retailers to move away from changing their inventory with each of the four seasons toward high-frequency design innovation and small-batch just-in-time production (McFarlan *et al.* (2012); Taplin (2014)).

In some cases new production runs are obtained via through long-standing relationships with manufacturers, but more frequently the buyer-seller partnerships are short-lived. Terry (2008) reports that "Apparel companies' relationships with contract manufacturers in low-cost countries have historically been transient. Deals sometimes last only a few months as brands continuously pursue the lowest cost. On average, one-third to three-quarters of an apparel company's contractor portfolio turns over every year." This is consistent with our calculations of relationship duration which are summarized in Figure 8. By far, the most common relationship lasts a year or less. (Note the log scale on the vertical axis.)

Given the short duration of a typical match, there is a great deal of flux in the patterns of buyer-seller connections. There is also a lot of year to year fluctuation in the number of foreign business partners that individual buyers and sellers deal with. This is documented in

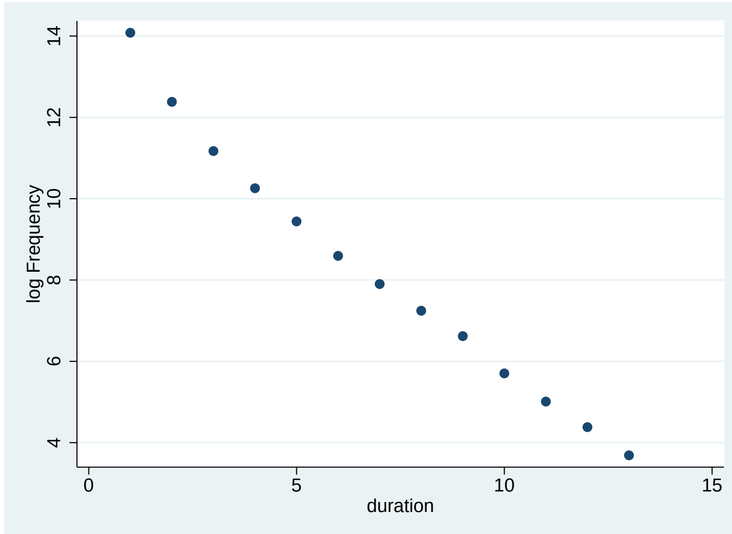


**Figure 8: Frequency distribution for relationship duration**

**Table 1: Year-to-year transition rates: buyers per seller**

| year t, year t+1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | >10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.65 | 0.27 | 0.05 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.32 | 0.31 | 0.21 | 0.09 | 0.03 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.19 | 0.22 | 0.23 | 0.17 | 0.09 | 0.05 | 0.02 | 0.01 | 0.01 | 0.00 | 0.01 |
| 4 | 0.13 | 0.15 | 0.18 | 0.18 | 0.14 | 0.09 | 0.05 | 0.03 | 0.02 | 0.01 | 0.02 |
| 5 | 0.10 | 0.10 | 0.13 | 0.16 | 0.16 | 0.12 | 0.08 | 0.05 | 0.03 | 0.02 | 0.04 |
| 6 | 0.08 | 0.07 | 0.10 | 0.13 | 0.14 | 0.13 | 0.11 | 0.08 | 0.05 | 0.03 | 0.07 |
| 7 | 0.07 | 0.06 | 0.08 | 0.09 | 0.12 | 0.13 | 0.12 | 0.10 | 0.07 | 0.05 | 0.11 |
| 8 | 0.07 | 0.05 | 0.05 | 0.07 | 0.10 | 0.11 | 0.11 | 0.11 | 0.09 | 0.07 | 0.16 |
| 9 | 0.06 | 0.05 | 0.05 | 0.06 | 0.08 | 0.09 | 0.10 | 0.10 | 0.10 | 0.08 | 0.24 |
| ≥10 | 0.05 | 0.03 | 0.03 | 0.03 | 0.04 | 0.04 | 0.05 | 0.05 | 0.06 | 0.06 | 0.56 |

Table 1 and Table 2, which report annual transition rates for sellers' buyer counts and buyers' seller counts respectively.

### 2.3.3 Degree distributions

One regularity in Tables 1 and 2 is the tendency to lose business partners, regardless of current number of partners. (For any row, compare the sum of probabilites to the left of the diagonal with the sum of probabilities to the right.) An implication is that it is unusual for firms to sustain large portfolios of foreign partners. This is apparent in the associated frequency distributions of the sellers (a.k.a. exporters) per buyer (a.k.a. importer) and buyers per

**Table 2: Year-to-year transition rates: sellers per buyer**

| year t, year t+1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | >10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.58 | 0.26 | 0.09 | 0.04 | 0.02 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 | 0.01 |
| 2 | 0.34 | 0.24 | 0.19 | 0.10 | 0.05 | 0.03 | 0.02 | 0.01 | 0.01 | 0.00 | 0.02 |
| 3 | 0.25 | 0.16 | 0.18 | 0.14 | 0.09 | 0.06 | 0.03 | 0.02 | 0.02 | 0.01 | 0.03 |
| 4 | 0.21 | 0.11 | 0.14 | 0.14 | 0.13 | 0.08 | 0.06 | 0.04 | 0.03 | 0.02 | 0.06 |
| 5 | 0.19 | 0.07 | 0.10 | 0.12 | 0.12 | 0.11 | 0.07 | 0.06 | 0.04 | 0.03 | 0.09 |
| 6 | 0.17 | 0.06 | 0.08 | 0.09 | 0.11 | 0.11 | 0.09 | 0.07 | 0.05 | 0.04 | 0.13 |
| 7 | 0.16 | 0.05 | 0.05 | 0.07 | 0.09 | 0.10 | 0.09 | 0.09 | 0.06 | 0.06 | 0.19 |
| 8 | 0.15 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.08 | 0.08 | 0.07 | 0.06 | 0.25 |
| 9 | 0.15 | 0.03 | 0.03 | 0.04 | 0.06 | 0.07 | 0.08 | 0.08 | 0.07 | 0.07 | 0.32 |
| ≥10 | 0.12 | 0.01 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 | 0.02 | 0.03 | 0.03 | 0.71 |

seller.[16] Figure 9 and Figure 10 depict these degree distributions with an OLS regression line superimposed for use later. On the horizontal axis, we have the number of connections of a buyer or seller. On the vertical axis, we report the inverse empirical CDF. Both axes are in log scale, so the near-linear relationships imply that the data come close to obeying a "power law."[17]

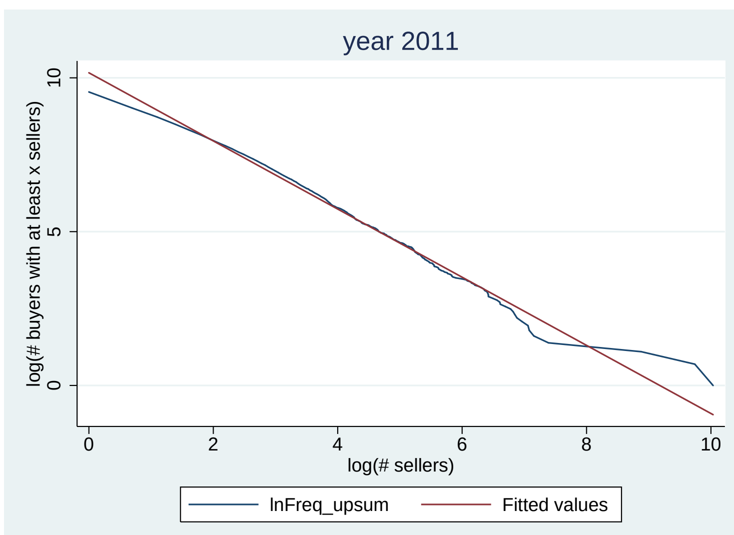


**Figure 9: Degree distribution: sellers per buyer, 2011**

While the power law shape is a robust pattern in the data, we note for later that the position of these degree distributions evolves through time with the set of market participants. For

[16]We will use "buyer" interchangeably with "importer" and "exporter" interchangeably with "seller."
[17]This feature of buyer-seller relationships has been noted in data for other countries, including Colombia (Eaton *et al.*, 2008, 2014; Bernard *et al.*, 2014b) and Norway (Bernard *et al.*, 2014b).

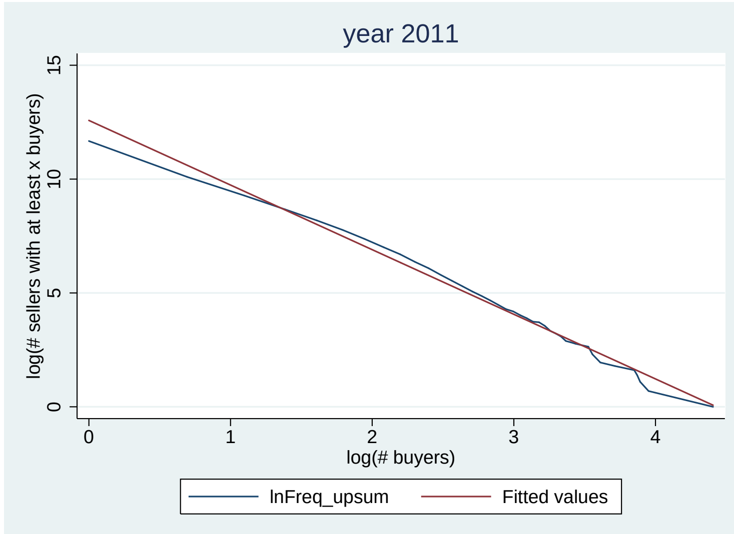


**Figure 10: Degree distribution: buyers per seller, 2011**

example the intercept and slope of the "buyers per seller" curve go from 12.18 and -1.99 in 2000 to 12.60 and -2.84 in 2011. That is, the increase in the number of sellers depicted in Figure **??** was largely due to an increase in exporters that supplied just one or several buyers. On the buyer side, the intercept and slope of the "sellers per buyer" distribution remained relatively stable between 2000 and 2011, going from 10.41 and -1.08 to 10.16 and -1.10. This is consistent with the fact that the total number of buyers didn't change much.

### 2.3.4 Search costs

With buyer-seller partnerships frequently disolving, the costs of maintaining a network of business connections are significant.[18] This is true regardless of whether an importer uses its own sourcing agents, a third-party sourcing firm, or some combination. What form do these costs take?

In a case study of U.S. apparel import intermediaries, one respondent stated it was im-

[18] WalMart's sourcing budget was $10 billion circa 2011 (McFarlan et al., 2012), while its gross income was $110 billion in 2012 (downloaded December 27, 2016 from http://www.marketwatch.com/investing/stock/wmt/financials ). Neither figure is specific to apparel. [**JT: it would be great to get more info. like this so that we could target search costs directly.**]

portant to visit manufacturers' factories and learn their capabilities: "You know, [go] into the factory and see what they're making for other people, or what their lines do, and then basically [give] them that type of products. . . . [T]o go to somebody who makes cotton underpants, and give them synthetic with charms, it's not the right thing to do because they're not gonna be the best of that" (Ha-Brookshire and Dyer, 2008). Retailers also wish to avoid factories that fall short in terms of shop floor safety, child labor standards, and environmental impact[19]. And since each importing firm has its own standards regarding acceptable practice, the industry norm is to perform an audit of each factory they deal with before placing any orders.[20]

On the manufacturers' side, we know less about the nature of search efforts. Clearly, some manufacturers invest in attending trade fairs, as mentioned above. Beyond this, interviews with manufacturers of plastics products in Colombia suggest that the costs of finding foreign buyers can include maintenance of an appealing web site in English, web searches for firms abroad that buy one's type of product, maintenance of a marketing staff, and maintenance of sales offices in destination markets (Dominguez *et al.*, 2008).

#### 2.3.5 Pricing

A final issue of relevance is how wholesale and retail prices are set. We could find no hard data on this, so what follows reflects the impressions of the industry insiders we spoke with. First, a large fraction of the buyer-seller relationships in the apparel sector are between retailers

[19]In a 2016 survey of U.S. apparel importers, "33 percent rated 'unmet social and environmental compliance' as having a high or very high impact on their supply chain, much higher than concerns for other supply chain risks such as 'labor disputes,' 'political unrest,' and 'lack of resources to manage supply chain risks.'" (Lu, 2016).

[20]This observation is based on a telephone interview with the president of the U.S. Fashion Industry Association, December 14, 2016.

and manufacturers. Whese these occur, the parties to the agreement generally negotiate over wholesale prices, product characteristics, and delivery time. Second, while relationships that involve three or more parties might in principle involve more complex bargaining procedures, multilateral bargaining is not typically observed. For example, when an importing retailer uses a sourcing agent to link up with manufacturers, the importer is likely to tell the sourcing firm what contract terms it is looking for, and the sourcing firm is likely to then negotiate with the potential manufacturers it identifies, keeping a margin between the factory-gate price and the price the importer agreed to pay.

# 3 A model of buyer-seller networks

Motivated by the stylized facts described above, we now develop a continuous-time two-sided search model. As mentioned above, we keep this exercise tractable by treating the importing agents in Figure 4 as a single type of firm, which we will hereafter refer to interchangably as retailers or buyers. The resulting structure is depicted in Figure 11.

Consumers acquire goods exclusively through retailers, who offer different but possibly overlapping menus of products, depending upon the set of suppliers they are currently partnered with. Retailers are also different in terms of the amenities they offer, like locational convenience, ambiance, and service. Consumers allocate their expenditures across retailers in a way that reflects their preferences for amenities and product menus.[21]

The dimensions of retailer heterogeneity are publicly observable, so consumers' expenditure patterns are characterized by a standard static optimization problem with full information. However, buyers and sellers in the wholesale market are unable to costlessly match with one

[21]Our setup draws equivalence to a nested Logit discrete choice model where indirect utility function of individual consumers depends on log price, see Verboven (1996)

another. Rather, each type of agent must invest in costly search to establish new business partnerships.

Because it is costly to find new business partners, buyers and sellers create rents when they meet one another. They bargain continuously and bilaterally over these rents, and the expected outcomes of these bargaining games determine the expected returns to successful search for each party.

Other things equal, the more intensively an agent searches, the higher the hazard rate with which she finds new partners and reaps her share of the associated rents. But these hazard rates depend upon other things as well.

First, matching hazards are influenced by market tightness. For example, when many buyers are searching for new suppliers, but not many suppliers are searching for new buyers, matching hazards will tend to be low for buyers and high for suppliers. As we will discuss shortly, the precise way in which search intensities on both sides of the market influence aggregate market tightness is determined by the matching function in our model, which we adopt from the labor-search literature.

Second, the ease with which agents find new business partners depends upon their previous successes. That is, agents who have already accumulated a large portfolio of business partners find it relatively easy to locate still more. This feature of our model, taken from Eaton *et al.* (2014), helps us to capture the "fat-tailed" distributions of buyers across sellers and sellers across buyers discussed above.

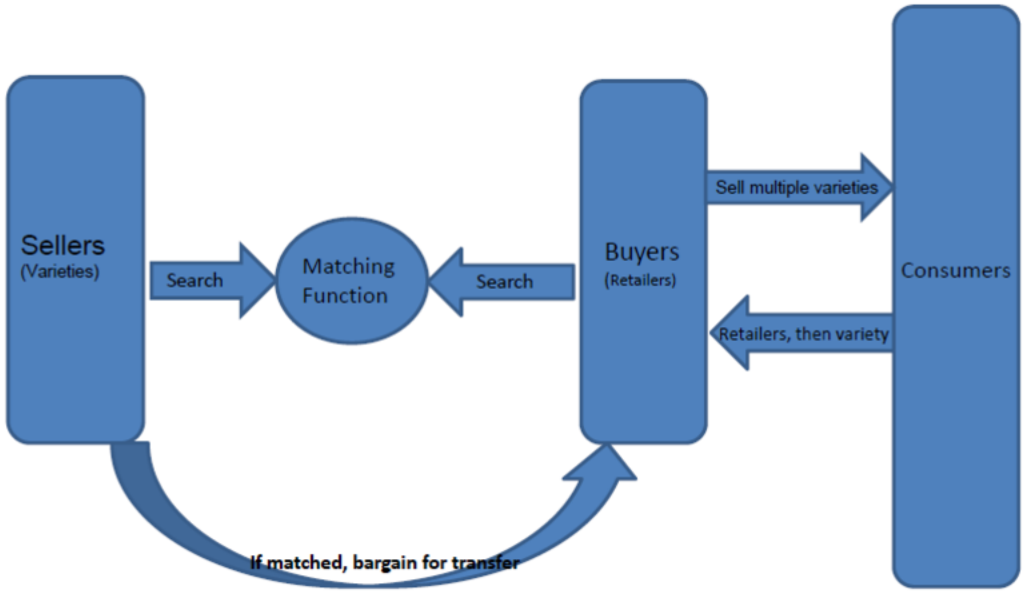


**Figure 11: Model diagram**

## 3.1 The Retail Market

**Preferences and pricing:** We now turn to model specifics. As in Akin *et al.* (forthcoming) and Bernard and Dhingra (2015), we start from a nested CES demand structure in which consumers have preferences over retailers, and within retailers, over products. Specifically, assume the retail market is populated by a measure-$B$ continuum of stores, and suppose consumers view these stores as imperfect substitutes, both because they offer distinct amenities and because they carry different—but not necessarily disjoint—sets of products. More precisely, indexing stores by $b$ and products (or exporting firms) by $x$, let consumers' preferences over retailers be given by the utility function:

$$C = \left[ \int_{b \in B} (\mu_b C_b)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $C_b$ measures consumption of the set of products, $J_b$, offered at store $b$,

$$C_b = \left[ \sum_{x \in J_b} \left( \xi_{xb} C_b^x \right)^{\frac{\alpha-1}{\alpha}} \right]^{\frac{\alpha}{\alpha-1}},$$

and $\mu_b$ and $\xi_{xb}$ are exogenous parameters that measure the inherent appeal or quality of retailer $b$ and product $x$, respectively.[22] This characterization of preferences implies that the exact price index for retailer $b$ is $p_b = \left[ \sum_{x \in J_b} \left( \frac{p_{xb}}{\xi_{xb}} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha}}$ and the exact price index for retailers as a group is $P = \left[ \int_b \left( \frac{p_b}{\mu_b} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}$.

Because of search frictions, retailers cannot instantaneously adjust the set of products they offer consumers. Rather, at each point in time they take their current offerings as given and engage in price competition in the retail market. It follows that the optimal retail prices at store $b$ satisfy

$$q_{xb} + \sum_{j' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b, \tag{1}$$

where $c_{x'b}$ is the marginal cost of supplying product $x'$ to final consumers through retailer $b$, including the manufacturing and shipping costs incurred by the producer of $x'$ and the retailing costs incurred by $b$.[23]

**Operating profits:** Equation (1) implies the standard result that the within-retailer cannibalization effect exactly offsets the cross-store substitution effect, so the mark-up rule is simply (Atkeson and Burstein, 2008; Hottman *et al.*, forthcoming; Bernard and Dhingra,

[22] Alternative nesting structures are possible. In particular, consumers might have preferences over bundles of types of goods, each of which is a CES aggregation over the bundles available from alternative retailers. That is, consumers first allocate spending across product categories, then across retailers in each category. This formulation is used in Atkin, et al (2015). Which specification is preferable depends upon the importance of transport and shopping time costs to consumers.

[23] Note that here since buyer-seller pairs set retail prices to maximize the value of the surplus generated by their business relationships, $c_{xb}$ is the cost for the pair to offer the product in retail market, which can include seller acquisition cost, transportation cost, and retailer inventory cost.

2015):

$$\frac{p_{xb} - c_{xb}}{p_{xb}} = \frac{1}{\eta}.$$

And since each retailer perceives the elasticity of demand for each of the products it offers to be $\eta$, the instantaneous profit flow jointly generated by retailer $b$ and its suppliers is[24]

$$\pi_b^T = \frac{E}{\eta P^{1-\eta}} \left[ \sum_{x \in J_b} \left( \frac{\eta}{\eta - 1} \right)^{1-\alpha} \tilde{c}_x^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_b^{\eta-1}, \tag{2}$$

where $\tilde{c}_x = \frac{c_{xb}}{\xi_{xb}}$ is the quality-adjusted marginal cost incurred by buyer-seller pair $x - b$ per unit supplied in the retail market.

## 3.2 The Wholesale Market and Payoff Functions

**Buyer-seller transfers:** We can now describe the flow pay-off functions for buyers (retailers) and sellers (foreign exporters) in the wholesale market. Suppose there are $I$ intrinsic buyer types indexed by $i \in \{1, 2, ..., I\}$, so that if buyer $b$ is a type$-i$ retailer, $\mu_b = \mu_i$. Similarly, suppose there are $J$ intrinsic seller types indexed by $j \in \{1, 2, .., J\}$, so that if seller $x$ is type$-j$ exporter, matches between this seller and a type$-i$ buyer generate a quality-adjusted marginal cost of $\tilde{c}_j$. Finally, let $\mathbf{s} = \{s_1, s_2, ..., s_J\}$ be a vector of counts of the number of sellers of each type currently matched to a particular buyer. Then by equation (2), the gross profit flow accruing to a type-$i$ buyer and its portfolio of suppliers $\mathbf{s}$ is:

$$\pi_i^T(\mathbf{s}) = \frac{E}{\eta P^{1-\eta}} \left[ \sum_{j}^{J} \left( \frac{\eta}{\eta - 1} \right)^{1-\alpha} s_j \tilde{c}_j^{1-\alpha} \right]^{\frac{1-\eta}{1-\alpha}} \mu_i^{\eta-1} \tag{3}$$

Note that when the elasticity of substitution across retailers exceeds the elasticity of substitution across products ($\alpha > \eta > 1$), this surplus exhibits diminishing returns with respect to

[24]See appendix A for details.

the number of suppliers of any type. That is, buyers who add additional sellers reduce total surplus per supplier.

To determine the division of this profit flow between a particular buyer and her portfolio of sellers, we assume that the total surplus associated with a particular buyer-seller match is divided up according to the Stole and Zwiebel (1996) bargaining protocol.[25] As demonstrated in the appendix, this implies that at each point in time the profit flow transferred to each type $j$ seller is

$$\begin{aligned} \tau_{ji}(\mathbf{s}) &\approx \beta \frac{\partial \pi_i^T(\mathbf{s})}{\partial s_j} \\ &= \frac{\beta}{\alpha-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta} \frac{E}{P^{1-\eta}} \left[\sum_j^J s_j \tilde{c}_j^{1-\alpha}\right]^{\frac{\alpha-\eta}{1-\alpha}} \tilde{c}_j^{1-\alpha} \mu_i^{\eta-1} \end{aligned} \tag{4}$$

where $\beta \in [0,1]$ is a parameter measuring the bargaining strengh of the seller, and the equality is approximate because we have used a derivative to describe a discrete one-unit change in $s_j$.

**Expressing the transfer function in observables:** Equation (4) provides a basis for estimating some key parameters of our model, but several tranformations are necessary in order to bring it to the data. First, since $\tau_{ji}(\mathbf{s})$ is not observable, we need to convert it to an expression describing the flow of export payments from a type$-i$ buyer to a type$-j$ seller in state $\mathbf{s}$. Recognizing that exports payments include both exporter profits, $\tau_{ji}(\mathbf{s})$, and compensation for the exporter's production costs, this is straightforward. As shown in the appendix, if some fraction $\lambda$ of the variable costs $c_j q_{ji}$ incurred by an $i-j$ partnership is

[25] Under the Stole and Zwiebel (1996) bargaining protocol, buyers bargaining continuously with each of the sellers they are matched with, treating each as the marginal supplier.

attributable to the seller, her flow export revenues from the partnership are:

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\sum_{\ell=1}^{J} s_\ell \tilde{c}_\ell^{1-\alpha}\right]^{\frac{\alpha-\eta}{1-\alpha}} \tilde{c}_j^{1-\alpha}\mu_i^{\eta-1}\left[\frac{\beta}{\alpha-1}+\lambda(\frac{\eta}{\eta-1})^{\eta-1}\right]. \qquad (5)$$

Second, neither quality-adjusted marginal costs, $\tilde{c}_\ell$, nor counts of the different types of sellers, $s_\ell$, are observable. However, we can eliminate the sum in square brackets by using the within buyer $i$ revenue share of a type$-j$ seller:

$$h_{j|i} = \frac{\tilde{c}_j^{1-\alpha}}{\sum_{\ell=1}^{J} s_\ell \tilde{c}_\ell^{1-\alpha}} \qquad (6)$$

Thus we can rewrite equation (5) in terms of observables and fixed effects:

$$r_{ji}(\mathbf{s}) = (h_{j|i})^{\frac{\alpha-\eta}{\alpha-1}}\frac{E}{P^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{\mu_i}{\tilde{c}_j}\right)^{\eta-1}\left[\frac{\beta}{\alpha-1}+\lambda(\frac{\eta}{\eta-1})^{\eta-1}\right] \qquad (7)$$

An even simpler expression obtains in the special case where cost per unit quality does not vary across products within retailers: $\tilde{c}_j = \tilde{c}$. Then equation (5) collapses to

$$r_{ji}(\mathbf{s}) = \frac{E}{P^{1-\eta}}\left(\frac{\eta}{\eta-1}\right)^{-\eta} s^{\frac{\alpha-\eta}{1-\alpha}}\left(\frac{\mu_i}{\tilde{c}}\right)^{\eta-1}\left[\frac{\beta}{\alpha-1}+\lambda(\frac{\eta}{\eta-1})^{\eta-1}\right] \qquad (8)$$

where $s = \sum_{\ell=1}^{J} s_\ell$ is the total number of sellers matched to the buyer, an observable variable.

## 3.3 Search and Matching

### 3.3.1 Market aggregates and Market Slackness

Next we characterize matching patterns in wholesale markets. For expositional clarity, we focus on the case of a single type of seller, and thereby reduce the vector $\mathbf{s}$ to the scaler, $s$. The more general case of multiple seller types is treated in our appendix.

First, we introduce variables that measure agents' "visibility." The key feature of these objects is that, for any two agents or groups of agents on the same side of the market, the ratio of their visibilities is also the ratio of their hazards for meeting a new business partner.

Let $M_i^B(s)$ be the measure of type-$i$ buyers with $s$ sellers, and define these buyers' visibility to be:

$$H_i^B(s) = \sigma_i^B(s) M_i^B(s)$$

where $\sigma_i^B(s)$ measures the search intensity of any one of these buyers. Aggregating over types and partner counts, the overall visibility of buyers is measured by:

$$H^B = \sum_{i=1}^{I} \sum_{s=0}^{s_{\max}} H_i^B(s)$$

Analogously, let $M_j^S(n)$ be the measure of type $j$ sellers with $n$ buyers, and suppose each of these sellers searches with intensity $\sigma_j^B(n)$. Then this group's visibility is measured by:

$$H_j^S(n) = \sigma_j^S(n) M_j^S(n),$$

and the overall visibility of sellers to buyers is:

$$H^S = \sum_{j=1}^{J} \sum_{n=0}^{n_{\max}} H_j^S(n)$$

Following much of the labor search literature, we assume a matching function that is homogeneous of degree one in the visibility of buyers and sellers. Specifically we assume that the measure of matches per unit time is given by (Petrongolo and Pissarides, 2001):[26]

$$X = f(H^S, H^B) = H^B \left[ 1 - (1 - \frac{1}{H^B})^{H^S} \right] \approx H^B \left[ 1 - e^{-H^S/H^B} \right] \tag{9}$$

From buyers' perspective, we can then define market slackness in a manner analogous to

[26]Other matching functions are of course feasible here. We have also experimented with $x = \frac{H^B H^S}{[(H^B)^\alpha + (H^S)^\alpha]^{1/\alpha}}$.

random search models:

$$\theta^B = \frac{f(H^S, H^B)}{H^B}.$$

The larger is $\theta^B$, the more matches take place for a given amount of buyer visibility. Likewise, market slackness from sellers' perspective is:

$$\theta^S = \frac{f(H^S, H^B)}{H^S}. \tag{10}$$

Finally, assuming random matching, the share of matches involving buyers of type $i$ with $s > 0$ sellers is:

$$\frac{\sigma_i^B(s)M_i^B(s)}{H^B} \tag{11}$$

and the share of matches involving sellers of type $j$ with $n > 0$ buyers is:

$$\frac{\sigma_j^S(n)M_j^S(n)}{H^S}.$$

In the absence of $\tilde{c}$ heterogeneity across seller types, sellers' payoffs do not depend upon $j$. And if sellers' search cost functions do not depend upon their type either, we can drop the $j$ subscript from $\sigma_j^S(n)$. For the time being we do so.

### 3.3.2 Optimal search

It remains to characterize the policy functions $\sigma_i^B(s)$ and $\sigma^S(n)$ that maximize the values of agents' expected payoff streams. To do this we introduce buyer and seller search cost functions, which measure the flow cost of sustaining search intensities $\sigma^B$ and $\sigma^S$, respectively:

$$\begin{aligned} k^B\left(\sigma^B, s\right) &= \frac{\left(\sigma^B\right)^{\nu_B}}{(s+1)^{\gamma^B}} \\ k^S\left(\sigma^S, n\right) &= \frac{\left(\sigma^S\right)^{\nu_S}}{(n+1)^{\gamma^S}} \end{aligned}$$

By assumption, search costs are positive and convex in search intensity: $\nu_B, \nu_S > 1$. Also, network effects may reduce the costs of forming new matches as agents' partner counts grow: $\gamma^B, \gamma^S \geq 0$.

**Buyer's problem:** Given that type-$i$ buyers enjoy profit flow $\pi_i^B(s)$ when they are matched with $s$ suppliers, such buyers choose their search intensity to solve:

$$V_i^B(s) = \max_{\sigma^B} \left\{ \frac{\pi_i^B(s) - k^B\left(\sigma^B\right) + s\delta V_i^B(s-1) + \sigma^B\theta^B V_i^B(s+1)}{\rho + s\delta + \sigma^B\theta^B} \right\} \tag{12}$$

where $\rho$ is the rate of time preference and $V_i^B(s)$ is the present value of a type$-i$ buyer that is currently matched with $s$ sellers. Intuitively, the seller reaps profit flow $\pi_i^B(s) - k^B\left(\sigma^B\right)$ until the next event occurs. With hazard $s\delta$ this event is an exogenous termination of one of the $s$ relationships, and with hazard $\sigma^B\theta^B$ it is a new match.

The optimal search policy for type-$i$ buyers with $s$ sellers, $\sigma_i^B(s)$, therefore satisfies

$$\frac{\partial k^B\left(\sigma^B, s\right)}{\partial \sigma^B} = \theta^B\left[V_i^B(s+1) - V_i^B(s)\right]. \tag{13}$$

**Sellers' problem:** Since sellers have constant marginal costs, the number of buyers they currently supply does not affect their expected returns from adding another one. On the other hand, the seller's payoff function from a particular match, $\tau_i(s)$, depends upon the buyer's type, $i$, and the buyer's current seller count, $s$, so ex post, it matters whom sellers match with. The value to any seller of matching with a type$-i$ buyer who has $s$ suppliers is:[27]

$$V_{i,s}^{S}=\frac{\tau_{i}(s)+(s-1)\delta V_{i,s-1}^{S}+\sigma_{i}^{B}(s)\theta^{B}V_{i.s+1}^{S}}{\rho+s\delta+\sigma_{i}^{B}(s)\theta^{B}}. \tag{14}$$

Intuitively, a business relationship with a type-$i$ buyer who has $s$ suppliers will terminate with exogenous hazard $\delta$, become a relationship with a type-$i$ buyer who has $s-1$ suppliers with hazard $(s-1)\delta$. Analogously, it will become a relationship with a type-$i$ buyer who has $s+1$ suppliers with hazard $\sigma_i^B(s)\theta^B$.

Taking expectations over the population of buyers that sellers might meet, the *ex ante* value of a new relationship is:

$$V^{S}=\sum_{i}\sum_{s=0}^{\infty}V_{i,s+1}^{S}P_{i}^{B}(s),$$

where $P_i^B(s) = H_i^B(s)/H^B$ is the relative visibility of buyers who are type$-i$ and have $s$ sellers. So the optimal search intensity for any seller with $n$ buyers satisfies:

$$\frac{\partial k^{S}\left(\sigma^{S},s\right)}{\partial\sigma^{S}}=\theta^{S}V^{S}. \tag{15}$$

#### 3.3.3 Equilibria and Transition Dynamics

**Equations of motion:** Given that all relationships end with exogenous hazard $\delta$, the equation of motion for the measure of buyers of type $i$ with $s$ sellers is:

[27]The destruction hazard $\delta$ is weighted by $(s-1)$ to adjust for the fact that the seller's own relationhip with the buyer may die, in which case the continuation value of this relationship for this seller is zero. Of course $V_s^S$ makes sense only if $s>0$, as a seller can't have a connection with a buyer with zero sellers.

$$\begin{aligned}
\dot{M}_i^B(s) &= \sigma_i^B(s-1)\theta^B M_i^B(s-1) + \delta(s+1)M_i^B(s+1) \\
&\quad - \left(\sigma_i^B(s)\theta^B M_i^B(s) + \delta s M_i^B(s)\right). \\
s &= 1, ..., s_{\max};\ i = 1, ..., I
\end{aligned} \tag{16}$$

This group gains a member whenever any of the $M_i^B(s-1)$ buyers with $s-1$ suppliers adds a supplier, and the hazard of this happening is $\sigma_i^B(s-1)\theta^B$. Similarly, it gains a member whenever any of the $M_i^B(s+1)$ buyers with $s+1$ suppliers loses a supplier because of exogenous attrition, and this occurs with hazard $\delta(s+1)$. By analogous logic, the group loses existing members that either successfully add a supplier (with hazard $\sigma_i^B(s)\theta^B$) or loses one (with hazard $\delta$). Finally, the measure of buyers of type $i$ with $s = 0$ sellers evolves according to:

$$\dot{M}_i^B(0) = \delta M_i^B(1) - \sigma_i^B(0)\theta^B M_i^B(0) \quad i = 1, ..., I \tag{17}$$

Replacing $B$ with $S$ and $s$ with $n$ in (16) and (17), the equations of motion for seller measures $M_j^S(n)$ obtain.

**Steady state:** To characterize the steady state of this system, we set $\dot{M}_i^B(s) = \dot{M}_j^S(n) = 0$ and solve the system of $I \cdot (s_{\max}+1) + J \cdot (n_{\max}+1)$ equations implied by both versions of (16) and (17)–for buyers and sellers. In doing so we, treat the measures of each type of intrinsic type as exogenous constants and impose the adding-up constraints:

$$M_i^B = \sum_{s=0}^{s_{\max}} M_i^B(s) \tag{18}$$

$$M_j^S = \sum_{n=0}^{n_{\max}} M_j^S(n), \tag{19}$$

**Transition dynamics:** Solving for transition dynamics is more involved. Suppose we wish to find the transition path from one market environment to a new one under perfect foresight. We begin by finding the steady distribution of buyers and sellers across types for the new regime, as well as the associated value functions. We then guess the trajectory of endogenous market-wide aggregates $\{\theta^B(t), \theta^S(t), P(t)\}$ from the initial state to this steady state, and solve for buyer and seller distribution functions using backward induction and finite differencing. Appendix C provides details.

### 3.4 Introducing assortative matching

Thus far, our model does not allow for the possibility that some retailers specialize in atheletic shoes, while others are more about dress shoes, and still others do both types of business. Nor it does it provide a mechanism through which assortative matching on the basis of product quality might be accomodated. These features of the model can be relaxed by introducing a compatibility function. This exercise is tangential to our purposes, so we relegate details of this extension to the appendix.

## 4 Fitting the model to data

In this section, we calibrate the model to the data and assess the quality of the fit.

### 4.1 Transfer function estimates

Our data allow us to calculate annual payments from each Colombian footwear importer to each of its foreign suppliers. These bilateral payment records provide a means for estimating equation (7), which we re-state below in log form:

$$\ln\ r_{jit} = \left(\frac{\alpha-\eta}{\alpha-1}\right)\ln h_{j|i,t} + \xi_{ij} + d_t + \varepsilon_{jit} \tag{20}$$

Here we have replaced $\ln\left\{\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\beta}{\alpha-1}+\lambda\right]\right\}+\ln\left(\frac{\mu_i}{\tilde{c}_j}\right)^{\eta-1}$ with match effects, $\xi_{ij}$, and we have absorbed temporal variation in $\ln\frac{E}{P^{1-\eta}}$ with time effects, $d_t$. We have also added a time-varying match-specific shock, $\varepsilon_{jit}$, to recognize that factors outside our model influence the data-generating process.

What might these factors be? Although our model implies that within-match variation in $h_{j|i}$ is driven by random matching patterns, the data may partly reflect variation in $r_{jit}$ due to transitory match-specific demand or marginal cost shocks. Second, to the extent that buyers source some of their merchandise from domestic suppliers, there is likely to be measurement error in $h_{j|i,t}$ that isn't completely absorbed by the fixed effects. Finally, intertemporal variation in sellers' shares of match-specific marginal costs ($\lambda$) might also introduce noise into the relationship between exporter revenues and their shares in buyer payments.

Transitory shocks to $r_{ijt}$ are likely to induce covariation in $h_{j|i,t}$, and measurement error in $h_{j|i,t}$ is likely to induce attenuation bias. Accordingly, we adopt an instrumental variable approach to estimating $\left(\frac{\alpha-\eta}{\alpha-1}\right)$. To construct our instrument, we exploit information on the rivals of seller $j$ who also supply buyer $i$ in period $t$. Specifically, we use the cross-buyer average log sales of these rival firms to buyers *other than* buyer $i$. This variable, denoted $\ln\bar{r}^c_{-j|-i,t}$, should reflect the marginal costs of the rival sellers and thus be correlated with $j$'s share of buyer $i$'s imports. Yet, since their sales to buyer $i$ are excluded from the average, it is arguably uncorrelated with idiosyncratic shocks to buyer $i$. Similarly, since all of seller $j$'s sales are excluded, $\ln\bar{r}^c_{-j|-i,t}$ should be orthogonal to idiosyncratic shocks that affect seller

**Table 3: Transfer function estimates**

| | 1st Stage IV-FE (dependent variable: $\ln h_{j\|i,t}$) | | 2nd stage IV-FE (dependent variable: $\ln r_{jit}$) | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $\ln \hat{h}_{j\|i,t}$ | – | – | 0.1937 (0.0083) | 0.0725 (0.0108) |
| $\ln \overline{r}^c_{-j\|-i,t}$ | −0.2388 (0.0024) | – | – | – |
| $\ln \overline{r}^c_{-j\|-i}$ | – | −0.3810 (0.0044) | – | – |
| instrument | – | – | $\ln \overline{r}^c_{-j\|-i,t}$ | $\ln \overline{r}^c_{-j\|-i}$ |
| match effects | yes | yes | yes | yes |
| year effects | yes | yes | yes | yes |
| $R^2$ | 0.8896 | 0.9045 | 0.8615 | 0.8314 |
| obs. | 771,200 | 1,256,000 | 771,200 | 1,256,000 |

$j$. A variant of this instrument, which we denote $\ln \overline{r}^c_{-j,-i}$ averages over rivals' sellers to not-$i$ buyers in *all* years we observe, not just year $t$.

Both instruments should be negatively correlated with $\ln h_{j|i,t}$ because rivals that sell large quantities to non-$i$ buyers are likely to sell large quantities to buyer $i$ too, driving down $j$'s share of $i$'s total imports. This is indeed the case, as can be seen in the first two columns of Table 3. The results for the two instruments are qualitatively similar, though the instrument based on rival sales in all years, $\ln \overline{r}^c_{-j,-i}$, yields a stronger negative relationship. Note that these regressions control for fixed match effects, and thus isolate variation in $\ln h_{j|i,t}$ over the course of seller $j$'s relationship with buyer $i$, as rival sellers come and go. This is the type of variation our model is designed to describe.

The last two columns of Table 3 report IV estimates of the transfer function coefficient $\left(\frac{\alpha-\eta}{\alpha-1}\right)$. Column (3) corresponds to the instrument based exclusively on rivals' contemporaneous sales, $\ln \overline{r}^c_{-j|-i,t}$, and column corresponds to the instrument based on rival sales in all years they are observed, $\ln \overline{r}^c_{-j,-i}$. The results imply that there are modestly diminishing returns to adding additional sellers of any type, or put differently, the elasticity of substitution across varieties within a store ($\alpha$) exceeds the elasticitiy of substitution across stores ($\eta$).

## 4.2 A preliminary calibration

We now move to a preliminary calibration of the dynamic structural model.We allow for thirty buyer types. The complexity of our numerical problem scales quickly with the number of seller types we allow, so we only allow for two types of seller. The model is parameterized by the elasticity of substitution across products, $\alpha$, the dispersion in the buyer types, $var(\mu)$, the search cost parameters $(k_0^B, k_0^S, \nu_V, \nu_S, \gamma^B, \gamma^S)$, the exogenous separation hazard, $\delta$, and the discount rate, $\rho$.Some of these we fix *ex ante*. First, based on estimates in Hottman *et al.* (forthcoming), we set $\alpha = 4.35$. Next, following the macro literature, we assume a discount rate of $\rho = 0.05$. Since there is no endogenous dropping of clients, we calibrate the match death hazard $\delta$ to 0.81 to reflect the probability of match death observed in our data. This implies an average duration of 1.25 years. Finally, we impose symmetry across buyers and sellers in the search cost scalars, $k_0^B = k_0^S = k_0$, and we assume that both cost functions are quadratic in search intensity: $\nu_V = \nu_S = 2$.

Given our assumption that $\alpha = 4.35$, we can infer from $\frac{\alpha-\eta}{\alpha-1} \approx 0.2$ that $\eta \approx 3.68$. On the seller side, we assume that the distribution of seller types is highly skewed with the quality adjusted cost of five percent of sellers half as high as the other 95%. We will estimate the cost scalar later, so for now we center the seller distribution at one, giving us a variance in the log seller cost distribution of 0.01. We estimate the variance of the fixed effect in Table 3, $var(\ln\left(\frac{\mu_i}{\tilde{c}_j}\right)^{\eta-1})$, to be 2.2. Due to our random matching assumption, the seller and buyer type distributions are independent, so we can calculate the log variance of the buyer type as approximately 0.30. We discretize the associated distribution of buyer effects using the

| Coef. | Sellers | Model | Buyers | Model |
|---|---|---|---|---|
| ln partners | -1.90 | -2.05 | -1.21 | -2.26 |
| (ln partners)2 | -0.20 | -0.59 | 0.01 | -1.27 |
| R2 | 0.99 | 0.99 | 0.99 | 0.99 |

**Table 4: Degree Distribution Regression Coefficients**

| | Data | | Model | |
|---|---|---|---|---|
| | 1 | 2 | 1 | 2 |
| 1 | 0.58 | 0.26 | 0.42 | 0.04 |
| 2 | 0.34 | 0.24 | 0.45 | 0.22 |
| 3 | 0.25 | 0.16 | 0.35 | 0.31 |
| 4 | 0.21 | 0.11 | 0.25 | 0.32 |
| 5 | 0.19 | 0.07 | 0.16 | 0.27 |
| 6 | 0.17 | 0.06 | 0.10 | 0.21 |
| 7 | 0.16 | 0.05 | 0.06 | 0.15 |
| 8 | 0.15 | 0.04 | 0.03 | 0.11 |
| 9 | 0.15 | 0.03 | 0.01 | 0.05 |

**Table 5: Estimated Transition Matrix:Sellers per buyer**

method suggested in Kennan (2006).

It remains to discuss the search cost level parameter $k_0$ and the network effects in search cost $\gamma^B$ and $\gamma^S$. To calibrate these parameters, we regress the log share of firms with one partner, two partners, etc, on log number of partners for both buyers and sellers in both our simulation and our data, then minimize distance between regression coefficients from the simulation and the data. We also match the first several columns of the partner transition matrices.

This exercise yields a search cost level parameter $k_0 = 15.0$, and more interestingly, network effects of $\gamma^S = 1.3$ and $\gamma^B = 1.7$. These parameters imply that network effects play an

| | Data | | Model | |
|---|---|---|---|---|
| | 1 | 2 | 1 | 2 |
| 1 | 0.65 | 0.27 | 0.42 | 0.43 |
| 2 | 0.32 | 0.31 | 0.45 | 0.46 |
| 3 | 0.19 | 0.22 | 0.35 | 0.37 |
| 4 | 0.13 | 0.15 | 0.25 | 0.27 |
| 5 | 0.10 | 0.10 | 0.16 | 0.18 |
| 6 | 0.08 | 0.07 | 0.10 | 0.12 |
| 7 | 0.07 | 0.06 | 0.06 | 0.07 |
| 8 | 0.07 | 0.05 | 0.03 | 0.04 |
| 9 | 0.06 | 0.05 | 0.01 | 0.01 |

**Table 6: Estimated Transition Matrix:Buyers per seller**

important role in the model's ability to match the data. In particular, as in Eaton *et al.* (2014), reductions in search costs due to high visibility allow the model to explain the very large firms that populate the right-hand tails of the client distributions (Figures 9 and 10).

The model does a better job estimating the buyer per seller distribution than it does estimating the seller per buyer distribution (Table 4). On the other hand, considering we only have three degrees of freedom, the model captures the overall data features fairly well. It implies a fat tail in the buyers per seller distribution and also captures the general shape of the partner transition matrix (Figure **??**), including the general tendency to lose clients over time. **Need to redo this figure**

# 5 Putting the model to work

## 5.1 preliminary counterfactuals

In this section we run counterfactual experiments with the model.

Our first experiment is to reduce the search cost parameter $k_0$ by 30 percent, scaling back costs proportionately at all levels of search intensity. Roughly speaking, we think of this exercise as approximating improvements in global communications, and perhaps also the effects of better access to intermediaries in Panama.[28] Again, we see that the tail of the degree distribution of sellers per buyer gets fatter (Figure 13).

For this experiment, we show the full transition from the estimated steady state to eight years after the shock (Figure 12). One implication is that it takes 6-7 years for the welfare benefits of lower costs to be realized. These amount to more that 10 percent per year. Here the gains are driven partly by the increase in the number of varieties available to consumers,

[28]**Note:** this experiment will be replaced by an exercise in which the reduction in $k_0$ is chosen to replicate the growth of trade flows, conditioned on the observed increase in the number of exporters.

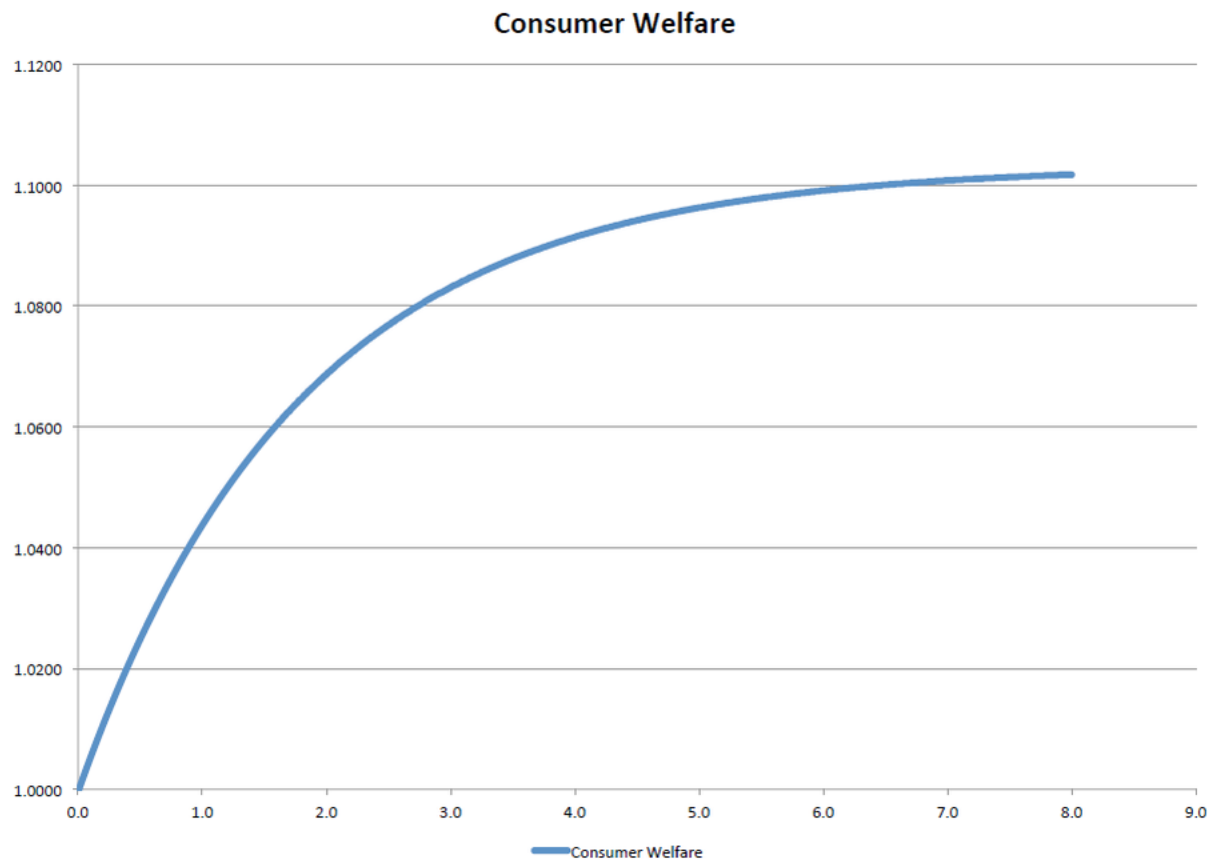


**Figure 12: 30% reduction in search cost: Welfare**

and partly by the fact that varieties are spread across more retailers.

## 5.2 Interpreting the value functions

Exploiting the structure of our model, we can measure the intangible capital stocks that retailers accumulate as their build international business relationships. Figure 14 depicts $V_i^B(s)$ values for the different intrinsic buyer types, and shows how these values vary as firms add and lose clients. To reduce clutter, here have averaged the value functions across the firms with $\mu_i$ values in the lowest tercile (denoted "low $\mu$"), the middle tercile (denoted "medium $\mu$"), the upper tercile (denoted "high $\mu$"). All values are normalized by the cross-importer average annual value of imports, US$ 7,665.

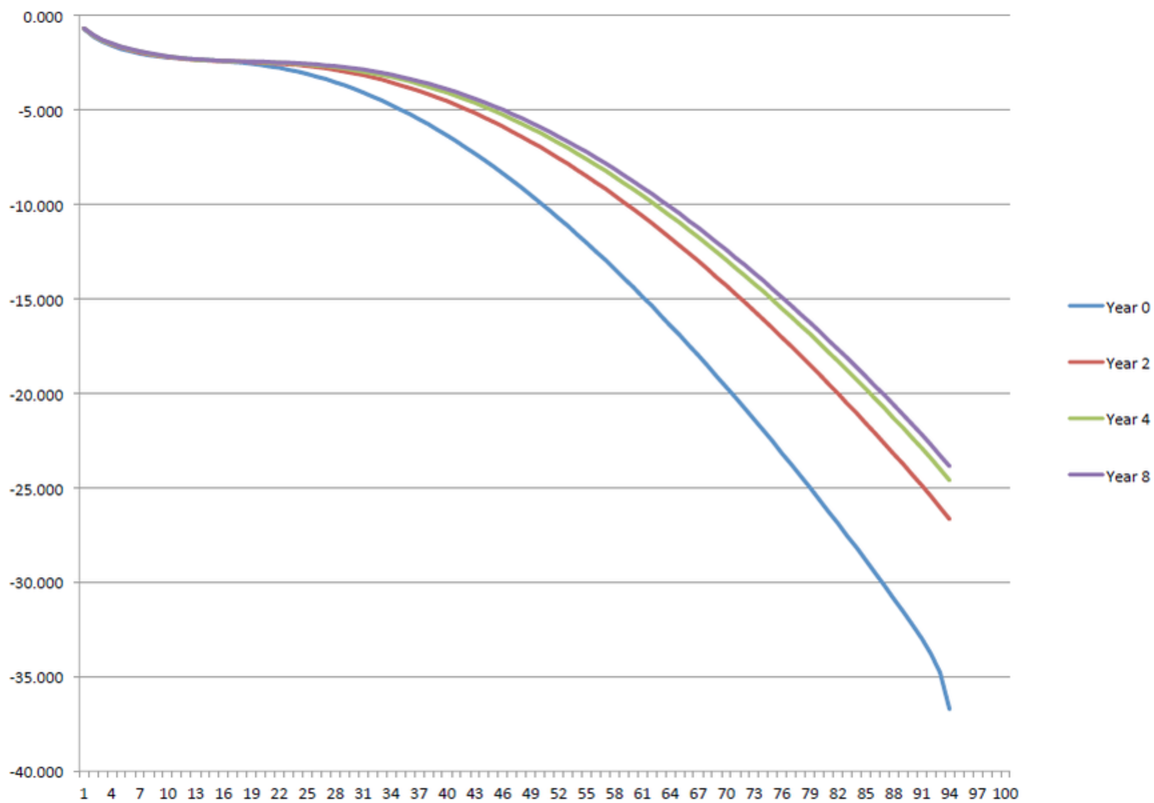


**Figure 13: 30% reduction in search cost: Sellers per buyer**

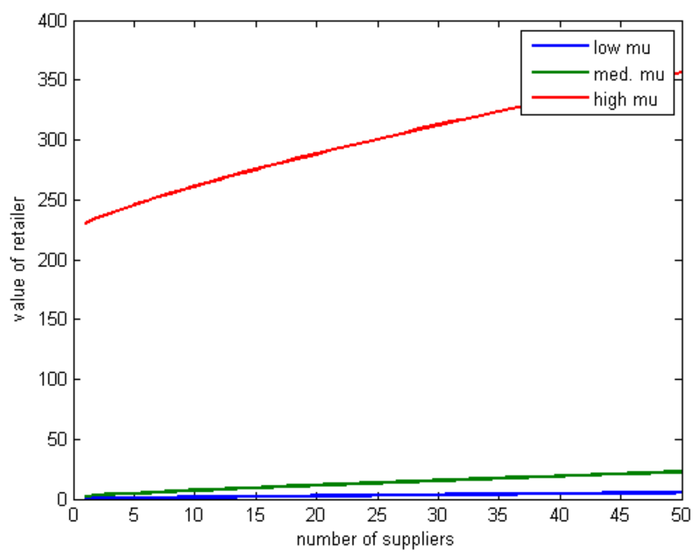


**Figure 14: Retail firm values by type and state**

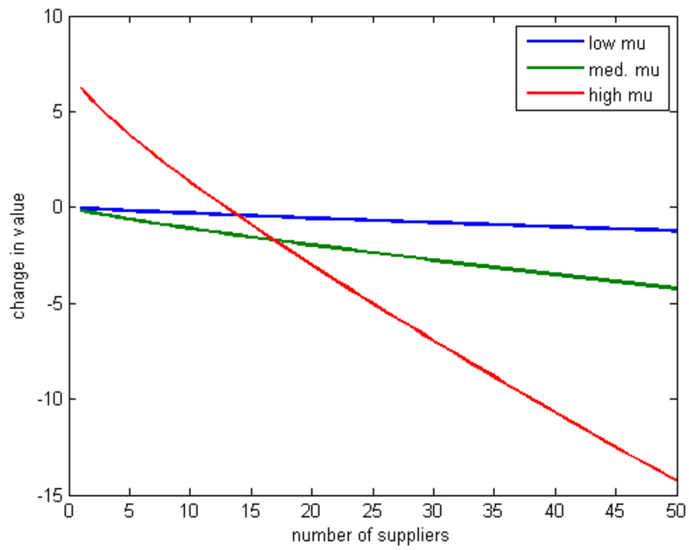


**Figure 15: Capital gains for retailers with search cost reduction**

Several features of this figure merit note. First, $V_i^B(s)$ increases with the number of clients because each client adds value to the retailer by increasing the flow of rents. This is especially true at high-$\mu$ retailers, where relatively large sales volumes are generated per variety sold. Second, however, because of diminishing returns to varieties ($\left(\frac{\eta-1}{\alpha-1}\right) < 1$) and convex search costs, $V_i^B(s)$ is concave in $s$. Finally, the international business connections of high-$\mu$ shoe retailers are quite valuable. Consider, for example, a high-$\mu$ retailer with 30 suppliers. If we took these suppliers away but permitted it to search for replacements, its value would drop by $(300 - 230) \times \$7,665 \approx \$540,000$. Or, if we took these connections away and did not let it replace them, its value would drop by $300 \times \$7,665 \approx \$2,300,000$.

Whatever portfolio of sellers a retailer happens to have, the value of its international business relationships is sensitive to market conditions. Returning to our counterfactual experiments, we now ask how they adjust as search costs fall. Figure 15 depicts the change in value for each of our three classes classes of retailers as a function of the number of suppliers they have.

Two forces are in play in this graph. First, reductions in international search costs generate capital losses because the value of any business relationship is bounded by the costs of replacing it. Firms that have invested in building an extensive portfolio of foreign suppliers therefore lose more value than firms of the same intrinsic appeal that have not. Second, however, reductions in search costs make it less costly for firms to expand, and this is particularly important to high-$\mu$ firms that currently have just a few business partners. These firms are net beneficiaries of search cost reductions. Put differently, high-quality start-ups prefer a world with low search frictions, while established retailers would rather see their investments in suppliers maintain their values.

# 6 Summary

We have developed a dynamic model of international buyer-seller matching in which search intensities are optimally chosen on both sides of the markets, and we have shown that it nicely captures key cross-sectional and dynamic features of international business relationships. Counterfactual exercises based on the model yield several basic messages. First, changes in the population of foreign suppliers—especially in China—led to substantial welfare improvements among Colombian consumers. Second, reductions in search frictions also have the potential to generate large welfare gains. Third, however, search frictions spread firms' adjustments to market shocks over subtantial periods, so that the full benefits of greater market participation by foreign suppliers may take 8-10 years to accrue. Finally, because of these search frictions, connections with foreign business partners are an important component of retailers' intangible capital stock. For the largest retailers, these can be worth millions of dollars.

The empirical application we report is preliminary. In future drafts we plan to incorporate seller-side heterogeneity and to exploit a larger set of moments in the estimation exercise. We also hope to explore applications to other markets, including the U.S. market for apparel.

# References

J. Ahn, A. Khandelwal, and S. Wei. The role of intermediaries in facilitating trade. *Journal of International Economics*, 84:73–85, 2011.

David Akin, Benjamin Faber, and Marco Gonzalez-Navarro. Retail globalization and household welfare: evidence from mexico. *Journal of Political Economy*, forthcoming.

Facundo Albornoz, Hector F Calvo Pardo, Gregory Corcos, and Emanuel Ornelas. Sequential exporting. *Journal of International Economics*, 88(1):17–31, 2012.

Treb Allen. Information frictions in trade. *Econometrica*, 86(6):2041–2083, 2014.

Plunkett Analytics. Apparel and clothing brands, designers, importers and distributors industry (u.s.) analytics. Technical report, Plunkett Research, 2015.

P. Antras and A. Costinot. Intermediated trade. *Quarterly Journal of Economics*, pages 1318–1374, 2011.

P. Antras and D. Shor. Organizing the global value chain. *Econometrica*, pages 2127–2204, 2013.

Costas Arkolakis. A unified theory of firm selection and growth. *Quarterly Journal of Economics*, 131(1):89–155, 2016.

A. Atkeson and A. Burstein. Pricing-to-market, trade costs, and international relative prices. *American Economic Review*, pages 1998–2031, 2008.

Andrew B Bernard and Swati Dhingra. Contracting and the division of the gains from trade. Technical report, National Bureau of Economic Research, 2015.

A. Bernard, A. Moxnes, and Y. Saito. Production networks, geography, and firm performance. Technical report, Tuck School of Business at Dartmouth, 2014.

Andrew B Bernard, Andreas Moxnes, and Karen Helene Ulltveit-Moe. Two-sided heterogeneity and trade. Technical report, National Bureau of Economic Research, 2014.

G. Bertola and P. Garibaldi. Wages and the size of firms in dynamic matching models. *Review of Economic Dynamics*, pages 335–368, 2001.

B. Blum, S. Claro, and I. Horstmann. Intermediation and the nature of trade costs: Theory and evidence. Technical report, University of Toronto, Rottman School of Management, 2009.

Bernardo S Blum, Sebastian Claro, and Ignatius Horstmann. Facts and figures on intermediated trade. *The American Economic Review*, pages 419–423, 2010.

Alberto Cavallo. Are online and offline prices similar? evidence from large multi-channel retailers. *American Economic Review*, pages 283–303, 2016.

Thomas Chaney. The Network Structure of International Trade. *American Economic Review*, 104(11):3600–3034, 2014.

Dominguez, Juan Camilo, Jonathan Eaton, Marcela Eslava, and James Tybout. Search and learning in export markets: Case studies for colombia. 2008.

Lukasz A Drozd and Jaromir B Nosal. Understanding international prices: Customers as capital. *The American Economic Review*, 102(1):364–395, 2012.

Jonathan Eaton, Marcela Eslava, Maurice Kugler, and James Tybout. The margins of entry into export markets: evidence from colombia, 2008.

Jonathan Eaton, Marcela Eslava, David Jinkins, C.J. Krizan, and Jim Tybout. A search and learning model of export dynamics. Technical report, Working paper, 2014.

J. Fernandez-Blanco. A directed search model of intermediated trade. *European Economic Review*, pages 1481–1494, 2012.

Dorieann Fitzgerald, Stephanie Haller, and Yaniv Yedid-Levi. How exporters grow. Technical report, Unpublished working paper, 2016.

Lucia Foster, John Halitwanger, and Chad Syverson. The slow growth of new plants: learning about demand? Technical report, Unpublished working paper, 2015.

Gary Gereffi and Olga Memedovic. The global apparel value chain: What prospects for upgrading by developing countries. Technical report, United Nations Industrial Development Organization, 2003.

F. Gourio and L. Rudanko. Customer capital. *Review of Economic Studies*, 81(3):1102–1136, 2014.

Jung Ha-Brookshire and Barbara Dyer. Apparel import intermediaries: The impact of a hyper-dynamic environment on u.s. apparel firms. *Clothing and Textiles Research Journal*, pages 66–90, 2008.

Jung Ha-Brookshire and Barbara Dyer. Framing a descriptive profile of a transformed apparel industry apparel import intermediaries in the united states. *Journal of Fashion Marketing and Management*, pages 161–178, 2009.

C. Hottman, S. Redding, and D. Weinstein. Quantifying the sources of firm heterogeneity. *Quarterly Journal of Economics*, forthcoming.

John Kennan. A note on discrete approximations of continuous distributions. *University of Wisconsin Madison*, pages 211–251, 2006.

Kevin Lim. Firm-to-firm trade in sticky production networks. Technical report, Princeton University, Dept. of Economics, 2016.

Sheng Lu. 2016 fashion industry benchmarking study. Technical report, United States Fashion Industry Association, 2016.

Erzo GJ Luttmer. Selection, growth, and the size distribution of firms. *The Quarterly Journal of Economics*, 122(3):1103–1144, 2007.

Erzo GJ Luttmer. On the mechanics of firm growth. *The Review of Economic Studies*, 78(3):1042–1068, 2011.

F. McFarlan, Michael Shih-Ta Chen, and Keith Chi-Ho Wong. Li and fung 2012: Hbs case study 9-312-102. Technical report, Harvard Business School, 2012.

Barbara Petrongolo and Christopher Pissarides. Looking into the black box: a survey of the matching function. *Journal of Economic Literature*, 39:390–431, 2001.

Paul Piveteau. An empirical dynamic model of trade with customer accumulation. Technical report, unpublished working paper, 2015.

James E. Rauch and Vitor Trindade. Ethnic Chinese Networks in International Trade. *The Review of Economics and Statistics*, 84(1):116–130, 2002.

James E Rauch and Joel Watson. Starting small in an unfamiliar environment. *International Journal of Industrial Organization*, 21(7):1021–1042, 2003.

J. Rauch and J. Watson. Network intermediaries in international trade. *Journal of Economics and Management Science*, 2004.

James E Rauch. Business and social networks in international trade. *Journal of economic literature*, pages 1177–1203, 2001.

Claudia Steinwender. Information frictions and the law of one price. Technical report, working paper, 2014.

L.A. Stole and J. Zwiebel. Intra-firm bargaining under non-binding contracts. *Review of Economic Studies*, pages 375–410, 1996.

Yoichi Sugita, Kensuke Teshima, and Enrique Seira. Assortative matching of exporters and importers. Technical report, Working Paper, 2014.

Ian Taplin. Global commodit chains and fast fashion: How the apparel industry continues to re-invent itself. *Competition and Change*, pages 246–64, 2014.

Lisa Terry. Snapshot: Sewing up the apparel supply chain. *Inbound Logistics*, 2008.

James Tybout, Zi Wang, and Yingyan Zhou. Are customs records consistent across countries? evidence from the u.s. and colombia. Technical report, Pennsylvania State University, 2016.

# Appendix

# A Demand and Pricing

Using standard CES results, we begin by characterizing prices and market shares for a particular retailer $b$ offering a particular subset of product varieties in the group, $x \in J_b$:

$$C_b = \left(\sum_{x\in J_b} \left(\xi_x C_b^j\right)^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha}{\alpha-1}}, \qquad C = \left(\int_b \left(\mu_b C_b\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{A-1}$$

$$P_b = \left[\sum_{x\in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{1/(1-\alpha)}, \quad P = \left[\int_b \left(\frac{P_b}{\mu_b}\right)^{(1-\eta)}\right]^{1/(1-\eta)} \tag{A-2}$$

$$h_b = \frac{\left(\frac{P_b}{\mu_b}\right)^{1-\eta}}{P^{1-\eta}}, \qquad h_{x|b} = \frac{\left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}}{P_b^{1-\alpha}}$$

These expressions imply the revenue generated by retail sales of product $x$ at store $b$ is:

$$\begin{aligned} R_{xb} &= P_{xb} q_{xb} \\ &= h_{x|b} h_b E \\ &= \mu_b^{\eta-1} \xi_x^{\alpha-1} P_{xb}^{1-\alpha} P_b^{\alpha-\eta} P^{\eta-1} E \end{aligned} \tag{A-3}$$

Since we assume a continuum of buyers, $\frac{\partial \ln P}{\partial \ln P_b} = 0$. Also,

$$\begin{aligned} \frac{\partial \ln P_b}{\partial \ln P_{ib}} &= \frac{\partial P_b}{\partial P_{ib}} \frac{P_{ib}}{P_b} \\ &= 1/(1-\alpha) \left[\sum_{x\in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{1/(1-\alpha)-1-1/(1-\alpha)} \left[(1-\alpha)\left(\frac{P_{ib}}{\xi_i}\right)^{1-\alpha}\right] \\ &= \left[\sum_{x\in J_b} \left(\frac{P_{xb}}{\xi_x}\right)^{1-\alpha}\right]^{-1} \left[\left(\frac{P_{ib}}{\xi_i}\right)^{1-\alpha}\right] = h_{i|b} \end{aligned}$$

Bertrand-Nash pricing therefore implies:

$$
\begin{aligned}
\frac{\partial \ln R_{xb}}{\partial \ln P_{xb}} &= (1-\alpha) + h_{x|b}\,(\alpha-\eta) \\
\frac{\partial \ln R_{xb}}{\partial \ln P_{x'b}} &= h_{x'|b}\,(\alpha-\eta) \qquad \forall x' \neq x
\end{aligned}
$$

Plugging these expressions into the first-order conditions for pricing,

$$
q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}}(p_{x'b} - c_{x'b}) = 0 \quad \forall x \in J_b,
$$

we obtain:

$$
\begin{aligned}
\frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}}\frac{p_{xb}}{E}(\frac{p_{xb}-c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}}\frac{p_{x'b}}{E}(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0 \\
\frac{q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}}\frac{1}{q_{xb}}\left(\frac{p_{xb}q_{xb}}{E}\right)(\frac{p_{xb}-c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}}\frac{1}{q_{x'b}}\left(\frac{q_{x'b}p_{x'b}}{E}\right)(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0 \\
\frac{p_{xb}q_{xb}}{E} + \frac{\partial q_{xb}}{\partial p_{xb}}\frac{p_{xb}}{q_{xb}}\left(\frac{p_{xb}q_{xb}}{E}\right)(\frac{p_{xb}-c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}}\frac{p_{xb}}{q_{x'b}}\left(\frac{q_{x'b}p_{x'b}}{E}\right)(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0 \\
h_{xb} + \frac{\partial q_{xb}}{\partial p_{xb}}\frac{p_{xb}}{q_{xb}}(h_{xb})(\frac{p_{xb}-c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \frac{\partial q_{x'b}}{\partial p_{xb}}\frac{p_{xb}}{q_{x'b}}h_{jb}(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0 \\
h_{xb} + \big(-\alpha + (\alpha-\eta)h_{x|b}\big)(h_{xb})(\frac{p_{xb}-c_{xb}}{p_{xb}}) + \sum_{x' \in J_b,\, x' \neq x} \big((\alpha-\eta)h_{x'|b}\big)\,h_{xb}(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0 \\
1 - \alpha(\frac{p_{xb}-c_{xb}}{p_{xb}}) + (\alpha-\eta)\sum_{x' \in J_b} h_{x'|b}(\frac{p_{x'b}-c_{x'b}}{p_{x'b}}) &= 0
\end{aligned}
$$

where $h_{xb} = h_{x|b}h_b = \frac{p_{xb}q_{xb}}{E}$. From this we can infer $\epsilon^b_{xx} = \frac{\partial \ln h_{xb}}{\partial \ln P_{xb}} - 1 = -\alpha + h_{x|b}(\alpha-\eta)$, and by analogous logic, $\epsilon^b_{xx'} = (\alpha-\eta)h_{x|b}$.

Next, plugging these expressions into the first-order conditions for pricing, we obtain:

$$\begin{aligned} q_{xb} + \sum_{x' \in J_b} \frac{\partial q_{x'b}}{\partial p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \quad \forall j \in J_b, \\ q_{xb} + \sum_{x' \in J_b} \epsilon^b_{xx'} \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) &= 0 \\ q_{xb} + \sum_{\substack{x' \neq x \\ x' \in J_b}} \left[ h_{x|b}(\alpha - \eta) \right] \frac{q_{x'b}}{p_{xb}} (p_{x'b} - c_{x'b}) + \left[ (\alpha - \eta) h_{x|b} \right] \frac{q_{xb}}{p_{xb}} (p_{xb} - c_{xb}) &= 0 \end{aligned}$$

Since this relationship holds for all $x \in J_b$, the mark-up for each product must be the same. Call it $m = \frac{p_{xb} - c_{xb}}{p_{xb}}$ and reduce this equation to $1 - \alpha m + (\alpha - \eta) m = 0$, or

$$m = \frac{1}{\eta}.$$

Essentially the same result can be found in Atkeson and Burstein (2008) and Hottman *et al.* (forthcoming).

# B Value functions with heterogenous buyers

Let $\mathbf{s} = \{s_1, s_2, ..., s_J\}$ be a vector of counts of the number of sellers of each type $j \in \{1, 2, ...J\}$ who are attached to a particular buyer, and let $\mathbf{s}_{-j} = \{s_1, s_2, .s_{j-1}, s_{j+1,}.., s_J\}$ be the same vector without its $j^{th}$ element, so that $(s_j, \mathbf{s}_{-j})$ is one way to indicate that a seller is in state $\mathbf{s}$.

The buyer-to-seller transfer function $\tau^S_{ji}(\mathbf{s})$ and the type-$i$ buyer payoff function $\pi^B_i(\mathbf{s})$ are chosen to satisfy the surplus sharing rule

$$(1-\beta) V^S_i(s_j, \mathbf{s}_{j-1}) = \beta \left[ V^B_i(s_j, \mathbf{s}_{-j}) - V^B_i(s_j - 1, \mathbf{s}_{-j}) \right], \ s_j \in \{1, 2, ..., s^{\max}\}, \ i \in \{1, 2, ..., N^B\}, \tag{A-4}$$

We now derive closed-form expressions for the surplus shares implied by (A-4). The logic is similar to that found in Bertola and Garibaldi (2001), though it is adapted to our discrete state space.

Suppressing buyer-type indices, the flow value of a type-$i$ buyer who is currently in state $\mathbf{s}$ is:

$$\begin{aligned} \rho V_i^B(\mathbf{s}) &= \pi_i^B(\mathbf{s}) - k_s^B(\sigma_i^B(\mathbf{s})) + \sigma_i^B(\mathbf{s}) \sum_j^J \theta_j^B \left[V_i^B(s_j+1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})\right] \\ &\quad + \delta \sum_j^J s_j \left[V_i^B(s_j-1, \mathbf{s}_{-j}) - V_i^B(\mathbf{s})\right] \end{aligned} \tag{A-5}$$

Likewise the value to a type-$j$ seller of being matched with a type-$i$ buyer in state $\mathbf{s}$ is:

$$\begin{aligned} \rho V_{ji}^S(\mathbf{s}) &= \tau_{ji}(\mathbf{s}) + \sigma_i^B(\mathbf{s}) \sum_k^J \theta_k^B \left[V_{ji}^S(s_k+1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})\right] \\ &\quad + \delta \sum_{k=1}^J (s_k - \mathbf{1}_{k=j}) \left[V_{ji}^S(s_k-1, \mathbf{s}_{-k}) - V_{ji}^S(\mathbf{s})\right] \end{aligned} \tag{A-6}$$

Finally, the ex ante expected value of a new business relationship for a type-$j$ seller is:

$$V_j^S = \sum_i \sum_{s \in S} P_i^B(\mathbf{s}) V_{ji}^S(\mathbf{s})$$

where $P_i^B(\mathbf{s}) = H_i^B(\mathbf{s})/H^B$ is the relative visibility of type-$i$ buyers in state $\mathbf{s}$.

# C Bargaining

Differencing the buyer's value function and suppressing buyer type $i$, we have:

$$
\begin{aligned}
& \rho(V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j})) \\
= \ & [\pi^B(s_j,\mathbf{s}_{-j}) - \pi^B(s_j-1,\mathbf{s}_{-j})] - [k^B(s_j,\mathbf{s}_{-j}) - k^B(s_j-1,\mathbf{s}_{-j})] \\
& +\sigma^B(s_j,\mathbf{s}_{-j})[\sum_{k\neq j}\theta_k^B V^B(s_j,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j+1,\mathbf{s}_{-j}) - \theta^B V^B(s_j,\mathbf{s}_{-j})] \\
& -\sigma^B(s_j-1,\mathbf{s}_{-j})[\sum_{k\neq j}\theta_k^B V^B(s_j-1,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j,\mathbf{s}_{-j}) - \theta^B V^B(s_j-1,\mathbf{s}_{-j})] \\
& +\delta[\sum_{k\neq j} s_k V^B(s_j,s_k-1,\mathbf{s}_{-j,k}) + s_j V^B(s_j-1,\mathbf{s}_{-j}) - \bar{s}V^B(s_j,\mathbf{s}_{-j})] \\
& -\delta[\sum_{k\neq j} s_k V^B(s_j-1,s_k-1,\mathbf{s}_{-j,k}) + (s_j-1)V^B(s_j-2,\mathbf{s}_{-j}) - (\bar{s}-1)V^B(s_j-1,\mathbf{s}_{-j})]
\end{aligned}
$$

Now we simplify this equation in two steps. First apply a discrete approximation of the first order condition at $(s_j-1,s_{-j})$ for the buyer search:

$$
\begin{aligned}
[k^B(s_j,\mathbf{s}_{-j}) - k^B(s_j-1,\mathbf{s}_{-j})] \ \approx \ & (\sigma^B(s_j,\mathbf{s}_{-j}) - \sigma^B(s_j-1,\mathbf{s}_{-j}))[\sum_{k\neq j}\theta_k^B V^B(s_j-1,s_k+1,\mathbf{s}_{-j,k}) \\
& +\theta_j^B V^B(s_j,\mathbf{s}_{-j}) - \theta^B V^B(s_j-1,\mathbf{s}_{-j})]
\end{aligned}
$$

Using the above, we can simplify

$$
\begin{aligned}
& -[k^B(s_j,\mathbf{s}_{-j}) - k^B(s_j-1,\mathbf{s}_{-j})] \\
& +\sigma^B(s_j,\mathbf{s}_{-j})[\sum_{k\neq j}\theta_k^B V^B(s_j,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j+1,\mathbf{s}_{-j}) - \theta^B V^B(s_j,\mathbf{s}_{-j})] \\
& -\sigma^B(s_j-1,\mathbf{s}_{-j})[\sum_{k\neq j}\theta_k^B V^B(s_j-1,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V^B(s_j,\mathbf{s}_{-j}) - \theta^B V^B(s_j-1,\mathbf{s}_{-j})] \\
= \ & \sigma(s_j,\mathbf{s}_{-j})[\sum_{k\neq j}\theta_k^B(V^B(s_j,s_k+1,\mathbf{s}_{-j,k}) - V^B(s_j-1,s_k+1,\mathbf{s}_{-j,k})) \\
& +\theta_j^B(V^B(s_j+1,\mathbf{s}_{-j}) - V^B(s_j,\mathbf{s}_{-j})) - \theta^B(V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j}))]
\end{aligned}
$$

Second, we can also simplify the destruction side using

$$
\begin{aligned}
& \delta[\sum_{k \neq j} s_k V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) + s_j V^B(s_j - 1, \mathbf{s}_{-j}) - \bar{s} V^B(s_j, \mathbf{s}_{-j})] \\
& -\delta[\sum_{k \neq j} s_k V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k}) + (s_j - 1) V^B(s_j - 2, \mathbf{s}_{-j}) - (\bar{s} - 1) V^B(s_j - 1, \mathbf{s}_{-j})] \\
= \; & \delta[\sum_{k \neq j} s_k (V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k})) \\
& +(s_j - 1)(V^B(s_j - 1, \mathbf{s}_{-j}) - V^B(s_j - 2, \mathbf{s}_{-j})) - \bar{s}(V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j}))]
\end{aligned}
$$

To summarize, the above gives us:

$$
\begin{aligned}
& \rho[V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j})] \\
= \; & [\pi^B(s_j, \mathbf{s}_{-j}) - \pi^B(s_j - 1, \mathbf{s}_{-j})] \qquad \text{(A-7)} \\
& +\sigma^B(s)[\sum_{k \neq j} \theta_k^B (V^B(s_j, s_k + 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k + 1, \mathbf{s}_{-j,k})) \\
& +\theta_j^B (V^B(s_j + 1, \mathbf{s}_{-j}) - V^B(s_j, \mathbf{s}_{-j})) - \theta^B (V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j}))] \\
& +\delta[\sum_{k \neq j} s_k (V^B(s_j, s_k - 1, \mathbf{s}_{-j,k}) - V^B(s_j - 1, s_k - 1, \mathbf{s}_{-j,k})) \\
& +(s_j - 1)(V^B(s_j - 1, \mathbf{s}_{-j}) - V^B(s_j - 2, \mathbf{s}_{-j})) - \bar{s}(V^B(s_j, \mathbf{s}_{-j}) - V^B(s_j - 1, \mathbf{s}_{-j}))]
\end{aligned}
$$

By the definition of the type $j$ seller's value function, we have

$$
\begin{aligned}
\rho V_j^S(\mathbf{s}) \; = \; & \tau^j(\mathbf{s}) + \sigma^B(s)[\sum_{k \neq j} \theta_k^B V_j^S(s_j, s_k + 1, \mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j + 1, \mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s})] + \qquad \text{(A-8)} \\
& \delta[(\sum_{k \neq j} s_k V_j^S(s_k - 1, \mathbf{s}_{-k}) + (s_j - 1) V_j^S(s_j - 1, \mathbf{s}_{-j})) - \bar{s} V_j^S(\mathbf{s})]
\end{aligned}
$$

Finally, using equations (A-7), (A-8) and (A-4), we have

$$
\begin{aligned}
& \beta\rho[V^B(s_j,\mathbf{s}_{-j}) - V^B(s_j-1,\mathbf{s}_{-j})] - (1-\beta)\rho V_j^S(s_j,\mathbf{s}_{-j}) \\
= \ & \beta[\pi^B(s_j,\mathbf{s}_{-j}) - \pi^B(s_j-1,\mathbf{s}_{-j})] + \\
& (1-\beta)\sigma^B(s)\left[\sum_{k\neq j}\theta_k^B(V_j^S(s_j,s_k+1,\mathbf{s}_{-j,k})) + \theta_j^B V^S(s_j+1,\mathbf{s}_{-j})] - \theta^B V^S(s_j,\mathbf{s}_{-j})\right] + \\
& \delta(1-\beta)\left[\sum_{k\neq j}s_k(V^S(s_j,s_k-1,\mathbf{s}_{-j,k})] + (s_j-1)(V^S(s_j-1,\mathbf{s}_{-j}) - \bar{s}(V^S(s_j,\mathbf{s}_{-j})\right] \\
& -(1-\beta)\left[\tau^j(\mathbf{s}) + \sigma^B(s)[\sum_{k\neq j}\theta_k^B V_j^S(s_j,s_k+1,\mathbf{s}_{-j,k}) + \theta_j^B V_j^S(s_j+1,\mathbf{s}_{-j}) - \theta^B V_j^S(\mathbf{s})]\right] \\
& -(1-\beta)\delta\left[[(\sum_{k\neq j}s_k V_j^S(s_k-1,\mathbf{s}_{-k}) + (s_j-1)V_j^S(s_j-1,\mathbf{s}_{-j})) - \bar{s}V_j^S(\mathbf{s})]\right] \\
= \ & 0
\end{aligned}
$$

Or, cancelling terms and re-arranging, the flow transfer to a type$-j$ seller by a type-$i$ buyer in state $\mathbf{s}$ is share $\beta$ of the total flow surplus generated by their match:

$$\tau^j(\mathbf{s}) = \beta[\pi^B(s_j,\mathbf{s}_{-j}) - \pi^B(s_j-1,\mathbf{s}_{-j}) + \tau^j(\mathbf{s})] \tag{A-9}$$

The total flow surplus created by the marginal match between a type$-j$ seller by a type-$i$ buyer in state $\mathbf{s}$ must equal the sum of the flow surpluses reaped by the buyer and the seller:

$$\pi_i^B(s_j,\mathbf{s}_{-j}) - \pi_i^B(s_j-1,\mathbf{s}_{-j}) + \tau^j(\mathbf{s}) = \pi_i^T(\mathbf{s}) - \pi_i^T(s_j-1,\mathbf{s}_{-j})$$

So we can re-state (A-9) as:

$$\tau_{ji}(\mathbf{s}) = \beta\left[\pi_i^T(\mathbf{s}) - \pi_i^T(s_j-1,\mathbf{s}_{-j})\right] \tag{A-10}$$

# D Transition dynamics

Details to come

# E Adding assortative matching

It is straightforward to modify the model so that particular sellers tend to specialize in particular types of goods. To do so, continue to assume that buyers and sellers encounter each other through an undirected search process. But now suppose that shipments only take place between *compatible* buyers and sellers who meet, and let any randomly selected pair of type$-i$ buyer and type$-j$ seller be compatible with probability $d_{ij} \in [0, 1]$. Finally, assume that buyers and sellers know these probabilities and choose their search intensities accordingly. With these additional assumptions, we are able keep the random search aspects of the model while accomodating the fact that we observe particular types of businesses doing business with one other with greater or lesser frequency than pure randomness would imply.

**Success rates:** For type-$i$ buyers, the expected share of encounters that result in business partnerships is now:

$$a_i^B = \frac{\sum_j \sum_{n=0}^{n_{\max}} d_{ij} \sigma_j^S(n) P_j^S(n)}{\sum_j \sum_{n=0}^{n_{\max}} \sigma_j^S(n) P_j^S(n)} \tag{A-11}$$

where $P_j^S(n) = H_j^S(n)/H^S$ is the share of matches that involve type$-j$ sellers with $n$ buyers. Similarly, for type $j$ sellers, the expected share of meetings that result in business partnerships is:

$$a_j^S = \frac{\sum_i \sum_{s=0}^{s_{\max}} d_{ij} \sigma_i^B(s) P_s^B(i)}{\sum_i \sum_{s=0}^{s_{\max}} \sigma_i^B(s) P_s^B(i)} \tag{A-12}$$

where, recall, $P_i^B(s) = H_i^B(s)/H^B$ is the share of matches that involv type-$i$ buyers who have $s$ sellers. Thus, for a type-$i$ buyer with $s$ suppliers, the hazard of finding another compatible seller is $\sigma_i^B(s)a_i^B\theta^B$. Likewise, for a type-$j$ seller with $n$ buyers, the hazard of finding another compatible buyer is $\sigma_j^S(n)a_j^S\theta^S$.

**Policy functions:** Incorporating compatibility, the programming problem for a type-$i$ buyer with $s$ sellers becomes:

$$V_i^B(s) = \max_{\sigma_i^B(s)} \left\{ \frac{\pi_i^B(s) - c_B\left(\sigma^B\right) + s\delta V_i^B(s-1) + \sigma_i^B(s)a_i^B\theta^B V_i^B(s+1)}{\rho + s\delta + \sigma_i^B(s)a_i^B\theta^B} \right\} \tag{A-13}$$

Accordingly, the new buyer policy functions, $\sigma_s^B(i)$, solve the first order conditions:

$$c_B'\left(\sigma_i^B(s)\right) = a_i^B\theta^B\left[V_i^B(s+1) - V_i^B(s)\right]. \tag{A-14}$$

Similar modifications apply on the sellers' side. The value to a seller of an existing compatible relationship with a type$-i$ buyer in state $s$ now depends on $a_i^B$. This is because the hazard of this buyer adding another seller depends upon her compatibility:

$$V_{i,s}^S = \frac{\tau_i(s) + (s-1)\delta V_{i,s}^S(s-1) + \sigma_i^B(s)a_i^B\theta^B V_{i,s}^S(s+1)}{\rho + s\delta + \sigma_i^B(s)a_i^B\theta^B} \tag{A-15}$$

And the ex-ante potential value of a new relationship with a compatible buyer is:

$$V_j^S = \sum_{i=1}^{I}\sum_{s=0}^{s_{\max}} V_i^S(s+1)\frac{d_{ij}P_i^B(s)}{\sum_{i,s} d_{ij}P_i^B(s)}$$

The associated seller policy functions, $\sigma_j^S(n)$, therefore solve:

$$c_S'\left(\sigma_j^S(n)\right) = a_j^S\theta^S V_j^S. \tag{A-16}$$

**Empirical implementation:** The $d_{ij}$ 's can be solved for using observed shares of different product categories at different firms, so this extension adds no new parameters to identify.

(Details to come.)