Asset Liquidity and Home Bias

Athanasios Geromichalos

University of California - Davis

Ina Simonovska

University of California - Davis, NBER and CESifo

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ABSTRACT

We study optimal asset portfolio choices in a two-country model of costly international trade. We allow assets to not only represent claims on future consumption, but to also serve as media of exchange. In the model, trading in a certain country involves the exchange of locally produced goods for a portfolio of assets. Assuming foreign assets trade at a cost, we characterize equilibria in which different countries' assets arise as media of exchange in different types of trades. More frequent trading opportunities at home result in agents holding proportionately more domestic over foreign assets. As international trade becomes more integrated, agents demand higher amounts of foreign assets. Moreover, foreign assets turn over faster than home assets because the former have desirable liquidity properties, but unfavorable returns over time. Our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

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Email: ageromich@ucdavis.edu, inasimonovska@ucdavis.edu.

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1 Introduction

In this paper, we explore how liquidity properties of asset shape countries' international diversification patterns. A concrete concept of asset liquidity is, therefore, required. To that end, we employ a model in the tradition of modern monetary theory, extended to include trade between countries and real assets that serve a double role. Assets are claims to future consumption, as is standard in finance, but they can also be used as means of payments in decentralized (non-Walrasian) markets. It is this second-new role that captures the notion of asset liquidity.

Assuming foreign assets trade at a cost in centralized markets, we characterize equilibria in which different assets arise as media of exchange in different types of decentralized meetings. We provide sufficient conditions for each of three types of equilibria to arise. In the first case, assets circulate as media of exchange in their respective domestic markets, regardless of asset prices. This result endogenizes the commonly assumed existence of currency areas in international economics. In the second case, dubbed international currencies equilibrium, agents use domestic assets to trade both at home and abroad, regardless of asset prices. The third equilibrium allows either case to arise, depending on equilibrium asset prices.

As long as agents have more opportunities to trade in domestic over foreign decentralized markets, they hold more domestic assets than suggested by the world asset portfolio. Thus, agents' portfolios exhibit home bias. Since agents hold larger amounts of domestic over foreign assets, they have larger claims to domestic over foreign consumption goods, which gives rise to consumption home bias. Hence, the novel liquidity mechanism positively links asset and consumption home bias due to costly cross-border trade. Moreover, foreign assets turn over faster than home assets because, while the former have desirable liquidity properties, they yield lower future consumption and are undesirable savings tools. Thus, our mechanism offers an answer to a long-standing puzzle in international finance as described in Lewis (1999): a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

On the empirical side, Amadi and Bergin (2008) document that turnover rates of foreign assets are twice as high as those of domestic assets for a set of four countries. Yet, as we document in this paper, the average country holds 86 percent of its wealth in domestic equities. Moreover, we observe that bilateral equity investment shares are strongly positively correlated with bilateral import shares. Models based on frictions in international asset and/or goods markets can capture the last two facts, but at the cost of generating counterfactual predictions about turnover rates.

The liquidity mechanism described in this paper can rationalize the three regularities discussed above; however, in order to remain tractable, the model assumes away the risky nature of equity returns. Thus, the mechanism is complementary to existing theories that relate asset and consumption home bias in models of aggregate uncertainty. For example, Heathcote and

Perri (2007) use the international business cycle framework of Backus, Kehoe, and Kydland (1992) and Backus, Kehoe, and Kydland (1993) and show that, when preferences are biased toward domestic goods, asset home bias arises because endogenous international relative price fluctuations make domestic stocks a good hedge against non-diversifiable labor income risk. Their model generates a tight link between openness to trade and the level of asset diversification. This positive link is also explored by Collard, Dellas, Diba, and Stockman (2009) in an endowment economy with a separable utility function between traded and non-traded goods. In their model, an agent's optimal portfolio includes the entire stock of home firms that produce domestic non-traded goods and a fully diversified portfolio of equities of firms that produce tradable goods. The authors show that, if the share of non-traded goods in consumption is large, the model can generate substantial portfolio home bias.

Like the literature discussed above, this paper studies asset portfolio choices in an international framework. However, the model used relates more closely to the growing literature that focuses on the liquidity properties of objects other than fiat money. In their pioneering work, Lagos and Rocheteau (2008) assume that part of the economy's physical capital can be used as a medium of exchange along with money. Their goal is to study the issue of over-investment and how it is affected by inflation. In other related literature, Geromichalos, Licari, and Suarez-Lledo (2007) introduce a real financial asset and study the co-existence of money and the asset as media of exchange, with special focus on the relationship between asset prices and monetary policy. Lester, Postlewaite, and Wright (2008) take this framework one step further and endogenize the acceptability (of various media of exchange) decisions of agents. Finally, Lagos (2006) considers a similar framework, enriched with uncertainty, in order to address the equity premium and risk-free rate puzzles. The present paper, however, is the first to introduce liquidity of assets in a multi-country environment, and explore the implications of liquidity on the distribution of asset holdings across countries.

Finally, our model relates to the literature of money-search models applied to international frameworks. In their pioneering work, Matsuyama, Kiyotaki, and Matsui (1993) employ a two-country, two-currency money-search model, with indivisible money and goods, and study conditions under which the two currencies serve as media of exchange in different countries. Wright and Trejos (2001) extend this framework by introducing a model with indivisible goods, and endogenize prices using bargaining theory. Head and Shi (2003) also consider a two-country two-currency search model and show that the nominal exchange rate depends on the stocks and growth rates of the two monies. Finally, Camera and Winkler (2003) use a search-theoretic model of monetary exchange in order to show that the absence of well integrated international goods markets does not necessarily imply a violation of the law of one price. In this paper, we employ a model with divisible assets and goods. Moreover, by endowing assets, other than fiat money, with certain liquidity properties, we bring the money-search literature closer to questions related to international portfolio diversification.

The remainder of the paper is organized as follows. Section 2 describes the modeling environment. Section 3 characterizes the media of exchange that arise in a symmetric two-country model of international trade in goods and assets. Section 4 describes the model's predictions regarding asset and consumption home bias as well as asset turnover rates and provides empirical support for the mechanism. Section 5 concludes.

2 Model

2.1 Description of the Physical Environment

Time is discrete with an infinite horizon. Each period consists of two sub-periods. During the first sub-period trade occurs in decentralized markets (DM), while in the second sub-period economic activity takes place in centralized (Walrasian) markets (CM). Sometimes we refer to the DM as the day market and the CM as the night market. There is no aggregate uncertainty. There are two countries, A and B. During the first sub-period, a distinct decentralized market opens within each country and anonymous bilateral trade takes place. We refer to these markets as DM_i, $i = A, B$. Without loss of generality, assume that DM_A opens first. Each country has a unit measure of buyers and a measure ξ of sellers who live forever. The identity of agents (as sellers or buyers) is permanent. Sellers from country i are immobile; they can neither visit nor produce the special good in country j. Buyers are mobile. Therefore, in DM_i sellers who are citizens of country i meet buyers who could be citizens of any of the two countries.

All agents discount the future between periods (but not sub-periods) at rate $\beta \in (0, 1)$. Buyers consume in both subperiods, and supply labor in the CM. Their preferences, which are independent of their citizenship, are given by $\mathcal{U}^B(x_A, x_B, X, H)$ where x_i is consumption in DM_i, $i = A, B$, X is consumption in the CM, and H is labor in the CM. Sellers consume only in the CM, and they produce in both the DM and the CM. Sellers' preferences are given by $\mathcal{U}^S(h, X, H)$, where the only new variable is h, which stands for hours worked in the DM. In line with Lagos and Wright (2005) (LW henceforth), we use the functional forms

$$\begin{aligned}\mathcal{U}^B(x_A, x_B, X, H) &= u(x_A) + u(x_B) + U(X) - H,\\ \mathcal{U}^S(h, X, H) &= -c(h) + U(X) - H,\end{aligned}$$

We assume that u and U are twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, $U' > 0$, $u'' < 0$, and $U'' \leq 0$. For simplicity we assume that $c(h) = h$, but this is not important for any of our results. Let $q^* \equiv \{q : u'(q^*) = 1\}$, and suppose that there exists $X^* \in (0, \infty)$ such that $U'(X^*) = 1$ with $U(X^*) > X^*$.

In the day markets, sellers and buyers are matched randomly, according to a matching technology that is identical in both DM's. Let B, S be the total number of buyers and sellers in some DM and define market tightness as $\theta = B/S$. The total number of matches in this market is given by $M(B, S)$, where M is assumed to be increasing in both arguments, concave, and homogeneous of degree one. The arrival rate of buyers to an arbitrary seller is $a_S = M(B, S)/S = M(B/S, 1) \equiv f(\theta)$. Homogeneity of degree one implies that the arrival rate of sellers to an arbitrary buyer is $a_B = M(B, S)/B = a_S/\theta$.[1] In both DM's $S = \xi$. Buyers get to visit the domestic DM with probability σ_H and the foreign DM with probability σ_F, which can be thought of as the degree of economic integration. The measure of *active* buyers in both DM's is given by $B = \sigma_H + \sigma_F$ and the market tightness by $\theta = \xi(\sigma_H + \sigma_F)^{-1}$. In a meeting in any DM, the buyer always makes a take-it-or-leave-it offer to the seller. Any sale to a buyer from country j will count as imports of country j from country i.

During the second sub-period agents trade in centralized markets, CM_i, $i = A, B$. All agents consume and produce a general good or *fruit* which is identical in both countries. Agents are located in the home-country and have access to a technology that can transform one unit of labor into one unit of the fruit. Following Lucas (1978), we assume that there are two trees, one in each country, that produce fruit. Shares of the tree in country i are traded in CM_i, but due to perfect financial integration, agents from country j can place any order and buy shares of this tree at the ongoing price ψ_i. Let $T_i > 0$ denote the total supply of the tree in country i and R_i the dividend of tree i. Since in this paper we focus on symmetric equilibria, we assume that $T_A = T_B = T > 0$ and $R_A = R_B = R > 0$. T and R are exogenously given and constant.

Except from consuming the fruit of the trees and trading their shares in the CM's, buyers can also carry some claims into the DM's in order to trade them for the special good, if they meet a seller. Hence, in this model the assets serve not only as stores of value, but also as media of exchange. The necessity for a medium of exchange arises due to anonymity and a double coincidence of wants problem that characterizes trade in the decentralized markets (see Kocherlakota (1998) for an extensive discussion). The assets can play this role as long as they are portable, storable, divisible, and recognizable by all agents. We assume that all these properties are satisfied. As we explain in more detail below, we do not place any *ad hoc* restrictions on which assets can serve where as means of payments.

In the absence of any frictions in the physical environment, the model would predict that agents' share of foreign assets in their portfolios is anywhere between 0 and 100%. In order to derive sharper predictions, additional assumptions are necessary. One friction that is very common in the international macroeconomics literature is the so-called *currency areas* assumption. According to this assumption, when trading in country i only the currency (in our case the asset) of that country can be used as a medium of exchange. This assumption is extremely

[1] Clearly, the assumption here is that the arrival rate for buyers does not depend on the citizenship of the buyer. Equivalently, the probability with which a seller meets a local buyer is equal to the probability with which she meets a foreign buyer. There are examples in the literature where this is not necessarily true. See for instance Blanchard and Diamond (1994), where the authors consider matching with ranking.

appealing, and results in very intuitive predictions about international portfolio choices. However, since our paper is in the spirit of modern monetary theory, we consider such an assumption undesirable, and insist that agents should choose for themselves which objects (assets) will be used as media of exchange and which will not.

We now introduce the main friction of our model. We assume that whenever an agent from country i holds one share of t_j, she has a claim to $R-\kappa$ units of fruit, with $\kappa \in (0, R)$, while the net return of one claim to the local tree is still given by R. Sometimes, we refer to κ as the cost of liquidating the foreign asset. The most straightforward way to interpret κ is the following: When an agent from country i holds one share of t_j, R units of the general good have to be physically delivered from country j to its claimant in country i. Therefore, κ represents a transportation cost. Alternatively, κ can be thought of as an information friction or cost that agents have to pay in order to participate in the foreign asset market. Finally, κ may capture policy frictions such as tariffs on goods imports or taxes on foreign dividend returns, both of which are commonly observed across many countries. It is important to highlight that all the results presented in this paper hold for even tiny values of κ.

2.2 Value Functions and Optimal Behavior

We begin with the description of the value functions in the Walrasian market. For a seller from country $i = A, B$, the Bellman's equation is given by

$$\begin{aligned} &W_i^S(t) = \max_{X,H,\hat{t}} \left\{U(X) - H + \beta V_i^S(\hat{t})\right\} \\ \text{s.t.} \quad &X + \psi_i \hat{t}_i + \psi_j \hat{t}_j = H + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j, \end{aligned}$$

where $t \equiv (t_i, t_j)$ and t_l denotes the amount of asset $l = A, B$ held by the agent when she enters CM_i. Variables with hats denote next period's choices. It can be easily verified that, at the optimum, $X = X^*$. Using this fact and replacing H from the budget constraint into W_i^S yields

$$\begin{aligned} W_i^S(t) \quad &= \quad U(X^*) - X^* + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j \\ &+ \quad \max_{\hat{t}} \left\{-\psi_i \hat{t}_i - \psi_j \hat{t}_j + \beta V_i^S(\hat{t})\right\}. \end{aligned} \tag{1}$$

Some results are worth highlighting here. First, the choice of $\hat{t}$ does not depend on t. In other words, there are no wealth effects, which follows from the quasi-linearity of $\mathcal{U}$. Second, the function W_i^S is linear and we can write[2]

$$W_i^S(t) = \Lambda_i^S + (\psi_i + R)\, t_i + (\psi_j + R_\kappa)\, t_j,$$

[2] The term Λ_i^S consists of constant terms (like X^*, $U(X^*)$) and terms that depend on $\hat{t}$. By the no wealth effect property, the latter does not depend on t. Hence, W_i^S is linear.

where the definition of Λ_i^S is obvious.

Next, consider a buyer from country i. The state variables for this agent are summarized by $t \equiv (t_{ii}, t_{ij}, t_{ji}, t_{jj})$, where t_{ij} is the amount of asset i that is used for trade in DM_j. In other words, we allow buyers to choose any amount of assets they wish, but we do not allow them to use t_{ij} and t_{jj} for trade in DM_i. This assumption makes the agent's maximization problem more tractable.[3] The Bellman's equation for a buyer from i is given by

$$W_i^B(t) = \max_{X,H,\hat{t}} \left\{U(X) - H + \beta V_i^B(\hat{t})\right\}$$
$$\text{s.t.} \quad X + \psi_i\left(\hat{t}_{ii} + \hat{t}_{ij}\right) + \psi_j\left(\hat{t}_{ji} + \hat{t}_{jj}\right) = H + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}).$$

Once again, $X = X^*$ at the optimum. This allows us to write

$$\begin{aligned} W_i^B(t) &= U(X^*) - X^* + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}) \\ &+ \max_{\hat{t}} \left\{-\psi_i\left(\hat{t}_{ii} + \hat{t}_{ij}\right) - \psi_j\left(\hat{t}_{ji} + \hat{t}_{jj}\right) + \beta V_i^B(\hat{t})\right\}. \end{aligned} \tag{2}$$

As in the case of sellers, there are no wealth effects in the choice of $\hat{t}$, and W_i^B is linear:

$$W_i^B(t) = \Lambda_i^B + (\psi_i + R)(t_{ii} + t_{ij}) + (\psi_j + R_\kappa)(t_{ji} + t_{jj}),$$

where the definition of Λ_i^B is obvious.

We now turn to the terms of trade in the DM's. As explained earlier, we place no restrictions on which assets can be used as means of payments. Consider meetings in DM_i, and as a first case let the buyer be a citizen of country i (a local). Suppose that the asset holdings of the buyer are t and those of the seller are $\tilde{t}$. The solution to the bargaining problem is a list (q_i, x_{ii}, x_{ji}), where q_i is the amount of special good, x_{ii} is the amount of asset i, and x_{ji} is the amount of asset j that changes hands. With take-it-or-leave-it offers by the buyer, the bargaining problem we need to solve is

$$\begin{gathered} \max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) + W_i^B(t_{ii} - x_{ii}, t_{ij}, t_{ji} - x_{ji}, t_{jj}) - W_i^B(t)\right], \\ s.t. \;\; -q_i + W_i^S(\tilde{t}_i + x_{ii}, \tilde{t}_j + x_{jj}) - W_i^S(\tilde{t}) = 0, \\ and \;\; x_{ii} \le t_{ii}, x_{ji} \le t_{ji}. \end{gathered} \tag{3}$$

[3] Moreover, given our model specification, if agents only choose some (t_i, t_j), which they can trade in any market, the order in which markets open clearly matters. We consider this an undesirable feature.

Exploiting the linearity of the W's, we can re-write this problem as follows

$$\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) - (\psi_i + R)x_{ii} - (\psi_j + R_\kappa)x_{ji} \right],$$
$$s.t.\ \ q_i = (\psi_i + R)x_{ii} + (\psi_j + R_\kappa)x_{ji},$$
$$and\ \ x_{ii} \leq t_{ii}, x_{ji} \leq t_{ji}.$$

The following lemma describes the bargaining solution in detail.

Lemma 1. *Define* $\pi_i \equiv (\psi_i + R)t_{ii} + (\psi_j + R_\kappa)t_{ji}$.

$$If \quad \pi_i \geq q^*, \quad then \quad \begin{cases} q_i = q^*, \\ (\psi_i + R)x_{ii} + (\psi_j + R_\kappa)x_{ji} = q^*. \end{cases}$$

$$If \quad \pi_i < q^*, \quad then \quad \begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii},\ x_{ji} = t_{ji}. \end{cases}$$

Proof. It can be easily verified that the suggested solution satisfies the necessary and sufficient condition for maximization. □

Since the buyer is from country i, seller and buyer have the same valuation for both assets. Hence, all that matters for the bargaining solution is whether the buyer's real balances of assets used for trade in DM_i, i.e. π_i, are enough to buy the first best quantity. If the answer to that question is yes, then $q_i = q^*$, and the buyer spends amounts of assets t_{ii}, t_{ji} such that $\pi_i = q^*$. Notice that in this case t_{ii}, t_{ji} cannot be pinned down separately.[4] On the other hand, if $\pi_i < q^*$, the buyer gives up all her asset holdings, and purchases as much q as her real balances allow.

Now consider a meeting in DM_i, when the buyer is a foreigner with asset holdings t. Again, denote the solution as (q_i, x_{ii}, x_{ji}). The bargaining problem to be solved is the same as in (3), after replacing W_i^B, with W_j^B. Using the linearity of the value functions, one can re-write the

[4] Later, we will see that this does not cause any indeterminacy issues, because in equilibrium the agent will set $t_{ji} = 0$.

bargaining problem as[5]

$$
\begin{aligned}
&\max_{q_i, x_{ii}, x_{ji}} \left[u(q_i) - (\psi_i + R_\kappa) x_{ii} - (\psi_j + R) x_{ji} \right], \\
&\textit{s.t.}\;\; q_i = (\psi_i + R) x_{ii} + (\psi_j + R_\kappa) x_{ji}, \\
&\textit{and}\;\; x_{ii} \le t_{ii}, x_{ji} \le t_{ji}.
\end{aligned}
$$

Substituting for $(\psi_i + R)x_{ii} + (\psi_j + R)x_{ji}$ from the constraint into the objective, we can re-write the latter as $u(q_i) - q_i - \kappa x_{ji} + \kappa x_{ii}$. Notice that for every unit of asset i that goes from the buyer to the seller, two positive effects are generated. First, trade is facilitated, i.e. the seller produces q for the buyer in exchange for the asset. Second, the social surplus increases because the asset goes to the hands of the agent that has a higher valuation for it (because the seller does not have to incur the liquidation cost κ). These facts determine the spirit of the solution to the bargaining problem, which is now stated in detail.

Lemma 2. *Define $\overline{q}(\psi) \equiv \left\{ q : u'(q) = \frac{\psi + R_\kappa}{\psi + R} \right\}$ and $\underline{q}(\psi) \equiv \left\{ q : u'(q) = \frac{\psi + R}{\psi + R_\kappa} \right\}$, with $\overline{q}(\psi) > q^* > \underline{q}(\psi)$, for all $\psi < \infty$. The bargaining solution is the following:*

a) If $t_{ii} \ge \frac{\overline{q}(\psi_i)}{\psi_i + R}$, then $\begin{cases} q_i = \overline{q}(\psi_i), \\ x_{ii} = \frac{\overline{q}(\psi_i)}{\psi_i + R}, \\ x_{ji} = 0. \end{cases}$

b) If $t_{ii} \in \left[\frac{\underline{q}(\psi_j)}{\psi_i + R}, \frac{\overline{q}(\psi_i)}{\psi_i + R} \right)$, then $\begin{cases} q_i = t_{ii}(\psi_i + R), \\ x_{ii} = t_{ii}, \\ x_j = 0. \end{cases}$

c1) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i + R}$ and $\pi_i \ge \underline{q}(\psi_j)$, then $\begin{cases} q_i = \underline{q}(\psi_j), \\ x_{ii} = t_{ii}, \\ x_{ji} = \frac{\underline{q}(\psi_j) - (\psi_i + R)t_{ii}}{\psi_j + R_\kappa}. \end{cases}$

c2) If $t_{ii} < \frac{\underline{q}(\psi_j)}{\psi_i + R}$ and $\pi_i < \underline{q}(\psi_j)$, then $\begin{cases} q_i = \pi_i, \\ x_{ii} = t_{ii}, \\ x_{ji} = t_{ji}. \end{cases}$

Proof. See the Appendix. □

The solution to the bargaining problem is very intuitive, and it is driven by the amount of foreign assets held by the citizen of country j. The buyer should only use asset i, whenever

[5] Here we have silently assumed that the buyer and seller can only exchange good for assets and not assets for assets. Given that the two parties have different valuations for the two assets, it is not at all clear why this should be the case. The justification for our assumption is as follows. Since sellers have no bargaining power, it is always optimal to bring zero assets into the match, which is what they will do in equilibrium. Hence, although for the sake of generality we describe the bargaining solution for any asset holding $\tilde{t}$ by the seller, we know that in equilibrium $\tilde{t} = 0$. So our assumption simplifies the bargaining problem without affecting equilibrium results.

possible. If her asset-i holdings are unlimited, she should buy the quantity defined as $\overline{q}(\psi)$. This quantity is bigger than q^*, the maximizer of $u(q) - c(q)$, because of the second positive effect of using asset i, described above (the wedge between the buyer's and seller's valuation). In a similar spirit, if the buyer's balances allow her to buy $\underline{q}(\psi)$ or more, she should never use any asset j for trade. Positive amounts of asset j will change hands, only if the asset i holdings are such that the buyer cannot purchase $\underline{q}(\psi)$. In that case, the buyer will use the amount of asset j that, together with all her asset i, buys the quantity $\underline{q}(\psi)$, and nothing above that level.

We now proceed to the description of the value functions in the DM. Once these functions are known, we can plug them into the expressions for the CM value functions, and characterize the optimal behavior of agents. Consider first a seller from country i. Since buyers make take-it-or-leave-it offers, $V_i^S(t) = W_i^S(t)$. Use this result in (2), (3) to obtain

$$W_i^S(t) = K_i^S + \max_{\hat{t}} \left\{ J_i^S(\hat{t}) \right\},$$

where

$$J_i^S(\hat{t}) = \left[-\psi_i + \beta \left(\hat{\psi}_i + R \right) \right] \hat{t}_i + \left[-\psi_j + \beta \left(\hat{\psi}_j + R_\kappa \right) \right] \hat{t}_j, \tag{4}$$

and K_i^S is a term that does not depend on $\hat{t}$.[6] We refer to J_i^S as the objective function of seller i. Maximization of this function with respect to $\hat{t}$ fully describes the optimal asset holdings of this agent in every period. We will return to discuss the optimal choice after completing the description of the objective functions for all agents.

Consider a buyer from i. The value function for this agent, when she enters the decentralized trade round with asset holdings $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$ is given by[7]

$$\begin{aligned} V_i^B(t) &= \sigma_H \sigma_F a_B^2 \Big\{ u(q_i) + u(q_j) + W_i^B \left(\hat{t}_{ii} - x_{ii}, \hat{t}_{ij} - x_{ij}, \hat{t}_{ji} - x_{ji}, \hat{t}_{jj} - x_{jj} \right) \Big\} \\ &+ \sigma_H a_B (1 - \sigma_F a_B) \Big\{ u(q_i) + W_i^B \left(\hat{t}_{ii} - x_{ii}, \hat{t}_{ij}, \hat{t}_{ji} - x_{ji}, \hat{t}_{jj} \right) \Big\} \\ &+ \sigma_F a_B (1 - \sigma_H a_B) \Big\{ u(q_j) + W_i^B \left(\hat{t}_{ii}, \hat{t}_{ij} - x_{ij}, \hat{t}_{ji}, \hat{t}_{jj} - x_{jj} \right) \Big\} \\ &+ (1 - \sigma_H a_B)(1 - \sigma_F a_B) W_i^B(t), \end{aligned} \tag{5}$$

where the q and x terms are determined through the bargaining protocols described in Lemmas 1 and 2. Also, it is understood that (q_i, x_{ii}, x_{ji}) are functions of $(\hat{t}_{ii}, \hat{t}_{ji})$ and (q_j, x_{ij}, x_{jj}) are functions of $(\hat{t}_{ij}, \hat{t}_{jj})$. Following the same steps as in the case of a seller, one can conclude that

[6] More precisely, one can show that, $K_i^S = (U(X^*) - X^*)(1 + \beta) + (\psi_i + R)t_i + (\psi_j + R_\kappa)t_j + \beta \max_{\hat{\hat{t}}} \left\{ -\hat{\psi}_i \hat{\hat{t}}_i - \hat{\psi}_j \hat{\hat{t}}_j + \beta V_i^S(\hat{\hat{t}}) \right\}$. By the no-wealth effect result, the choice of $\hat{\hat{t}}$ does not depend on $\hat{t}$.

[7] The first line represents the case in which the buyer is mathced in both DM's. The second and third line represent the cases in which she matches only in the home and foreign DM, respectively. The last line stands for the case in which the buyer does not trade in any DM.

the objective function for the buyer is given by

$$J_i^B(\hat{t}) = J_{i,L}^B(\hat{t}) + J_{i,F}^B(\hat{t}), \tag{6}$$

where

$$\begin{aligned} J_{i,L}^B(\hat{t}_{ii}, \hat{t}_{ji}) &= \left[-\psi_i + \beta\left(\hat{\psi}_i + R\right)\right]\hat{t}_{ii} + \left[-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)\right]\hat{t}_{ji} \\ &+ \beta p_H\Big\{u(q_i(\hat{t}_{ii}, \hat{t}_{ji})) - \left(\hat{\psi}_i + R\right)x_{ii}(\hat{t}_{ii}, \hat{t}_{ji}) - \left(\hat{\psi}_j + R_\kappa\right)x_{ji}(\hat{t}_{ii}, \hat{t}_{ji})\Big\}, \qquad (7) \\ J_{i,F}^B(\hat{t}_{ij}, \hat{t}_{jj}) &= \left[-\psi_i + \beta\left(\hat{\psi}_i + R\right)\right]\hat{t}_{ij} + \left[-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)\right]\hat{t}_{jj} \\ &+ \beta p_F\Big\{u(q_j(\hat{t}_{ij}, \hat{t}_{jj})) - \left(\hat{\psi}_i + R\right)x_{ij}(\hat{t}_{ij}, \hat{t}_{jj}) - \left(\hat{\psi}_j + R_\kappa\right)x_{jj}(\hat{t}_{ij}, \hat{t}_{jj})\Big\}. \qquad (8) \end{aligned}$$

In the expressions above, we have defined $p_H \equiv a_B\sigma_H$ and $p_F \equiv a_B\sigma_F$, i.e. p_H is the probability of being an active and matched buyer in the local DM and p_F is the analogous expression for the foreign DM. $J_{i,n}^B$ is the part of the objective that reflects the choice of assets to be traded in DM_n, with $n = H$ for home or $n = F$ for foreign.

Equation (6) highlights that the optimal choice of asset holdings to be traded in the home and foreign DM ($(\hat{t}_{ii}, \hat{t}_{ji})$ and $(\hat{t}_{ij}, \hat{t}_{jj})$, respectively) can be studied in isolation. This leads to a much more tractable optimization problem, and it is the reason for choosing the state space of a buyer to be $t = (t_{ii}, t_{ij}, t_{ji}, t_{jj})$. The term $-\psi_i + \beta\left(\hat{\psi}_i + R\right)$ represents the net gain of carrying one unit of home asset from today's CM into tomorrow's CM. Sometimes we refer to the negative of this term as the cost of carrying the asset across periods. Similarly, $-\psi_j + \beta\left(\hat{\psi}_j + R_\kappa\right)$ is the net gain from carrying one unit of foreign asset across consequtive CM's. In both (7) and (8), the second line represents the expected (discounted) surplus of the buyer in each DM. The following Lemma states an important result regarding the sign of the cost terms that appear in the agents' objective functions.

Lemma 3. *In any equilibrium,* $\psi_k \geq \beta(\hat{\psi}_k + R)$, $k = A, B$.

Proof. A formal proof can be found in Geromichalos, Licari, and Suarez-Lledo (2007). To see the result intuitively, just notice that if $\psi_k < \beta(\hat{\psi}_k + R)$ for some k, agents of country k will have an infinite demand for this asset, and so equilibrium is not well defined. □

According to the Lemma, for any agent from country i, the net gain of carrying home assets across periods is non-positive, and the net gain of carrying foreign asset across periods is strictly negative, due to the term κ. The non-negative sign of the cost terms assigns a very intuitive interpretation to the objective functions of agents: a buyer wishes to bring assets with her in the DM in order to facilitate trade. However, she faces a trade-off, because carrying these assets is not free (equations (7) and (8)). On the other hand, sellers have no benefit from carrying assets into the DM. This is why in (4) only the cost terms are present. We are now ready to discuss the optimal portfolio choices of agents and, consequently, equilibrium.

3 Equilibrium in the Two-Country Model

We start this section with a general definition of equilibrium, and explain why focusing on symmetric, steady-state equilibria can make the analysis more tractable.

Definition 1. An equilibrium for the two-country economy is a list of solutions to the bargaining problems in DM_i, $i = A, B$ described by Lemmas 1 and 2, and bounded paths of ψ_A, ψ_B such that agents maximize their objective functions (described by (4), (7), and (8)) under the market clearing conditions $T_{_A} = \sum_{i=A,B} t^i_{_{Ak}} + \xi \sum_{i=A,B} t^i_{_A}$ and $T_{_B} = \sum_{i=A,B} t^i_{_{Bk}} + \xi \sum_{i=A,B} t^i_{_B}$. The term $t^i_{_{jk}}$ denotes demand of a buyer from country i for asset j to trade in $DM_{_k}$, and the term $t^i_{_j}$ denotes the demand of a seller from country i for asset j.

In the remainder of this paper we focus on steady-state equilibria. Moreover, the two-country environment we consider is completely symmetric in the following sense: $T_{_A} = T_{_B} = T$, $R_{_A} = R_{_B} = R$, and $\sigma_{_H}, \sigma_{_F}$ are the same in both countries. Given the concavity of buyers' objective functions, each buyer has a certain (degenerate) demand for the home asset and a certain (degenerate) demand for the foreign asset, but these demand functions do not depend on the agent's citizenship. In terms of equilibrium objects this implies $t^A_{_{AA}} = t^B_{_{BB}}$, $t^A_{_{AB}} = t^B_{_{BA}}$, $t^A_{_{BA}} = t^B_{_{AB}}$, and $t^A_{_{BB}} = t^B_{_{AA}}$.[8] The demand of sellers need not be degenerate, but given asset prices, we can always choose it conveniently in order to clear the market (see Lemma 4 below for details). Two important implications follow. First, both assets have equal aggregate supply (T) and aggregate demand. Therefore, their equilibrium price has to be equal, $\psi = \psi_A = \psi_B$. Second, by the bargaining protocols, the amount of special good that changes hands in any DM depends only on whether the buyer is a local or a foreigner, but not on the label of the DM. These facts lead to the following definition.

Definition 2. A symmetric steady-state equilibrium for the two-country economy can be summarized by the objects $\{t_{_{HH}}, t_{_{HF}}, t_{_{FH}}, t_{_{FF}}, t^S_{_H}, t^S_{_F}, q_{_H}, q_{_F}, \psi\}$. The term $t_{_{ij}}$ is the equilibrium asset holdings of the representative buyer (of any country) for asset i to be used for trade in DM_j, with $i, j = H$ for home (or local) or $i, j = F$ for foreign. For future reference also define the total home and foreign asset holdings of buyers, $t_{_H} = t_{_{HH}} + t_{_{HF}}$ and $t_{_F} = t_{_{FH}} + t_{_{FF}}$. The term $t^S_{_i}$, $i = H, F$, represents equilibrium asset i holdings for the representative seller, and $q_{_i}$ stands for

[8] For example, $t^A_{_{AA}} = t^B_{_{BB}}$ means that the demand for asset A of a buyer from A in order to trade in her local DM, is equal to the demand of a buyer from B for asset B used for trade in her local decentralized market, DM_B. The remaining equations admit similar interpretations.

the amount of special good that changes hands in any DM, when the buyer is local ($i = H$) or foreign ($i = F$). Finally, ψ is the symmetric, steady-state equilibrium asset price.

We now proceed to a more careful discussion of the optimal portfolio choice of agents and, consequently, equilibrium. Notice that the symmetric, steady-state version of Lemma 3 dictates that $\psi \geq \beta R/(1-\beta) \equiv \psi^*$. The term ψ^* is just the so-called *fundamental* value of the asset, i.e. the unique price that agents would be willing to pay for one unit of this asset if we were to shut down the DM's (in which case the model would coincide with a two-country Lucas-tree model). The optimal portfolio choice of a seller is straightforward.

Lemma 4. *For any given asset price ψ, the optimal choice of asset holdings for a seller are given by $t^S_{F} = 0$ and*

$$t^S_{H} = \begin{cases} 0, \ \textit{if} \ \ \psi > \psi^*, \\ \in \mathbb{R}_+, \ \textit{if} \ \psi = \psi^*. \end{cases}$$

Proof. The result follows directly from inspection of (4) and Lemma 3. □

The seller never wishes to hold a positive amount of the foreign asset, given that the cost of carrying this asset across periods is strictly negative, and sellers have no benefit from bringing assets into the DM. A seller might demand some home assets, if the holding cost is zero. This is true when $\psi = \psi^*$. We now turn to the optimal portfolio choice of the buyer. We will examine the choice of (t_{HH}, t_{FH}) and (t_{HF}, t_{FF}) separately, by looking at the symmetric, steady-state versions of (7) and (8).

Lemma 5. *For any given asset price ψ, a buyer's optimal asset holdings for trade in the local DM satisfy $t_{FH} = 0$ and*

$$\psi = \beta(\psi + R) \left\{1 + p_{H}[u'\left(q_{H}(t_{HH}, 0)\right) - 1]\right\}. \tag{9}$$

Proof. See the Appendix. □

Lemma 5 reveals that buyers never carry any foreign asset in order to trade in the local DM. The intuition behind this result is straightforward. Recall from Lemma 1, that in this type of meeting the bargaining solution depends only on the real asset balances that the buyer brings into the DM. In other words, any combination of assets for which $(\psi+R)t_{HH} + (\psi+R_\kappa)t_{FH} = \pi$, for some given π, buys the same amount of special good. However, the foreign asset has a higher holding cost. Hence, it is always optimal for the buyer to set $t_{FH} = 0$ and purchase any desired quantity using t_{HH} only. The optimal choice of t_{HH} is characterized by the first-order condition in (9). The term $q_H(t_{HH}, 0)$ is the quantity of special good that changes hands in a meeting with a local buyer who carries t_{HH} units of the home and zero units of the foreign asset.

We can characterize the amount of good that changes hands in the DM in more detail. Applying total differentiation in (9) yields

$$\frac{\partial q_H}{\partial \psi} = \frac{R}{\beta p_H (\psi + R)^2 u''(q_H)} < 0 \tag{10}$$

Since q_H is strictly decreasing in ψ, it will reach its highest value when ψ attends its lowest value, i.e. $\psi = \psi^*$. It is straightforward to verify that when $\psi = \psi^*$, $q = q^*$. When this is true, the cost of carrying the asset across periods is zero, and carrying any amount $t_{HH} \geq q^*/(\psi^* + R) = (1-\beta)q^*/R$ is optimal. Intuitively, since the cost of carrying assets is zero, optimality requires that the buyer brings any amount of assets that will buy her q^* in the local DM (because this maximizes the buyer's surplus). By Lemma 1, any $t_{HH} \geq (1-\beta)q^*/R$ does that job.

Next, we consider the optimal choice of assets used for trade in the foreign DM. There are three scenarios (or regimes), that depend on the values of the following parameters: β, p_F, and κ/R. In the first scenario, regardless of asset prices, the agent sets $t_{HF} = 0$ and uses only foreign assets for trade in the foreign DM. In a second scenario, again regardless of asset prices, the agent chooses $t_{FF} = 0$ and trades in the foreign DM with her home asset. Finally, there is a third scenario, under which t_{HF} or t_{FF} could be set equal to zero (except from a knife-edge case), depending on asset prices. The following Lemma describes the details. Figure 1 that follows below it summarizes the parameter values that make up the regions referred to in Lemma 6.

Lemma 6. *Case 1: Assume $(p_F, \beta) \in R_1$ or $(p_F, \beta) \in R_2$ and $\frac{\kappa}{R} \geq \frac{(1-2p_F)}{(1-p_F)(1-\beta)} \equiv \tilde{\kappa}$. Then the buyer's optimal asset holdings satisfy $t_{HF} = 0$ and*

$$\psi = \beta \left[(1-p_F)(\psi + R_\kappa) + p_F(\psi + R)\, u'\left(q_F(0, t_{FF})\right)\right]. \tag{11}$$

Case 2: Assume $(p_F, \beta) \in R_4$. The buyer's optimal asset holdings satisfy $t_{FF} = 0$ and

$$\psi = \beta \left[(1-p_F)(\psi + R) + p_F(\psi + R_\kappa)\, u'\left(q(t_{HF}, 0)\right)\right]. \tag{12}$$

Case 3: Assume $(p_F, \beta) \in R_3$ or $(p_F, \beta) \in R_2$ and $\kappa/R < \tilde{\kappa}$. Also, define $\psi_c \equiv \beta(1-p_F)(2R - \kappa)/[1 - 2\beta(1-p_F)]$. The buyer's optimal asset holdings are as follows: a) If $\psi > \psi_c$, then $t_{HF} = 0$ and t_{FF} satisfies (11). b) If $\psi < \psi_c$, then $t_{FF} = 0$ and t_{HF} satisfies (12). c) In the knife-edge case $\psi = \psi_c$, $t_{HF}, t_{FF} > 0$ and both (11) and (12) hold. The optimal choices t_{HF}, t_{FF} cannot be uniquely pinned down, *but the q_F purchased by the buyer is unique.*

Proof. See the Appendix. □

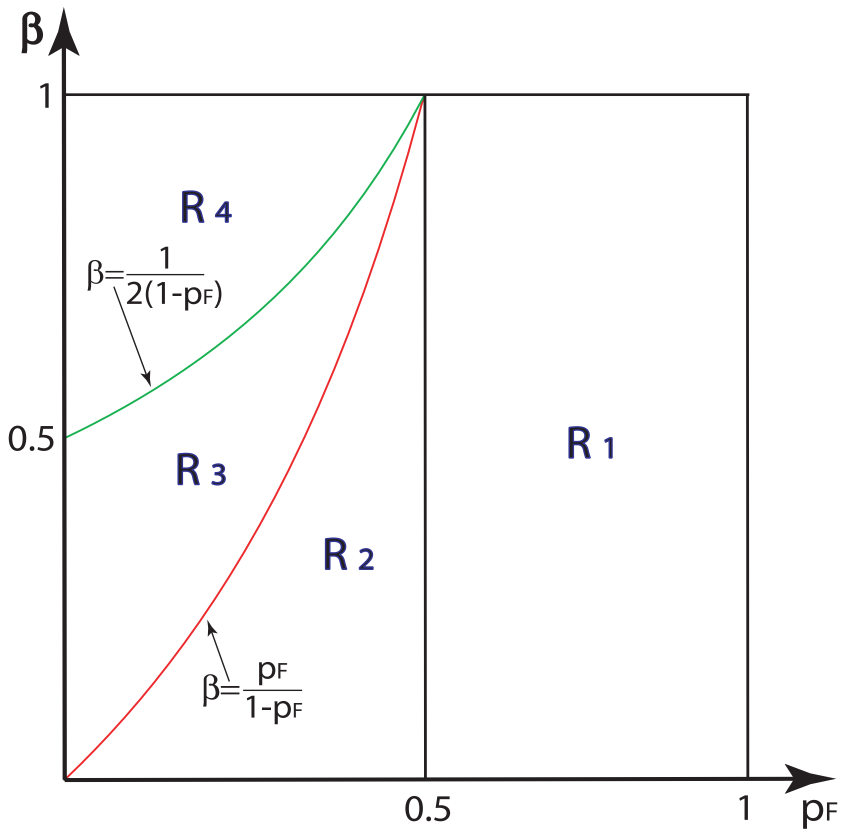

Figure 1: Parameter Values and Regions

Although Lemma 6 might seem complicated at first, it has an intuitive explanation. The liquidation cost κ creates a "wedge" between the asset valuations of a seller and a buyer who come from different countries. The buyer, who makes the interesting decisions here, knows that if she meets with a foreign seller, she will have a bigger purchasing power if she carries the foreign asset (which is home asset to the seller, i.e. t_{FF}). This follows directly from the bargaining solution. On the other hand, meeting the seller is not guaranteed, and if the buyer stays unmatched in the foreign DM, she will be stuck with the "toxic" asset that incurs a liquidation cost. A reverse story applies regarding the optimal choice of t_{HF}. If the buyer carries a big amount of this asset and matches in the foreign DM, she will have a small purchasing power, because for every unit of t_{HF} that she passes to the seller, the latter will suffer a liquidation cost in the forthcoming CM. Not surprisingly, when p_F is big, the buyer decides to carry out trades in the foreign DM using the foreign asset. On the other hand, when it is less likely to match abroad (small p_F), the buyer prefers to carry her home asset, even when trading in the foreign country. In intermediate cases, both regimes can arise, depending on the asset price, which affects the holding costs. We will discuss this in more detail after we have characterized equilibrium.

The relationship between the optimal t_{HF} and t_{FF} with p_F is straightforward. What is perhaps more surprising is the fact that in R_2, the buyer sets $t_{HF} = 0, t_{FF} > 0$ when κ is relatively high. This might seem counterintuitive at first, given that a high κ means high liquidation cost for a buyer who did not get rid of the foreign asset. But one should not forget that a high κ also implies a very small purchasing power for the buyer, if she carries only t_{HF}. These forces have the opposite direction. Whether it is optimal to trade with t_{HF} or t_{FF} depends on the value of β. When κ is high, the buyer realizes that she might suffer a big liquidation cost, but this will happen tomorrow. On the other hand, when the buyer is trying to buy some q and pay with t_{HF}, the seller will have to incur a cost in the current period's CM (tonight). Thus, a high κ actually dictates the use of t_{FF} in foreign meetings, as long as agents are not very patient. As an extreme case, consider points in R_2 that are close to the origin, and suppose κ is very high. Although p_F is tiny, the buyer chooses $t_{HF} = 0, t_{FF} > 0$ because, for the points in question, β is also tiny.

To examine q_F, one needs to consider different parameter values. First, focus on the values described in Case 1, in which $t_{HF} = 0$. Applying total differentiation in (11), one can obtain

$$\frac{\partial q_F}{\partial \psi} = \frac{R - \kappa\beta(1-p_F)}{\beta p_F(\psi + R)^2 u''(q_F)} < 0. \tag{13}$$

Since q_F is strictly decreasing in ψ, it will reach its highest value when the asset price is equal to the fundamental value. It is straightforward to verify that when $\psi = \psi^*$, $q_F = q^*_{F,1} \equiv \{q : u'(q) = 1 + (\kappa/R)(1-\beta)(1-p_F)/p_F\} \leq q^*$.[9] This highlights an interesting difference between the optimal q_L and q_F. As long as $\psi = \psi^*$, which implies that the cost of holding assets is zero, $q_L = q^*$ (see equation (9)). On the other hand, even if $\psi = \psi^*$ $q_F \leq q^*$, and $q = q^*$ only if $p_F = 1$, i.e. only if the buyer matches in the foreign DM with probability one. Another interesting observation is that even if $\psi = \psi^*$, the buyer brings an amount of t_{FF} which is just enough in order to purchase the desired q_F (compare this to our discussion of t_{HH} holdings after Lemma 5). The reason is that the cost of holding foreign asset, $\psi - \beta(\psi + R_\kappa)$, is strictly negative even when $\psi = \psi^*$.

Next, focus on parameter values such as in Case 2. Total differentiation in (12) yields

$$\frac{\partial q_F}{\partial \psi} = \frac{R - \kappa + \beta\kappa(1-p_F)}{(\psi + R_\kappa)^2 u''(q_F)} < 0. \tag{14}$$

Like before, we can find the maximum possible value of q_F by setting $\psi = \psi^*$ in (12). One can

[9] A comment on notation: when we write $q^*_{F,1}$, the asterisk refers to the fact that this is the highest value that q_F can attend and the number 1 refers to the case, in particular Case 1.

show that this value is given by $q^*_{F,2} = \underline{q}(\psi^*) = \{q : u'(q) = R/[R - \kappa(1-\beta)]\} < q^*$.[10] There is a similarity between the optimal q_F here, and the optimal q_H implicitly described in Lemma 5. Since in both cases the buyer is using home asset to carry out trades, when $\psi = \psi^*$, the cost of holding the asset is zero, and she can purchase the highest possible amounts of q allowed by the bargaining protocol, namely q^* and $\underline{q}(\psi^*)$.[11] This is not true in Case 1 above, where the buyer trades using the foreign asset. In general, $q_F \leq q^*$, which is strictly smaller than the highest amount of q admissible by the bargaining protocol, namely $\overline{q}(\psi^*)$.

Case 3 is just a set of economies where the optimal behavior of the buyer could look as in Case 1 or 2, depending on asset prices. If $\psi > \psi_c$, then $t_{HF} = 0$ and we are practically back in Case 1. If $\psi < \psi_c$, the buyer's optimal choice is the one described in Case 2. In the knife-edge case $\psi = \psi_c$, both (11) and (12) hold. One can easily verify that in this case $q_F = q_{F,c} \equiv \{q : u'(q) = (1-p_F)/p_F\}$. The optimal choices of t_{HF}, t_{FF} cannot be uniquely pinned down, since various combinations of asset holdings deliver the same value for the objective function in (8).

So far we have analyzed the optimal behavior of agents for given (symmetric and steady state) asset prices. To complete the description of the model, we need to treat ψ as an equilibrium object. More precisely, we need to incorporate the exogenous supply of the asset in the analysis, and connect asset prices with optimal behavior and market clearing. We do this in the Proposition 1 below. Given the discussion on agents' optimal behavior, most of the results in the proposition follow naturally, but a more formal treatment can be found in the Appendix. One issue that has to be handled is the following: when T is large, in a way that will be made precise below, $\psi = \psi^*$ and the cost of carrying assets is zero. Hence, sellers are willing to hold any amount of assets, and buyers are willing to hold any amount of assets in excess of the amount that buys them the desired quantity of special good during DM trade. There are many, equivalent, ways to break this indeterminacy. We assume that it is buyers who absorb the excess supply.[12] Hence, in all equilibria $t^S_H = 0$ (by assumption) and $t^S_F = 0$ (by Lemma 4).

For the reader's convenience, before stating the proposition we repeat the definitions of

[10] A similar comment on notation as in footnote 9 applies here.

[11] The claim regarding q^* is obvious. To see why the claim regarding $\underline{q}$ is true, just re-write the bargaining solution in DM_i (Lemma 2) setting $t_{ii} = 0$.

[12] This result could arise endogenously, if we imposed an $\epsilon > 0$ cost of participating in the asset markets. Since sellers get a zero payoff by holding any amount of assets in $\mathbb{R}_+$, they would choose to hold zero even for a tiny ϵ. On the other hand, buyers want to hold strictly positive amounts of assets in order to trade in the DM. Since the ϵ cost is sunk, if $\psi = \psi^*$ they would be happy to hold any amount of assets that exceeds the amount used for trade.

some objects already introduced above, and we also define a few new objects.

$$
\begin{aligned}
q^*_{F,1} &\equiv \Big\{q : u'(q) = 1 + \frac{\kappa}{R}\frac{(1-\beta)(1-p_F)}{p_F}\Big\}, \\
q^*_{F,2} &\equiv \underline{q}(\psi^*) = \Big\{q : u'(q) = \frac{R}{R-(1-\beta)\kappa}\Big\}, \\
q_{H,c} &\equiv \Big\{q : u'(q) = \frac{R(1-2p_F) - \kappa(1-\beta)(1-p_F)}{p_H[R-\beta\kappa(1-p_F)]}\Big\} \\
q_{F,c} &\equiv \Big\{q : u'(q) = \frac{1-p_F}{p_F}\Big\} \\
T^*_1 &\equiv \frac{(1-\beta)(q^* + q^*_{F,1})}{R}, \\
T^*_2 &\equiv (1-\beta)\left(\frac{q^*}{R} + \frac{\underline{q}(\psi^*)}{R-\kappa(1-\beta)}\right) \\
T_c &\equiv \frac{1-2\beta(1-p_F)}{R-\kappa\beta(1-p_F)}(q_{H,c} + q_{F,c}) < T^*_2, \\
\psi_c &\equiv \frac{\beta(1-p_F)(2R-\kappa)}{1-2\beta(1-p_F)}. \\
\tilde{\kappa} &\equiv \frac{(1-2p_F)}{(1-p_F)(1-\beta)}
\end{aligned}
$$

Proposition 1. *For any parameter values there exist a unique steady-state equilibrium with $t_{FH} = 0$. Moreover:*

a) If $(p_F, \beta) \in R_1$ or $(p_F, \beta) \in R_2$ and $\kappa/R \geq \tilde{\kappa}$, equilibrium is characterized by "local currency dominance", and agents trade in the foreign DM using only foreign assets, i.e. $t_{HF} = 0$. If $T \geq T^*_1$, then $\psi = \psi^*$, $q_H = q^*$, $q_F = q^*_{F,1} < q^*$. If $T < T^*_1$, then $\psi > \psi^*$, $q_H < q^*$, $q_F < q^*_{F,1}$ and for any T in this region $\partial\psi/\partial T < 0, \partial q_H/\partial T > 0, \partial q_F \partial T > 0$. Finally, if $T \geq T^*_1$, $t_H = T - T^*_1$.

b) If $(\beta, p_F) \in R_4$, an "international currencies" equilibrium arises, in the sense that that agents use their home assets in all DM's, i.e. $t_{FF} = 0$, and so $t_F = 0$. If $T \geq T^*_2$, then $\psi = \psi^*$, $q_H = q^*$, and $q_F = \underline{q}(\psi^*) < q^*$. If $T < T^*_2$, then $\psi > \psi^*$, $q_H < q^*$, and $q_F < \underline{q}(\psi^*)$. For all T in this region $\partial\psi/\partial T < 0, \partial q_H/\partial T > 0, \partial q_F \partial T > 0$.

c) If $(\beta, p_F) \in R_3$, or $(\beta, p_F) \in R_2$ and $\kappa/R < \tilde{\kappa}$ a "mixed regime" equilibrium arises, in the sense that both local currency dominance or international currencies could arise depending on T. If $T > T_c$, we are in the international currencies regime. For T's in this region, $\psi \in [\psi^*, \psi_c)$, $q_H \in (q_{H,c}, q^*]$, and $q_F \in (q_{F,c}, q^*_{F,2}]$. If $T < T_c$, we switch to a local currency dominance equilibrium, and $\psi > \psi_c$, $q_H < q_{H,c}$, $q_F < q_{F,c}$. In the knife-edge case $T = T_c$, buyers purchase $q_F = q_{F,c}$ in the foreign DM using any combination of home and foreign assets.

Proof. See the Appendix. □

A unique steady state equilibrium exists for all parameter values. Buyers never carry foreign assets in order to trade in their home DM. To say more one needs to look at different cases of parameter values. Under parameter values summarized as Case 1 in Lemma 6, an equilibrium with local currency dominance arises. Buyers choose to trade in the foreign DM using only foreign asset, because the positive effect (big purchasing power) of holding foreign assets dominates the negative (liquidation cost). There exists a critical level T_1^*, that captures the *liquidity needs* of the economy. For $T < T_1^*$, increasing T helps buyers purchase more special good in both DM's. Hence, the marginal valuation of one unit of the claim is bigger than ψ^*, i.e. the price of the asset in a world where the Lucas-tree serves only as a store of value. Sometimes, we refer to the distance $\psi - \psi^*$ as the *liquidity premium*, because it reflects a premium in the valuation of the asset that stems from its second role (as a medium of exchange). When $T \geq T_1^*$, the asset's liquidity properties have been exploited (increasing T does not help buyers purchase more good), and $\psi = \psi^*$. When this is true, the cost of carrying home asset is zero, so buyers absorb the excess supply $T - T_1^*$.

A similar analysis applies when parameter values are as in Case 2 in Lemma 6. We refer to this case as an international currencies equilibrium, because the home asset is used everywhere in the world as means of payment. The asset supply that captures the liquidity needs of the economy is given by T_2^*. When $T \geq T_2^*$, the asset price is down to its fundamental value, and $q_{_H}, q_{_F}$ have reached their upper bounds. Notice that in this case, buyers do not purchase any foreign assets in the CM, i.e. $t_{_F} = 0$. However, this does not mean that no agent ever holds any foreign assets. Sellers from country i, who got matched with foreign buyers, get payed with, and therefore hold some, asset j (which is home asset to the buyers). This will be important in the next section, when we will calculate asset home bias.

When parameter values are as in Case 3 in Lemma 6, we can end up with local currency dominance or international currencies equilibrium, depending on T, which directly affects ψ. Equilibria with local currency dominance arise if and only if $T > T_c$. The intuition behind this result is as follows: when T is big, equilibrium prices are relatively low, which means that the cost of carrying the asset is relatively low. When the cost of carrying assets is low, the term κ becomes relatively more important. Recall from the discussion of Lemma 6, that it is precisely when κ is relatively big that agents choose to use the foreign asset in order to trade in the foreign DM (this becomes obvious when $(\beta, p_{_F}) \in R_2$). On the other hand, if T is very small, the cost of carrying assets is large, and so the term κ becomes less relevant. For intermediate values of $(\beta, p_{_F})$ (remember we are in Case 3), this leads the buyers to optimally choose their home asset to trade in all DM's.

We conclude this section by comparing the equilibrium values of $q_{_H}, q_{_F}$ and $t_{_H}, t_{_F}$, which will lead us to our discussion of asset and consumption home bias in the next section. Suppose first that we are in a local currency dominance regime.[13] For given ψ (which can be explicitly

[13] Hence, one of the following is true: i) $(p_{_F}, \beta) \in R_1$, or ii) $(p_{_F}, \beta) \in R_2$ and $\kappa/R \geq \tilde{\kappa}$, or iii) $T > T_c$ and either $(p_{_F}, \beta) \in R_3$ or $(p_{_F}, \beta) \in R_2$ and $\kappa/R < \tilde{\kappa}$.

determined if one knows T), the first-order conditions (9) and (11) implicitly define q_H, q_F as functions of the probabilities p_H, p_F. We have

$$q_H = G_H(p_H) \equiv \Big\{q : 1 + \frac{\psi - \beta(\psi + R)}{\beta p_H(\psi + R)}\Big\}, \quad (15)$$

$$q_F = G_F(p_F) \equiv \Big\{q : \frac{\psi - \beta(\psi + R_\kappa)(1 - p_F)}{\beta p_F(\psi + R)}\Big\}. \quad (16)$$

The following lemma summarizes some interesting properties of these functions.

Lemma 7. *a) For $p_H = p_F = 0$, $G_H(p) = G_F(p) = 0$. For every $p_H = p_F = p \in (0, 1]$, $G_H(p) > G_F(p)$, and the distance $G_H(p) - G_F(p)$ is an increasing function of κ.*

b) For any $p_F \in (0, 1]$ define $\tilde{p}_H(p_F) \equiv \{p \in (0, 1] : G_H(p) = G_F(p_F)\}$, and notice that $\tilde{p}_H(p_F) < p_F$. Then, for any $p_F \in (0, 1]$, $p_H > \tilde{p}_H(p_F)$ implies $q_H > q_F$.

c) For any $p_F \in (0, 1]$, $p_H > \tilde{p}_H(p_F)$ implies $t_H > t_F$.

Proof. See the Appendix. □

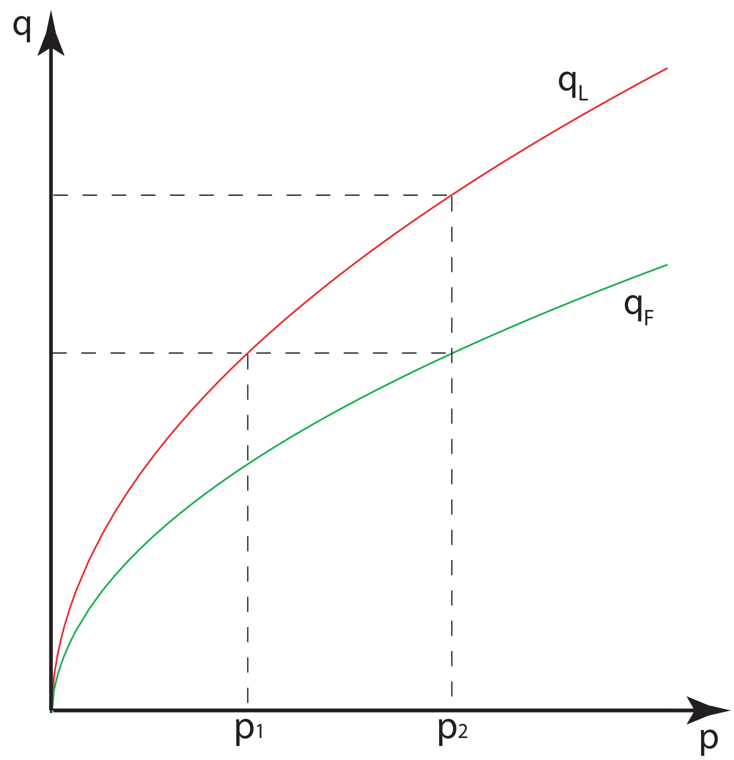

Figure 2: Trading Opportunities and Quantities Traded

The basic results of Lemma 7 are depicted in Figure 2. For any given $p_H = p_F = p > 0$, G_H is above G_F, and the distance between the two increases as the wedge induced by κ increases. Hence, for any given p_F, one can always find a critical $\tilde{p}_H(p_F)$, such that if $p_H > \tilde{p}_H(p_F)$, then

$q_H > q_F$. Of course, in the local currency dominance regime, $t_H = t_{HH}$ and $t_F = t_{FF}$. Also, from bargaining, $q_H = \min\{(\psi + R)t_{HH}, q^*\}$ and $q_F = (\psi + R)t_{FF}$. Hence, if $p_H > \tilde{p}_H(p_F)$, not only $q_H > q_F$, but also $t_H > t_F$. In words, agents buy greater amounts of home versus foreign assets, even when the probability of trading in the local DM is smaller (but not too much) than the probability of trading in the foreign DM. Notice that we do not claim that $t_H > t_F$ is equivalent to home asset bias (although it will turn out that this is a sufficient condition). In fact, we have not even defined home asset and consumption bias yet. This is the main subject of the next section.

4 Home Bias in Assets and Goods

In this section we explore three predictions of our model that are supported in the data. These include the existence of asset home bias, the positive correlation between asset and consumption home bias, and the higher turnover rates of foreign over home assets. We characterize the properties of the equilibrium corresponding to Case 1 above only, namely the currency area equilibrium. We find this equilibrium most interesting because it is most empirically relevant. We describe the properties of the remaining equilibria in the Appendix.

4.1 Asset Home Bias

In this section, we argue that the model's predicted asset portfolio is biased toward domestic assets. The result is summarized in Proposition 2.

Proposition 2. *For any T and for any $p_F \in (0, 1]$, let $p_H > \tilde{p}_H(p_F)$. Then agents' portfolios exhibit home bias, in the sense that the home asset's share in the entire portfolio is greater than 50 percent. Formally,*

$$\frac{2t_H + p_F t_F}{2(t_H + t_F)} > 0.5 \tag{17}$$

Proof. If for a given p_F, $p_H > \tilde{p}_H(p_F)$, then, by Lemma 7, $t_H > t_F$ and the inequality in (17) follows immediately. □

Proposition 2 states that as long as the trading opportunities at home are not significantly less than the trading opportunities abroad, a reasonable assumption for most countries engaged in international trade, the countries' portfolios will exhibit home bias. The proof of the proposition follows immediately from Lemma 7. The more interesting part of the proposition is to understand why the left-hand side of (17) represents the home asset's share in the agents' portfolio. To fix ideas, suppose we are focusing on country i. The term $t_H + t_F$, in the denominator, stands for the total asset holdings. It is multiplied by 2, because we count asset holdings in both the DM and CM (we assume an equal weight). The numerator represents the home asset holdings. The term $p_F t_F$ is the amount of asset i held by sellers who matched in the DM with foreign buyers who were carrying t_F units of asset i, which to them is foreign (recall that we are focusing on local currency dominance). For more details see the Accounting Appendix.

Home bias remains an empirical regularity in cross-country data. We document this fact using cross-country data on international asset and liability positions provided by Lane and Milesi-Ferretti (2007). During the years 1997-2007, the average domestic asset share among 78 countries is 86 percent. This artifact is not due to the large number of developing economies that comprise the dataset. In fact, the domestic share is as much as 74 percent for the average OECD country throughout the period.

In the Appendix, we summarize the average domestic asset share in each country over the past decade. To measure asset home bias, we employ a methodology similar to Collard, Dellas, Diba, and Stockman (2009). First, we compute international diversification as follows:[14]

$$\text{Int'l Dvrsf.} = \frac{\text{Foreign Portfolio Equity Assets}}{\text{Stock Mkt Cap + Foreign Portfolio Equity Assets - Foreign Portfolio Equity Liab.}}$$

Then, we capture asset home bias through the home asset share:

$$\text{HA} = 1 - \text{Int'l Dvrsf.} \tag{18}$$

With this definition in mind, the countries in our sample exhibit asset home bias, since the average home asset share (HA) is well in excess of the 50-percent benchmark that characterizes a fully-diversified portfolio in our model.

4.2 Consumption and Asset Home Bias

In this model, countries exhibit consumption home bias. Moreover, consumption and asset home bias go hand-in-hand. Proposition 3 states the result.

Proposition 3. *Define C_F, C_T, and C_H as the value of foreign (or imported) consumption, total consumption, and consumption produced at home, respectively. For any T and for any $p_F \in (0,1]$, let $p_H > \tilde{p}_H(p_F)$. Then $C_H > C_F$, implying $\frac{C_H}{C_T} > 0.5$.*

[14] Stock market capitalization data are from WDI.

Proof. See the Appendix. □

To understand Proposition 3, notice that for each agent in the model, imported consumption consists of special goods bought from foreign sellers as well as fruit obtained from abroad, when net claims to foreign trees are positive. On the contrary, domestic consumption includes special goods bought in domestic DM meetings as well as fruit obtained not only when net claims to the domestic tree are positive, but also via work. From Proposition 2, as long as trading opportunities abroad are not much higher than at home, the economy exhibits home bias in asset holdings. Since, at the country level, more domestic assets are held relative to foreign ones, net aggregate claims to foreign trees fall short of net claims to domestic trees. With respect to the DM, Lemma 7 ensures that the quantity exchanged (and consumed) in a domestic meeting exceeds the quantity exchanged in a foreign meeting. Since expenditure is increasing in the quantity purchased, the value of domestic consumption exceeds the value of imported consumption in the DM. Thus, for any non-negative work effort, domestic consumption exceeds imported consumption, ensuring that domestic goods' share of total consumption is well in excess of 50 percent.

The above discussion demonstrates that consumption and asset home bias are interrelated in the model. In particular, the correlation between the two biases arises not only due to the fact that domestic assets give agents higher claims to domestic consumption in Walrasian markets, but also because agents have higher purchasing power in domestic bilateral meetings. This means that our model potentially relates these two variables not only at at the aggregate, but also at the bilateral level.

To test this prediction, we obtain bilateral equity portfolio investment data for the year 2007 from the Coordinated Portfolio Investment Survey (CPIS) database provided by the World Bank. The advantage of this database is that it provides bilateral asset holdings for as many as 59 countries. However, among these countries, a number of positive bilateral observations are not made publicly available for a variety of reasons. Thus, the database cannot be used to compute domestic asset shares. After dropping missing observations, we are left with 2473 observations on bilateral asset holdings. Using stock market capitalization data for 2007, we compute bilateral investment shares.

We merge these data with bilateral goods import data for the same year, provided by Feenstra, Lipsey, Deng, Ma, and Mo (2005). We compute bilateral import shares using GDP data for 2007 from the WDI. The correlation between the bilateral equity investment and goods import shares is 0.2135 and it is highly statistically significant. Moreover, a simple OLS regression of asset investment shares on good import shares (and a constant term) yields a coefficient of 0.6312, with standard error of 0.0581. These findings provide strong evidence that consumption and asset home bias are positively related not only at the aggregate, but also at the bilateral level, much like predicted by the model.

4.3 Domestic and Foreign Asset Turnover Rates

The two predictions discussed above are in line with the existing literature (see Heathcote and Perri (2007) and Collard, Dellas, Diba, and Stockman (2009)). However, the additional prediction that relates domestic and foreign asset turnover rates discussed below is unique to the present model.

Proposition 4. *Define the turnover rates of home and foreign asset as*

$$\begin{aligned} TR_{H} &= \frac{2(p_{H}t + p_{F}t_{F})}{2t_{H} + p_{F}t_{F}}, \\ TR_{F} &= \frac{2p_{F}}{2 - p_{F}}. \end{aligned}$$

There exists a level of asset supply $\tilde{T} < \infty$, such that $T \geq \tilde{T}$ implies $TR_{F} > TR_{H}$.

Proof. See the Appendix. □

Since TR_{F} is a constant, it is unaffected by the asset supply. On the other hand, TR_{H} is decreasing in t_{H}, and since for $T > T_1^*$, $t_{L} = T - T_1^*$, it is also decreasing in T. It can be verified that one can always find a level of T large enough to ensure that $TR_{F} > TR_{H}$. Intuitively, when $T > T_1^*$, we have $q_{H} = q^*$, and increasing T further has no effect on the quantity of special good bought. Therefore, in this range, a higher T increases the home asset holdings of agents, but has small or zero effect on the amount of these assets that changes hands during a period, leading to a relatively small turnover ratio for the home asset.

An alternative, but equally intuitive interpretation is the following. Given local currency dominance, claims to the home Lucas-tree can serve both as store of value and as a medium of exchange. However, claims to the foreign tree are only valued for their services as media of exchange: agents do not hold these assets as a store of value (their rate of return is negative), but they do hold them in order to facilitate trade in the foreign DM. Therefore, when $T > T_1^*$, the cost of carrying home asset is zero, and agents keep the home asset as a store of value, which, reduces its turnover rate. On the other hand, agents unload foreign assets (to foreign sellers) at the first given opportunity, which leads to a relatively higher turnover rate.

Empirically, Amadi and Bergin (2008) document that turnover rates of foreign assets are twice as high as those of home assets for four countries over a large period of time. The coexistence of high turnover rates of foreign assets and bias toward domestic assets in countries' portfolios has been a long-standing puzzle in the international finance literature. As the results in this section establish, the liquidity mechanism, coupled with the assumption of costly foreign asset trade, can reconcile these observations.

5 Conclusion

In this paper, we study optimal asset portfolio choices in a two-country model of costly international trade. We allow assets to not only represent claims on future consumption, but to also serve as media of exchange. In the model, trading in a certain country involves the exchange of locally produced goods for a portfolio of assets. Assuming foreign assets trade at a cost, we provide sufficient conditions for the existence of currency-area and international-currencies equilibria.

We further argue that the novel liquidity mechanism can help rationalize the existence of asset home bias, which is a well-known empirical regularity in the data. According to the model, more frequent trading opportunities at home result in agents holding proportionately more domestic over foreign assets. As international trade becomes more integrated, agents demand higher amounts of foreign assets. Moreover, foreign assets turn over faster than home assets because the former have desirable liquidity properties, but unfavorable returns over time. Thus, our mechanism offers an answer to a long-standing puzzle in international finance: a positive relationship between consumption and asset home bias, coupled with higher turnover rates of foreign over domestic assets.

The model we propose abstracts away from risk diversification considerations and rather explores liquidity properties of assets. As such, it is complementary to the large literature on home bias and can be incorporated into existing frameworks in order to study asset and consumption home bias both qualitatively and quantitatively.

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A Theory Appendix

To be completed.