Optimal Unemployment Insurance with Endogenous UI Eligibility*

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Abstract

Many studies on optimal unemployment insurance (UI) assume universal benefits to the unemployed. This paper introduces UI eligibilitity rule, in a stylized way, into the framework by Hopenhayn and Nicolini (2009). The main contribution is to demonstrate that the consideration of the UI eligibility rule provides an additional incentive device to induce workers to work, and therefore, changes the nature of the optimal UI contract. Particularly, the presence of the UI eligibility rule generates entitlement effects, which mitigates moral hazard quits by eligible workers, and thus, helps fix the loophole highlighted in Hopenhayn and Nicolini (2009). Moreover, we find that strategic quits still show up in the optimal UI contract when monitoring on quits is weak and disutility of work is large. When this moral hazard behavior is taken into account by the UI agency, the consideration of the UI eligibility rule generates a *differentiated* UI contribution fee scheme for employed workers. Particularly, the optimal UI contract, which aims at implementing positive search efforts, promoting a valuable UI eligibility and eliminating moral hazard behavior, is featured with high and increasing UI contribution fees in the employment spell for the ineligible workers, while low and decreasing fees for the eligible ones. The differentiated UI fee scheme is missing in the existing literature, but is in line with the empirical evidence.

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1 Introduction

Many studies on optimal unemployment insurance (UI) contract assume that unemployed workers receive UI benefits unconditionally (see Shavell and Weiss, 1979; Hansen and Imrohoroglu, 1992; Pallage and Zimmerman, 1998; Davidson and Woodbury, 1998; Hopenhayn and Nicolini, 1997, 2009). However, in most existing UI programs in the world, the UI benefits are available to workers who are entitled to UI, and UI eligibility has to be earned with previous, but not too distant, employment. For example, in the United States, workers have to work for about 20 weeks to be entitled to the UI benefits. And the benefits do not last forever. Unemployed workers, on average, run out of the (regular) benefits in 26 weeks.[1] In this paper, we introduce the realistic UI eligibility rules, in a stylized way, into the framework by Hopenhayn and Nicolini (2009) in which workers have to earn the UI entitlement through employment and they might lose it if they leave their jobs voluntarily or decline job offers. The main contribution is to demonstrate that the consideration of the UI eligibility rule provides an additional incentive device to induce workers to work, and therefore, changes the nature of the optimal UI contract greatly.

We believe that introducing UI eligibility rule is *important* for at least three reasons. *Firstly*, it is a realistic feature of the UI systems in most countries, but receive little attention in the literature as mentioned above. *Secondly*, and more importantly, if the UI eligibility has to be earned through work and the duration of the benefit is finite, the incentives of workers change significantly, and terms of the optimal UI contract may change in nature. Suppose the current UI system in the United States is applied to the economy analyzed in Hopenhayn and Nicolini (2009). Then for workers who are not entitled to UI, the presence of the benefit would encourage them to be engaged in market activities in a hope of gaining the UI entitlement, which is the entitlement effect of the UI pointed out by Mortensen (1977)[2]. On the other hand, when the UI agency cannot perfectly monitor workers' behavior in job rejections and quits, generosity of UI induces the workers entitled to UI to quit from existing jobs or to reject job offers to collect benefits, which is the moral hazard effect of the UI stressed in Hopenhayn and Nicolini (2009) and Zhang and Faig (2012). The entitlement effect can mitigate the moral hazard effect if the UI agency can punish the workers who do so by taking away their UI entitlement with a positive probability. The reason is that the concern of losing the valuable UI entitlement lessens the opportunistic behavior. Obviously, the entitlement

[1] Most OECD countries have similar rules. For example, workers need to work for at least 420 hours in the past 52 weeks to earn UI entitlement in Canada, and the counterpart numbers in France and Germany are 6 and 12 months, respectively. As for the duration of benefits, Canadian, French and German unemployed workers are entitled to benefits for up to 11, 42, and 18 months, respectively.

[2] Also see Burdett (1979), Hamermesh (1979).

effect and its interaction with the moral hazard effect is missing from the model if the UI benefits are universally available as assumed in the standard literature. Thus, the corresponding results on the optimal UI contract would be highly misleading.

Thirdly, the UI entitlement proves empirically crucial to understand the impact of the UI on labor supply. Several analyses based on micro data reveal that workers tailor their labor supply decisions to the qualification and disqualification requirements of UI. For example, Card and Riddell (1996) and Christofides and McKenna (1996) both find that many job terminations happen when a worker approaches the duration that permits a UI eligibility. Andolfatto and Gomme (1996) shows that the 1972 liberalization of UI in Canada increases the labor market turnover greatly. Workers at the marginal jobs are more willing to leave jobs to collect UI and then quickly find another job after the exhaustion of benefits. Katz and Meyer (1990) reports that a sharp increase in the escape rate from unemployment is observed among UI recipients when the benefits are likely to expire.[3] The importance of the UI eligibility emphasized above motivates us to take this institutional feature seriously to further our understanding of the optimal UI programme.

To balance the trade-off between realism and tractability, we model the moral hazard effect of the UI by assuming imperfect monitoring by the UI agency; i.e., the UI agency monitors only a fraction of the UI claimants.[4] Therefore, workers who are not qualified (who quit their jobs or reject job offers) can collect benefit with a positive probability.[5] Thus, the complementary probability measures the monitoring power by the UI agency. We refer to imperfect monitoring on the job refusals and quits as moral hazard because this strategic behavior would have not taken place if the UI agency were able to monitor all UI claimants perfectly. In this contribution, both the UI agency and workers take the UI eligibility rule as given. An optimal UI contract that minimizes costs of promising a certain level of expected lifetime utilities to workers is solved along the dimension of the net transfer between the UI agency and workers: the profile of UI benefits for unemployed workers, and the profile of UI contribution fees for eligible and ineligible employed workers.

The consideration of UI eligibility is indeed important in the analysis. The nature of the optimal UI contract *differs* significantly from the one derived in Hopenhayn and Nicolini (2009). First of all, strategic quits highlighted in their paper do not necessarily take place in the presence of the endogenous UI eligibility. Hopenhayn and Nicolini find that with imperfect monitoring on quits by the UI agency, the optimal UI contract has a loophole in the sense that unemployed workers may improve their UI benefits by getting employed and quitting immediately. They show that this loophole exists as long as disutility of work is large. With the UI eligibility rule, we find that even with a large value of disutility of work, the loophole disappears when the monitoring power of the UI agency is not sufficiently weak. The rational for this finding is twofold. Firstly, when the UI

[3] Also see Ham and Rea (1987) and Andolfatto and Gomme (1996).

[4] Equalevantly, we can assume that some workers who quit to successfully pretend to have lost their jobs voluntarily as in Zhang and Faig (2012).

[5] These two probabilities could be different, and the main qualitative results will not be affected.

agency promotes the UI entitlement, the consideration of the UI eligibility rule generates entitlement effect and workers value UI. Secondly, the UI agency in our model can *punish* the workers who quit by taking away their UI entitlement with some positive probability. When the likelihood of punishment is large, the concern with losing the valuable UI deters the worker from behaving strategically. Apparently, this mechanism is absent when the UI benefits can be collected unconditionally. Of particular note is that in our model the introduction of the UI eligibility rule provides an important incentive device to rule out the moral hazard quits. As a result, the loophole is fixed with the help of the entitlement effect, rather than offering decreasing UI fees to the employed as used in the Hopenhayn and Nicolini (2009)[6].

However, the entitlement effect cannot rule out the moral hazard effect perfectly. Consistent with the findings in literature, the optimal UI contract derived in this work has to deal with the moral hazard problem. Our model shows that if the UI agency ignores the moral hazard effect of the UI, the loophole—eligible unemployed workers find a job and quit immediately to upgrade their benefits—shows up in the optimal contract when the following two conditions are satisfied simultaneously: the effort at work is high (high disutility); and the likelihood of collecting benefits upon quits is large (weak monitoring, or weak punishment). The intuition is straightforward. When it is unlikely to be caught from a job quit by the UI agency, or the punishment is not likely to be imposed on the workers who quit, they are not too much concerned with losing the UI entitlement. Therefore, the entitlement effect is too small to dominate the moral hazard effect of the UI.

Secondly, as a novel result, we find that when the UI agency takes into account the moral hazard problem, the optimal contract is featured with a *differentiated* UI fee scheme. Particularly, the model predicts that if the constraint binds that ensures a UI-ineligible worker to accept the UI eligibility upon gaining it, the optimal contract requires such a worker to pay a *high* and *increasing* UI contribution fee over the spell of employment. In sharp contrast, a UI-eligible employed worker pays a *low* and *decreasing* UI contribution fee if the constraint binds that induces him or her to stay in the current employment. The differentiated UI fee result contrasts with the findings in Hopenhayn and Nicolini (2009), where the profile of the UI fee is monotonically decreasing in the employment spell. The difference in the level of the UI contribution fees is resulted from the fact that we incorporate the UI eligibility rule which generate the entitlement effect of the UI; the high fee paid by the ineligible workers can be regarded as a fair price to buy the valuable UI entitlement to be obtained in the future, while the low fees paid by the eligible workers can be regarded as a reward used to keep this type of workers from leaving their jobs voluntarily. The increasing profile of the UI fees paid by the UI-ineligibles is driven by the UI agency's desire to value the UI eligibility in each period; to fulfill this objective, the UI agency punishes workers who renounce the UI eligibility upon earning

[6]Hopenhayn and Nicolini (2009) resolves the loophole by imposing a (No-Quit) constraint, which leads to a decreasing UI fee over the spell of employment.

it by charging them with higher fees.

More importantly, the prediction of the gap in the UI fee is in line with the empirical evidence, although it is novel in the literature on optimal UI. Note that the optimal UI fees are part of the solution to the optimal UI contract that minimizes the cost of granting a utility to a worker in an incentive compatible way. This, together with the assumption of constant wages paid by a passive firm, suggests that the after-tax wage—difference between the wage and the UI fee—is essentially a reservation wage for an unemployed worker in the sense that it is the minimal wage that attracts the worker to accept a job offer. Given this, the main theoretical insight of the differentiated UI fees is that the UI-ineligible unemployed workers should receive lower after-tax reservation wage compared to the UI-eligible ones, which is consistent with the empirical evidence. For example, Fishe (1982), by using the Continuous Wage and Benefit History (CWBH) for the state of Florida for the years 1971 to 1974, estimates the effect on reservation wages of the duration of unemployment. He finds that the reservation wage decreases, on average, 15 percent when UI benefits are exhausted. More recently, DellaVigna and Paserman (2005) compare self-reported reservation wages between workers who are receiving benefits and not, and find receiving benefits raises the reservation wage by 4.7 percent.[7]

It is worth stressing that the differentiated UI fee result holds in various extensions of the model. For instance, when the wage is determined endogenously, the result remains qualitatively unchanged. In this setup, the firm would respond to the UI-ineligible workers' desire for the UI entitlement by offering lower wages. However, as long as the UI contract is designed in a way such that the UI eligibility is strictly valued by the workers, the entitlement effect would exist, and ultimately, lead to a gap in the after-tax wage between the ineligible and eligible workers. Likewise, when we endogenize the UI eligibility rule and treat the policy parameters governing how to earn and lose UI part of the optimal contract, the result remains robust as long as the entitlement effect is present in the model.[8] Lastly, when we generalize the analysis to the case where both bad and good matches coexist as in Hopenhayn and Nicolini (2009), the different level of UI fees between eligible and ineligible workers still exists. To see this, Hopenhayn and Nicolini assume that a bad match that cannot be distinguished from a good match involves higher disutility of work, but comes along easier. In this case, if the disutility of work for the bad match is not sufficiently large, then UI-ineligible workers might take advantage of the easy access to the bad match to earn their UI eligibility. In this setup, the eligible/ineligible workers would pay the same UI fees no matter which type of match they take. However, the different level of UI fees between the eligible and ineligible workers is unchanged since the ineligible workers still value the UI eligibility. It is worth noting that the presence of the high UI fees by the UI-ineligibles serves as an additional punishment for taking the bad match, which helps keep the workers from taking it.

Thirdly, our finding reveals that ineligible workers voluntarily stay employed when

[7]See Table E1 in the appendix of DellaVigna and Paserman (2005).

[8]Admittedly, endogenizing the UI eligibility matters quantitatively.

the UI eligibility is strictly valuable to them. Here, the valuable UI eligibility essentially serves as an alternative *device* to rule out quits. The key insight for this finding is that keeping current jobs will be rewarded by a positive possibility of gaining the UI eligibility in the future, which is highly valued by the ineligible workers. This result also reflects the entitlement effect, and it squares well with the empirical findings that the strategic quits will not happen unless the ineligible workers work long enough to earn UI entitlement as documented in Card and Riddell (1996) and Christofides and McKenna (1996).

Apart from these dissimilarities, the introduction of the UI eligibility nicely preserves many features similar to those found in the existing studies. For example, the UI benefits to eligible unemployed workers are decreasing over the spell of unemployment, and the replacement ratio is smaller than one; the UI fees charged to eligible employed workers are decreasing over the spell of employment. The intuition is that with risk-aversion, workers value consumption smoothing, so a permanent consumption reward is required to make the employment status attractive for the eligible workers.

The model developed in this paper is related, in various ways, to diverse studies. Andolfatto and Gomme (1996) and Brown and Ferrall (2003) incorporate the UI eligibility rule in their analyses, however, both papers explore quantitative impacts of changes in the UI generosity on labor market. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), and Wang and Williamson (1996) focus on the moral hazard problem based on unobservability of job search effort as opposed to job quits and rejections in our model. Similar to our paper, Hansen and Imrohoroglu (1992) and Atkeson and Lucas (1995) study unobserved job refusals. But they assume universal UI benefits and emphasize quantitative effect of UI on economic welfare. Davidson and Woodbury (1997) and Fredriksson and Holmlund (2001) study the optimal UI in a search and matching framework that allows for endogenous wage determination, while our paper, staying within the dynamic principal-agent setup of Hopenhayn and Nicolini (2009), is a partial equilibrium model with assumed constant wages. In terms of the prediction of differentiated UI fees, the paper most related to this one is Zhang and Faig (2012). However, that work considers risk neutral workers and focuses on the labor market impact of UI.

The paper proceeds as follows. Section 2 lays out the cost minimizing problem for the UI agency when it is interested in implementing positive search efforts, eliminating moral hazard behavior, and promoting the valuability of the UI eligibility. A solution to this problem is discussed. Section 3 examines loophole in the optimal UI contract when the UI agency does not keep the moral hazard effect of UI in mind. Particularly, we derive conditions under which a loophole shows up in the optimal UI contract. A comparison is made between our results with those in Hopenhayn and Nicolini (2009) to illustrate the importance of the UI eligibility. Section 4 conducts a welfare analysis to numerically show that it is indeed optimal to rule out moral hazard quits in the UI contract. Section 5 studies the main properties of the optimal UI contract developed in Section 2 where the UI agency takes into account the moral hazard effect of UI. Section 6 concludes.

2 The Model

This section introduces the UI eligibility rule into the framework by Hopenhayn and Nicolini (2009). For the purpose of comparison, we adopt their notations in what follows as much as possible. Based on a worker's employment and UI eligibility status, workers in our model are categorized into four types, namely the UI-eligible employed e, the UI-ineligible employed $\hat{e}$, the UI-eligible unemployed u, and the UI-ineligible unemployed $\hat{u}$. Denote the worker's type as i, so $i \in \{e, \hat{e}, u, \hat{u}\}$.

Following Zhang and Faig (2012), earning and losing UI eligibility are introduced in the following way. Ineligible workers can only earn UI entitlement while employed, and the probability of a transition from being ineligible to being eligible during one period is g. Upon gaining UI eligibility, the workers are allowed to choose whether to take it or renounce it.[9] If the UI entitlement is renounced, the workers remain ineligible for UI and cannot collect UI upon losing their jobs. The benfits do not last forever, and the probability of running out of the benefits for unemployed workers, a transition from eligibility to ineligibility, is d during one period. Workers are allowed to quit from their current jobs and to reject job offers. Upon doing these, eligible workers could collect the benefits with a probability π due to the lack of perfect monitoring on quits and rejections by the UI agency. Lastly, the UI benefits are financed by a mandatory state dependent UI contribution fee, τ^i, where $i \in \{e, \hat{e}\}$, imposed on employed workers. Note that in this contribution, both the UI agency and workers take the UI eligibility rule as given, so the values of g, d and π are exogenous parameters. The UI agency aims to minimize its costs as will be explained below.

The preference of a worker of type i is given by

$$E \sum_{t=1}^{\infty} \beta^t \left[u\left(c_t^i\right) - a_t \right], \tag{1}$$

where $u(\cdot)$ is the flow utility function and c_t^i is consumption for a worker of type i in period t. Particularly, $u(\cdot)$ is assumed to be strictly increasing, strictly concave and unbounded above; and c_t^i is assumed to be equal to wages net of UI fees if the worker is employed, and unemployment insurance benefits or social assistance if the worker is unemployed as will be explained below. As commonly assumed in the literature, this implies that the worker cannot save.[10] With a bit abuse of the notation, a_t measures search efforts

[9] When workers are not allowed to renounce the UI eligibility, we find that the expected lifetime utility for ineligible employed workers could be higher than eligible employed ones in the optimal UI contract, which is counterintuitive. To avoid this unimportant results, we allow workers to choose whether to take the UI entitlement or not.

[10] The literature on optimal UI, including Shavell and Weiss (1979), Wang and Williamson (1996), Hopenhayn and Nicolini (1997, 2009), and Fredriksson and Holmlund (2001), typically assume inability to save, or alternatively, that any savings undertaken by workers can be perfectly observed, and thus, completely controlled by the UI agency.

when workers are unemployed, or efforts at work when workers are employed in period t. Following Hopenhayn and Nicolini (2009), the value of the search effort is assumed to unobserved by the UI agency and to be either one or zero; $a_t = 1$ indicates that the unemployed worker searches hard for a job. In this case, the worker finds a job with a probability f regardless of the worker's UI eligibility state. $a_t = 0$ indicates that the unemployed worker does not put any effort in the job search, and therefore, cannot find a job no matter whether he or she is entitled to UI. The value of the effort at work, or the disutility while employed, is assumed to be publicly observable. Denote by m, its value is set to be a non-negative constant. So $a_t = m$.

$$a_t = \begin{cases} \left.\begin{array}{l} 1 \text{ (search hard)}, \\ 0 \text{ (not search)}, \end{array}\right. \text{if unemployed} \\ m, \text{ if employed} \end{cases}$$

In period t, an unemployed worker entitled to UI receives benefits b_t while searching for jobs. For an unemployed worker ineligible for UI, he or she collects what we refer to as "social assistance", a non-negative value $c_{\min}$, over the spell of unemployment.[11] The social assistance payment has infinite duration, but is potentially lower than the UI benefit. Once an unemployed worker locates a job successfully with a probability f, he or she decides whether to take it or not. If the job is accepted, the worker receives an exogenously determined constant wage w, and pays the UI fee τ^i, where $i \in \{e, \hat{e}\}$ over the spell of employment. If the job is declined, the worker entitled to UI is able to continue collecting benefits with a positive probability π. An employed worker may lose his or her job either exogenously with a probability s, or endogenously by quitting from the current job. Like job rejections, the eligible worker who quits can receive the benefit with a positive probability π. A worker decides whether to quit at the beginning of the period t. If the worker decides to do so, for the sake of tractability, the job termination is assumed to take place at the end of the period.[12]

2.1 Recursive Contract

This section lays out the optimal UI contract in a recursive way. A UI contract associates an expected utility to a worker of type i and its corresponding cost to the UI agency. The cost is evaluated by the expected discounted net transfers from the UI agency to workers required to provide the worker with the expected utility V^i. The worker responds to the contract rationally by maximizing (1) via choosing the searching effort, deciding whether to accept a job offer, whether to quit the current job, and whether to accept the UI eligibility upon gaining it. Given any promised lifetime utility V^i, the optimal

[11] Different from UI, by the discussion in Atkinson and Micklewright (1991), social assistance is not contributory and is independent of employment history. Given these features, we assume it is available to the ineligible unemployed workers.

[12] In a common practice, workers are required to give a notice to firms ahead of quits, such as 30 days.

contract minimizes the cost of granting that utility to the worker of type i in an incentive compatible way.

In what follows, we set up the cost minimizing problem for each type of workers in a way such that the UI agency is interested in implementing the positive search effort, eliminating the moral hazard quits from current jobs, and promoting the valuability of the UI entitlement. As argued in Hopenhayn and Nicolini (2009), an incentive problem arises when the UI agency tempts to fulfill these objectives. For example, the cost of implementing the positive search effort increases with the initial lifetime utility. Thus, if the initial utility level is sufficiently high, it may not be optimal to implement the positive search effort. In the appendix, we solve the general problem and characterize the set of utilities such that it is optimal to promote high search effort, rule out moral hazard quits and value the UI entitlement. The analysis in this section corresponds to utility levels within this set.

Denote by V_t^i the expected utility promised by the UI agency to a worker of type i at the beginning of a period t. For instance, V_t^e denotes the expected utility promised to an eligible employed worker at the beginning of period t. In addition, we record the evolution of the worker's type across periods in the superscript of the notation. For example, V_{t+1}^{eu} stands for the expected utility of a worker at the beginning of period $t+1$, who was UI-eligible unemployed in period t, and UI-eligible employed in period $t+1$. Denote by $W(V_t^e)$, $\hat{W}(V_t^{\hat{e}})$, $C(V_t^u)$, and $\hat{C}(V_t^{\hat{u}})$ the cost functions for the UI-eligible employed, the UI-ineligible employed, the UI-eligible unemployed, and the UI-ineligible unemployed workers, respectively.

We assume that in any period the consumption of a worker of type i is not smaller than c_{min}. This assumption imposes a lower bound for V_t^i; that is, $V_t^i \geq V_{min}$, where $V_{min} = c_{min}/(1-\beta)$, the expected utility of receiving c_{min} in all periods.

2.1.1 Eligible Employed Workers

The cost minimizing problem for a UI-eligible employed worker is given by

$$
\begin{aligned}
W\left(V_t^e\right) &= \min_{\tau_t^e, V_{t+1}^{ee}, V_{t+1}^{ue}, V_{t+1}^{\hat{u}e}} -\tau_t^e + \beta\left[(1-s) W\left(V_{t+1}^{ee}\right) + sC\left(V_{t+1}^{ue}\right)\right]. && (2)\\
\textit{subject to} &: \quad V_t^e = u\left(c_t^e\right) - m + \beta\left[(1-s) V_{t+1}^{ee} + sV_{t+1}^{ue}\right], && (3)\\
&: \quad c_t^e = w - \tau_t^e \geq c_{min}, && (4)\\
\text{(No-Quit)} &: \quad (1-s) V_{t+1}^{ee} + sV_{t+1}^{ue} \geq \pi V_{t+1}^{ue} + (1-\pi) V_{t+1}^{\hat{u}e}, && (5)\\
&: \quad V_{t+1}^{ee} \geq V_{min}, && (6)\\
&: \quad V_{t+1}^{ue} \geq V_{min}, && (7)\\
&: \quad V_{t+1}^{\hat{u}e} \geq V_{min}. && (8)
\end{aligned}
$$

A UI-eligible employed worker with the promised utility V_t^e pays a UI fee τ_t^e and decides whether to quit in period t. If the worker chooses to quit, he or she gets an expected continuation value $\pi V_{t+1}^{ue} + (1-\pi) V_{t+1}^{\hat{u}e}$. Otherwise, the expected continuation value for such a workers is $(1-s) V_{t+1}^{ee} + sV_{t+1}^{ue}$. To avoid voluntary job terminations, the UI agency imposes a (No-Quit) constraint such that the worker is better off by keeping the current job as shown by (5).

Eq. (3) is the promise-keeping constraint with the (No-Quit) constrain imposed. Inequality (4) is the budget constraint which requires that the UI contribution fee can not be too high so that the consumption of the UI-eligible employed worker is not lower than c_{min}. Inequalities (6), (7), and (8) are regularity constraints which state that possible promised utilities offered by the UI contract are not lower than V_{min}.

It is easy to see that the cost functions $W(\cdot)$, $\hat{W}(\cdot)$, $C(\cdot)$, and $\hat{C}(\cdot)$ are increasing and strictly convex, as the corresponding return functions are linear, and the function u in the constraints is strictly concave. Thus, they are almost everywhere differentiable.[13] Let $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$, $\mu_{4,t}$ and $\mu_{5,t}$ be the Lagrangian coefficients to constraints (4) - (8). The FOCs of the problem of (2) are derived as follows:

$$
\begin{aligned}
W'(V_t^e) &= \frac{1-\mu_{1,t}}{u'(c_t^e)}. && (9)\\
W'(V_{t+1}^{ee}) &= \frac{1-\mu_{1,t}}{u'(c_t^e)} + \frac{\mu_{2,t}}{\beta} + \frac{\mu_{3,t}}{\beta(1-s)}. && (10)\\
C'(V_{t+1}^{ue}) &= \frac{1-\mu_{1,t}}{u'(c_t^e)} - \frac{(\pi-s)\mu_{2,t}}{\beta s} + \frac{\mu_{4,t}}{\beta s}. && (11)\\
0 &= -(1-\pi)\mu_{2,t} + \mu_{5,t}. && (12)
\end{aligned}
$$

For the purpose of composition, discussions of the first order conditions (FOCs) for the cost minimizing problems for four types of workers are provided in details in the appendix.

2.1.2 Ineligible Employed Workers

The cost minimizing problem for a UI-ineligible employed worker is

$$
\hat{W}\left(V_t^{\hat{e}}\right) = \min_{\tau_t^{\hat{e}}, V_{t+1}^{e\hat{e}}, V_{t+1}^{\hat{e}\hat{e}}, V_{t+1}^{\hat{u}\hat{e}}} -\tau_t^{\hat{e}} + \beta\left[(1-s)\left(gW\left(V_{t+1}^{e\hat{e}}\right) + (1-g)\hat{W}\left(V_{t+1}^{\hat{e}\hat{e}}\right)\right) + s\hat{C}\left(V_{t+1}^{\hat{u}\hat{e}}\right)\right]. \quad (13)
$$

$$
\begin{aligned}
\textit{subject to} \quad &: \quad V_t^{\hat{e}} = u\left(c_t^{\hat{e}}\right) - m + \beta\left[(1-s)\left(gV_{t+1}^{e\hat{e}} + (1-g)V_{t+1}^{\hat{e}\hat{e}}\right) + sV_{t+1}^{\hat{u}\hat{e}}\right], && (14)\\
&: \quad c_t^{\hat{e}} = w - \tau_t^{\hat{e}} \geq c_{min}, && (15)\\
\text{(No-Quit)} \quad &: \quad gV_{t+1}^{e\hat{e}} + (1-g)V_{t+1}^{\hat{e}\hat{e}} \geq V_{t+1}^{\hat{u}\hat{e}}, && (16)\\
\text{(Valuable-UI)} \quad &: \quad V_{t+1}^{e\hat{e}} \geq V_{t+1}^{\hat{e}\hat{e}}, && (17)\\
&: \quad V_{t+1}^{\hat{e}\hat{e}} \geq V_{min}, && (18)\\
&: \quad V_{t+1}^{\hat{u}\hat{e}} \geq V_{min}. && (19)
\end{aligned}
$$

[13] See more discussions in Hopenhayn and Nicolini (2009).

A UI-ineligible employed worker with the promised utility $V_t^{\hat{e}}$ pays a UI fee $\tau_t^{\hat{e}}$ and decides whether to keep the job or not. If the worker chooses to quit or loses the job due to the exogenous separation shock, he or she receives a continuation value $V_{t+1}^{\hat{u}\hat{e}}$. Otherwise, such a worker remains employed and may gain the UI eligibility with a probability g next period. Upon gaining the UI eligibility, the worker is allowed to *renounce* it. To guarantee that the UI eligibility is valued and accepted by the worker, a (Valuable-UI) constraint is imposed as shown by (17). To rule out the quits, the UI agency imposes the (No-Quit) constraint such that the expected utility from keeping the job is no lower than the one from a job quit as shown by (16). With the (Valuable-UI) and (No-Quit) constraints, the promised utility $V_t^{\hat{e}}$ is expressed by (14). Inequalities (15), (18), and (19) are regularity constraints.

The (Valuable-UI) constraint plays an important role in the analysis. As proved in Appendix 7.2.2, it is welfare improving to impose such a constraint when the expected utility $V_t^{\hat{e}}$ is not too large.[14] The benefits of imposing it come in two ways. Firstly, imposing this constraint makes the UI eligibility valuable to workers, which provides the UI agency an additional tool to prevent the UI-ineligibles from a job quit. Particularly, the UI agency can promise the UI-ineligible workers a possibility of obtaining the valuable UI entitlement in the future as long as they stay in the current jobs. Secondly, as to be shown in Section 5, in response to the UI-ineligible workers' desire to earn UI entitlement though working, the UI agency can charge them with higher UI contribution fees, and thus, can further reduce overall costs of UI.

Let $\phi_{1,t}$, $\phi_{2,t}$, $\phi_{3,t}$, $\phi_{4,t}$ and $\phi_{5,t}$ be the Lagrangian coefficients to constraints (15) - (19) and the FOCs of the problem (13) are given as follows:

$$\hat{W}'\left(V_t^{\hat{e}}\right) = \frac{1-\phi_{1,t}}{u'\left(c_t^{\hat{e}}\right)}. \tag{20}$$

$$\hat{W}'\left(V_{t+1}^{\hat{e}\hat{e}}\right) = \frac{1-\phi_{1,t}}{u'\left(c_t^{\hat{e}}\right)} + \frac{\phi_{2,t}}{\beta(1-s)} - \frac{\phi_{3,t}}{\beta(1-g)(1-s)} + \frac{\phi_{4,t}}{\beta(1-g)(1-s)}. \tag{21}$$

$$\hat{C}'\left(V_{t+1}^{\hat{u}\hat{e}}\right) = \frac{1-\phi_{1,t}}{u'\left(c_t^{\hat{e}}\right)} - \frac{\phi_{2,t}}{\beta s} + \frac{\phi_{5,t}}{\beta s}. \tag{22}$$

$$W'\left(V_{t+1}^{e\hat{e}}\right) = \frac{1-\phi_{1,t}}{u'\left(c_t^{\hat{e}}\right)} + \frac{\phi_{2,t}}{\beta(1-s)} + \frac{\phi_{3,t}}{\beta g(1-s)}. \tag{23}$$

[14] As proved in Appendix 7.2.2, when the expected utility of $V_t^{\hat{e}}$ is in the interval $\left(V_{\min}, \bar{V}_t^{\hat{e}}\right)$, it is optimal to impose the (Valuable-UI) constraint. Otherwise, one has $V_{t+1}^{e\hat{e}} < V_{t+1}^{\hat{e}\hat{e}}$, which suggests that nobody values the UI entitlement. When the expected utility of $V_t^{\hat{e}}$ is large than $\bar{V}_t^{\hat{e}}$, the (Valuable-UI) constraint becomes slack, and workers value UI eligibility automatically.

2.1.3 Eligible Unemployed Workers

The cost minimizing problem for a UI-eligible unemployed worker is

$$C\left(V_t^u\right) = \min_{b_t, V_{t+1}^{uu}, V_{t+1}^{\hat{u}u}, V_{t+1}^{eu}} b_t + \beta\left[(1-f)\left((1-d)\,C\left(V_{t+1}^{uu}\right) + d\hat{C}\left(V_{t+1}^{\hat{u}u}\right)\right) + fW\left(V_{t+1}^{eu}\right)\right]. \tag{24}$$

$$\begin{aligned}
subject\ to \quad &: \quad V_t^u = u\left(b_t\right) - 1 + \beta\left[(1-f)\left((1-d)\,V_{t+1}^{uu} + dV_{t+1}^{\hat{u}u}\right) + fV_{t+1}^{eu}\right] & (25)\\
&: \quad b_t \geq c_{min}, & (26)\\
\text{(No-Rejection)} \quad &: \quad V_{t+1}^{eu} \geq \pi V_{t+1}^{uu} + (1-\pi)\,V_{t+1}^{\hat{u}u}, & (27)\\
\text{(Search-Incentive)} \quad &: \quad \beta f\left(V_{t+1}^{eu} - dV_{t+1}^{\hat{u}u} - (1-d)\,V_{t+1}^{uu}\right) \geq 1, & (28)\\
&: \quad V_{t+1}^{uu} \geq V_{min}, & (29)\\
&: \quad V_{t+1}^{\hat{u}u} \geq V_{min}. & (30)
\end{aligned}$$

To induce a UI-eligible unemployed worker with the promised utility V_t^u to search hard for jobs, the UI agency imposes the (Search-Incentive) constraint (28), which implies that expected values from searching hard and forming a match successfully is large enough to offset the disutilities from search. With the positive search effort, the unemployed worker finds a job with a probability f and receives continuation value V_{t+1}^{eu} upon taking it. Otherwise, he or she obtains the expected continuation value $\pi V_{t+1}^{uu} + (1-\pi)\,V_{t+1}^{\hat{u}u}$. To prevent the job rejection, a (No-Rejection) constraint is imposed as shown by (27). If the worker does not succeed in locating a job, he or she remains unemployed and might lose the UI eligibility with a probability d. Eq. (25) is the promise-keeping constraint. Inequality (26) states that the UI benefits cannot go below c_{min}. Inequalities (29) and (30) are regularity constraints. The regularity constraint $V_{t+1}^{eu} \geq V_{min}$ is satisfied when the (No-Rejection) constraint (27) is imposed.

Of particular note is that the values of d and π are important to understand the worker's job search and job acceptance behavior. Particularly, it explains why some workers search hard for jobs, while still choose to turn down the job offers. We find, as will be shown in Section 5, that this happens exactly when the likelihood of being caught by the UI agency from declining job offers is *small* compared to the probability of running out of benefits exogenously, that is, $d > (1-\pi)$. In this case, the eligible workers have strong incentive to escape from the exogenous shock of losing UI by searching jobs and rejecting offers afterwards. Put it differently, they take their luck in the job search as a way to avoid the bad shock of losing UI that comes with a high probability.

Let $\eta_{1,t}$, $\eta_{2,t}$, $\eta_{3,t}$, $\eta_{4,t}$, and $\eta_{5,t}$ be the Lagrangian coefficients to constraints (26) - (30). Then the FOCs of the problem (24) are expressed as follows:

$$C'\left(V_t^u\right) = \frac{1-\eta_{1,t}}{u'\left(b_t\right)}. \tag{31}$$

$$W'\left(V_{t+1}^{eu}\right)=\frac{1-\eta_{1,t}}{u'\left(b_t\right)}+\frac{\eta_{2,t}+\eta_{3,t}}{\beta f}. \tag{32}$$

$$C'\left(V_{t+1}^{uu}\right)=\frac{1-\eta_{1,t}}{u'\left(b_t\right)}+\frac{-\eta_{2,t}\pi+\eta_{4,t}}{\beta\left(1-f\right)\left(1-d\right)}-\frac{\eta_{3,t}}{\beta\left(1-f\right)}. \tag{33}$$

$$\hat{C}'\left(V_{t+1}^{\hat{u}u}\right)=\frac{1-\eta_{1,t}}{u'\left(b_t\right)}+\frac{-\eta_{2,t}\left(1-\pi\right)+\eta_{5,t}}{\beta\left(1-f\right)d}-\frac{\eta_{3,t}}{\beta\left(1-f\right)}. \tag{34}$$

2.1.4 Ineligible Unemployed Workers

The cost minimizing problem for a UI-ineligible unemployed worker is given by

$$\begin{aligned}
\hat{C}\left(V_t^{\hat{u}}\right) &= \min_{V_{t+1}^{\hat{u}\hat{u}},V_{t+1}^{\hat{e}\hat{u}}} \beta\left[\left(1-f\right)\hat{C}\left(V_{t+1}^{\hat{u}\hat{u}}\right)+f\hat{W}\left(V_{t+1}^{\hat{e}\hat{u}}\right)\right]. && (35)\\
\textit{subject to} &: V_t^{\hat{u}}=u\left(c_{min}\right)-1+\beta\left[\left(1-f\right)V_{t+1}^{\hat{u}\hat{u}}+fV_{t+1}^{\hat{e}\hat{u}}\right], && (36)\\
\text{(search-incentive)} &: \beta f\left(V_{t+1}^{\hat{e}\hat{u}}-V_{t+1}^{\hat{u}\hat{u}}\right)\geq 1, && (37)\\
&: V_{t+1}^{\hat{u}\hat{u}}\geq V_{min}. && (38)
\end{aligned}$$

A UI-ineligible unemployed worker with the expected utility $V_t^{\hat{u}}$ receives a flow consumption $c_{\min}$ from outside of the UI system, so the UI agency incurs no cost. To ensure a positive search effort, the UI agency imposes the (Search-Incentive) constraint (37). Thus, the worker incurs the search efforts $a=1$ and is matched with a job with a probability f next period. If such a job offer is accepted, this worker gets a continuation value $V_{t+1}^{\hat{e}\hat{u}}$. If the worker fails in finding a job or rejects the job offer, he or she remains unemployed and obtains a continuation value $V_{t+1}^{\hat{u}\hat{u}}$. Note that if the (Search-Incentive) constraint (37) is satisfied, then the ineligible unemployed worker would *not* reject any job offer since $V_{t+1}^{\hat{e}\hat{u}}\geq V_{t+1}^{\hat{u}\hat{u}}$. So there is no need to impose a (No-Rejection) constraint. This result is consistent with the entitlement effect. Eq. (36) is the promise-keeping constraint, and inequality (38) is the regularity constraint.

Let $\varphi_{1,t}$ and $\varphi_{2,t}$ be the Lagrangian coefficients to constraints (36) and (37) in the problem of (35). The FOCs of ineligible unemployed workers are given by:

$$\begin{aligned}
\hat{W}'\left(V_{t+1}^{\hat{e}\hat{u}}\right) &= \hat{C}'\left(V_t^{\hat{u}}\right)+\varphi_{1,t}. && (39)\\
\hat{C}'\left(V_{t+1}^{\hat{u}\hat{u}}\right) &= \hat{C}'\left(V_t^{\hat{u}}\right)-\varphi_{1,t}+\varphi_{2,t}. && (40)
\end{aligned}$$

3 Loopholes in the Optimal UI Contract with Imperfect Monitoring on Quits and Rejections

This section studies whether there exists a loophole in the optimal UI contract in the context of imperfect monitoring on job quits and rejections by the UI agency. Like

Hopenhayn and Nicolini (2009), we refer to the loophole as the case where workers are willing to work for a short time and then quit to re-qualify for higher level of benefits. Specifically, we explore the existence of loophole when the UI agency does not take this issue seriously. That is, the (No-Quit) constraint (5) and the (No-Rejection) constraints (16) and (27) are absent from the optimizing problems set up in Section 2. Hence, one has $\mu_{2,t} = \phi_{2,t} = \eta_{2,t} = 0$.

The consideration of the UI eligibility proves important. We show that the loophole does not necessarily exist even when the value of disutility of work is large, which is in sharp contrast with the finding in Hopenhayn and Nicolini (2009). In our model, the loophole disappears when the monitoring power on job quits is not sufficiently weak, or alternatively, when the likelihood of being punished from job quitting is high. This result is driven by the entitlement effect in the sense that ineligible workers in the model value the UI eligibility so much that they cannot affford losing it by quitting. However, the loophole cannot be fixed completely by the entitlement effect. Similar to the finding in the literature, the optimal UI contract in our model also needs to deal with moral hazard quits. We prove that a loophole exists when the monitoring power is low and the disutility of work is large.

As proved in Lemma 7.9 in the appendix, as long as the (Search-Incentive) constraint for UI-eligible unemployed workers binds initially, it binds in the subsequent periods in the absence of the (No-Quit) and the (No-Rejection) constraints.[15] Hence, in what follows, we focus on the case where the (Search-Incentive) constraint always binds for the eligible unemployed workers.

Proposition 3.1 *If the (Search-Incentive) constraint (28) binds for a UI-eligible unemployed worker, then UI benefits decrease as long as current UI benefits $b_t > c_{min}$, and UI contribution fees after re-employment increase with the unemployment spell if consumptions after re-employment $c^{eu}_{t+1} > c_{min}$.*

Proof If the (Search-Incentive) constraint always binds for an eligible unemployed worker with $V^u_t > V_{min}$, then by Lemma 7.7 in the appendix and convexity of $C(\cdot)$, one has $V^{uu}_{t+1} < V^u_t$. Since $b_t > c_{min}$, Eq. (31) suggests that the UI benefits decrease over the spell of the UI-eligible unemployment.

Moreover, by Lemma 7.10 in the appendix, one has $V^{\hat{u}uu}_{t+2} \leq V^{\hat{u}u}_{t+1}$. Since the (Search-Incentive) constraint (28) binds for all periods, one immediately has $V^{euu}_{t+2} < V^{eu}_{t+1}$. So V^{eu}_{t+1} decreases with the spell of the previous unemployment.

In addition, if $c^{eu}_{t+1} > c_{min}$, one has $c^{eu}_{t+1} > c^{euu}_{t+2}$ as suggested by Eq. (9). Since w is constant, the conclusion follows. $\square$

When the UI agency lacks perfect information on job quits, the decreasing profile of the benefit established in Proposition 3.1 potentially opens up a door for loopholes in

[15] We can prove that if the Search-Incentive constraint is not binding initially, then it is not binding forever. The proof is available upon request.

the optimal contract. Intuitively, a UI-eligible unemployed worker has incentive to re-qualify for higher level of UI by finding a job and leaving it immediately. In the following two propositions, we derive the sufficient conditions under which a voluntary quit from a socially efficient job happens.

Proposition 3.2 *(**Loophole**) If the (Search-Incentive) constraint of an eligible unemployed worker with a promised utility $V_t^u > V_{min}$ binds in period t, then there exists a $\bar{\pi} \in (0,1)$ such that the worker would improve his or her UI benefits by getting employed and quitting immediately for $\pi \in (\bar{\pi}, 1]$.*

Proof Consider an eligible unemployed worker with a promised utility $V_t^u > V_{min}$ in period t. Suppose this worker gets employed in period $t+1$ with a continuation value V_{t+1}^{eu}, and quits in period $t+2$ with a continuation value $\pi V_{t+2}^{ueu} + (1-\pi) V_{t+2}^{\hat{u}eu}$. If it is optimal for such a worker to do so, one needs to show that $\pi V_{t+2}^{ueu} + (1-\pi) V_{t+2}^{\hat{u}eu} > V_t^u$.

Firstly, we show that $V_{t+2}^{ueu} > V_t^u$ always holds in the absence of the (No-Quit) and the (No-Rejection) constraints.

By Lemma 7.1 in the appendix, if the (No-Quit) constraint is absent ($\mu_{2,t+1} = 0$), one has $W'\left(V_{t+1}^{eu}\right) = C'\left(V_{t+2}^{ueu}\right)$. Since the (Search-Incentive) constraint binds in period t, it has $\eta_{3,t} > 0$, which, together with Lemma 7.7 in the appendix, leads to $C'\left(V_t^u\right) < W'\left(V_{t+1}^{eu}\right)$. By convexity of the function $C(\cdot)$, it follows that $V_{t+2}^{ueu} > V_t^u$.

Secondly, we show that $\pi V_{t+2}^{ueu} + (1-\pi) V_{t+2}^{\hat{u}eu} > V_t^u$ for $\pi \in (\bar{\pi}, 1]$.

By Lemma 7.2 in the appendix, one has $V_{t+2}^{\hat{u}eu} = V_{min}$. Since $V_t^u > V_{min}$, one has $V_t^u > V_{t+2}^{\hat{u}eu}$. Combining $\left(V_{t+2}^{ueu} > V_t^u\right)$ with $\left(V_t^u > V_{t+2}^{\hat{u}eu}\right)$ leads to the conclusion that one can always find a $\overline{\pi} \in (0,1)$ such that $V_t^u = \overline{\pi} V_{t+2}^{ueu} + (1-\overline{\pi}) V_{t+2}^{\hat{u}eu}$. Hence, one has $\pi V_{t+2}^{ueu} + (1-\pi) V_{t+2}^{\hat{u}eu} > V_t^u$ for $\pi \in (\bar{\pi}, 1]$. □

Proposition 3.2 suggests that the probability of collecting benefits upon quits is crucial to understand the moral hazard behavior. When it is likely to collect benefits upon quits, or equivalently, when the monitoring power by the UI agency is low, the loophole of the optimal UI contract would allow workers to behave strategically to take advantage of the UI generosity. In sharp contrast, if the likelihood is low, or the monitoring power is strong, then this moral hazard behavior no longer exists and the loophole disappears.

Corollary 3.3 *When the monitoring power on quits by the UI agency is sufficiently large, that is, $\pi \in (0, \bar{\pi})$, eligible employed workers would not quit to re-qualify for higher benefits.*

Proof The result follows directly from the proof of Proposition 3.2. □

The intuition behind this result is that the UI agency in our model can punish eligible workers who quit by taking away their UI eligibility with a probability $(1-\pi)$. The low value of π implies the likelihood of being caught, and therefore, being punished is rather high. In response, the eligible workers, who value the UI eligibility and are concerned with losing it upon quitting, choose to keep their jobs.

Note that it is the entitlement effect that fixes the loophole in the case where $\pi \in (0, \bar{\pi})$, rather than the (No-Quit) constraint as argued in Hopenhayn and Nicolini (2009). This difference is due to the introduction of the UI eligibility rule. In Hopenhayn and Nicolini (2009), the benefits are universally available and the UI agency cannot punish workers at all.

Proposition 3.2 simply assumes that an eligible employed worker quits from his or her current job immediately after he or she locates a job successfully. To validate this assumption, the next proposition derives the sufficient condition under which a job quit would happen.

Proposition 3.4 *For a UI-eligible employed worker, if* $m > \frac{1-\pi}{1-s}(\beta\frac{\pi-s^2}{(\pi-s)(1-\beta)}+1)(u(w)-u(c_{\min}))+\frac{\pi-s}{f(1-s)}$*, where* $\pi > s$*, then the worker would like to quit his or her job.*

See the Proof in Appendix. □

Propositions 3.2 and 3.4 jointly show that when the monitoring power by the UI agency is weak, strategic quits are indeed an optimal choice for workers when the disutility of work is large. Given this result, a natural question to ask is whether the optimal UI contract should prevent moral hazard quits or not. Section 4 provides an answer to this question.

4 Numerical Analysis

In this section, we carry out a numerical exercise to show that it is indeed cost-saving for the UI agency to eliminate moral hazard behavior in designing the optimal UI contract. Particularly, we illustrate that with the (No-Quit) constraint imposed, the overall cost of granting a moderate promised lifetime utility V_t^i to workers of type i is lower than the case when such a constraint is absent. Of particular note is that since we calculate the economy-wide cost, the overall costs include the cost of granting $c_{\min}$ to the UI-ineligible unemployed workers.

The model period is set to be one quarter. The utility function is assumed to take the form of CRRA, so $u(\cdot) = \frac{c^{1-\gamma}}{1-\gamma}$. Follow the literature, the value of γ is set to be 2. The discount rate is set to match the annual real interest rate 4.8 percent. The job finding rate and the exogenous job separation rate are chosen to fit the quarterly finding rate and separation rate in Shimer (2005). For the UI policy parameters d and g, they are determined in a way such that the model predicted duration of collecting the regular benefits and duration of employment required to be entitled to UI are the same as the statutory requirements in the United States, which are 26 and 20 weeks, respectively. We pin down the value of π according to the calibration result in Zhang and Faig (2012), which is about 0.6. As for the effort at work m, the value is arbitrarily set to be 1. The wage rate for the employed is normalized to one. Lastly, we set $c_{\min}$ to be 0.17 so that the average social assistance payment per recipient amounted to 17 percent of average earnings in 1991 (see Wang and Williamson, 1996 and Fredriksson and Holmlund, 2001). Table 1 reports the parameter values used in the baseline numerical exercise.

Table 1

Baseline Parameterization (Quarterly)

Variables		Parameterization Description	Value
Parameter in the CRRA utility function	γ	$\gamma > 1$	2
Discount factor	β	annual real interest rate, 0.048	0.012
Job finding rate	f	Shimer (2005)	0.450
Separation rate	s	Shimer (2005)	0.033
Prob. of losing UI exogenously	d	duration of collecting regular UI, 26 weeks	1/2
Prob. of gaining UI eligibility	g	duration of employment required for UI, 20 weeks	2/3
Prob. of collecting benefits upon quits	π	Zhang and Faig (2012)	0.6
Effort at work	m	arbitrarily chosen	1
Wage rate	w	normalization	1
Social assistance	$c_{\min}$	Wang and Williamson (1996)	0.17

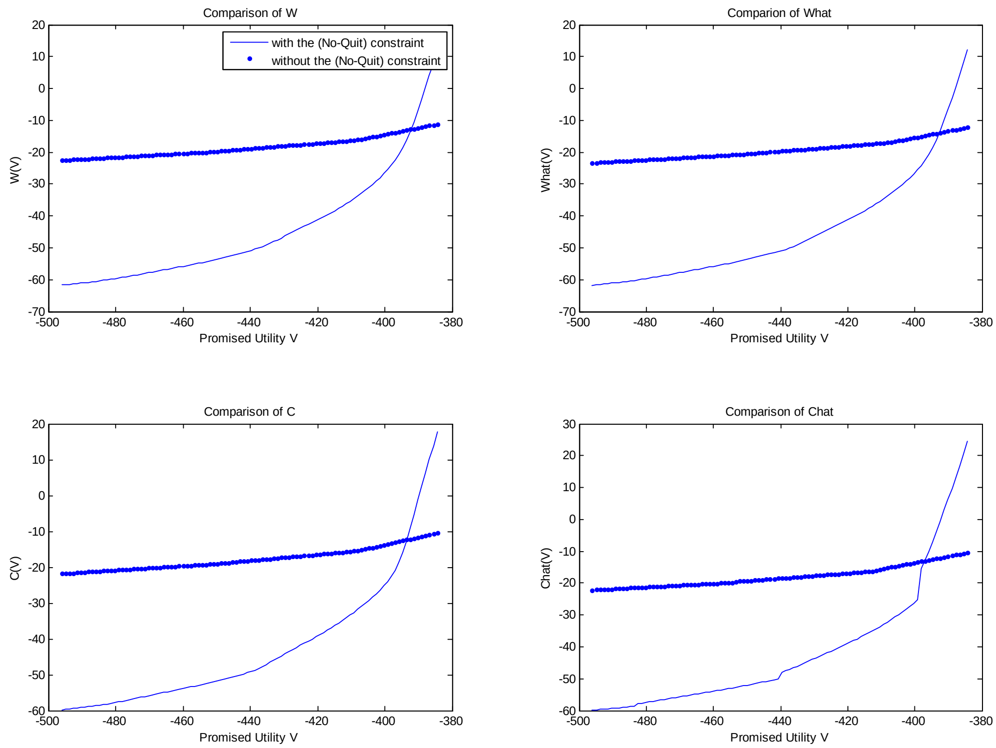
Figure 1 compares the overall costs of two different UI contracts that grant a moderate initial lifetime utility V^i to a worker of type i. In the figures, the vertical axis measures the economy-wide cost of the UI contracts, while the horizontal axis measures the initial lifetime utility V^i. The dotted line represents the UI contract where the (No-Quit) constraint is not imposed, while the solid line is associated with the UI contract where such a constraint is put in place. Figure 1 clearly shows that the overall cost of the UI contract that rules out moral hazard quits is lower when the promised lifetime value V^i is not too large, which is consistent with the analytical proof in the appendix. It is worth pointing out that since the respective cost of the UI contract for each type of workers is lower when the (No-Quit) constraint is imposed, the above result is independent of the worker's distribution across employment status and UI eligibility state.[16] In addition, we find that this result remains robust when we change values for the parameters $c_{\min}$, π, d, and g that govern the generosity of the UI system.

The policy implications of this result are twofolds. Firstly, for moderate promised utilities, the UI agency should take steps to mitigate moral hazard quits in designing the

[16] Unlike Atkeson and Lucas (1995), we do not make the assumption that the UI system runs balanced budget, which makes it impossible to compute the distribution of workers by employment status and UI eligibility state in our paper.

optimal UI. Secondly, it suggests that the UI contract is *suboptimal* in the countries where the moral hazard quits are present and severe.

Figure 1: Comparison of the Costs for Alternative UI Contracts



NOTE: Figure 1 compares the economy-wide costs between two UI contracts that grant a moderate promised lifetime utility V^i to a worker of type i. The dotted line corresponds to a UI contract where the (No-Quit) constraint is not imposed, while the solid line corresponds to a UI contract where such a constraint is present.

5 Properties of the Optimal UI Contract

The results in previous sections suggest that the UI contract that grants a moderate expected utility should be designed with the moral hazard problem in mind when the monitoring power is not strong enough. The analysis in this section, corresponding to the expected utility and the monitoring power within these sets, aims to explore properties of the optimal UI contract when the opportunistic behavior is taken into consideration.

Specifically, we examine the contract which imposes both the (No-Quit) and the (No-Rejection) constraints as stated in Section 2. It is worth pointing out that within this expected utility set it is welfare improving to impose the (Valuable-UI) constraint as proved in Appendix 7.2.2. The most important result in this section is that if the UI contract is designed in a way such that the positive search effort is implemented, the UI eligibility is valued, and the moral hazard quits are completely eliminated, then the optimal UI contract is featured with a differentiated UI fee scheme. This result is consistent with what we observe in reality although it is novel in the literature.

Proposition 5.1 *If the (Valuable-UI) constraint (17) does not bind, then the (No-Quit) constraint does not bind, and* $V_{t+1}^{\hat{e}\hat{e}} > V_{min}$.

Proof The proof follows directly from the discussions in Appendix 7.1.2. □

Proposition 5.1 shows that if the UI eligibility is strictly valuable to ineligible employed workers, then they would not choose to quit. Put it differently, the valuable UI eligibility essentially serves as an alternative *device* to rule out the job quit by the UI-ineligibles in this model. Intuitively, keeping the current job will be rewarded by a positive possibility of gaining UI eligibility in the future. This result reflects the entitlement effect in the early work by Mortensen (1977), and squares well with the empirical findings. For example, Christofides and McKenna (1996), by using the Canadian 1986-87 longitudinal Labor Market Activity Survey (LMAS), finds evidence that a significant number of jobs terminate when they have reached the duration that would permit a Ul claim. Moreover, they find that this result holds most clearly for initial UI users who are previously not eligible for UI. A similar finding is also established by Card and Riddell (1996).

Proposition 5.2 *If the (No-Quit) constraint (5) binds for an eligible employed worker with a promised utility* $V_t^e > V_{min}$ *in period* t*, then his or her expected utility increases over the spell of the UI-eligible employment when* $\pi > s$*. Meanwhile, if the (Valuable-UI) constraint (17) binds for an ineligible employed worker with a promised utility* $V_t^{\hat{e}} > V_{min}$ *in period* t*, then his or her expected utility decreases over the spell of the UI-ineligible employment.*

Proof By Lemmas 7.3 and 7.4 in the appendix, one has $W'(V_t^e) < W'(V_{t+1}^{ee})$. By convexity of $W(\cdot)$, one has $V_t^e < V_{t+1}^{ee}$. Given $V_t^{\hat{e}} > V_{min}$, by Lemmas 7.5 and 7.6 in the appendix, one has $\hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) < \hat{W}'(V_t^{\hat{e}})$. By convexity of $\hat{W}(\cdot)$, $V_{t+1}^{\hat{e}\hat{e}} < V_t^{\hat{e}}$. □

Proposition 5.2 implies that when the UI eligibility rule are explicitly modeled, expected utilities for the employed are no longer monotone over the spell of employment as in Hopenhayn and Nicolini (2009). Instead, if the employed worker is not entitled to UI upon forming an employment relation, the expected utility decreases on the job tenure. After gaining UI eligibility, the expected utility starts to increase over the spell of employment. This result, together with the assumption of constant wages, suggests a differentiated UI fees between the ineligible and eligible workers, which is established in the following corollary.

Corollary 5.3 ***(Differentiated UI Fees)*** *The UI contribution fee paid by an eligible employed worker with $c_t^e > c_{min}$ is decreasing over the duration of the UI-eligible employment if the (No-Quit) constraint binds in period t, while the fee paid by an ineligible employed worker with $c_t^{\hat{e}} > c_{min}$ is increasing over the duration of the UI-ineligible employment if the (Valuable-UI) constraint binds in period t, and the worker who newly gains the UI eligibility experiences a fall in the UI fee.*

Proof for the shape of the UI fees, by Proposition 5.2 and Eq. (20), one has $\frac{1-\phi_{1,t+1}}{u'(c_{t+1}^{\hat{e}\hat{e}})} < \frac{1-\phi_{1,t}}{u'(c_t^{\hat{e}})} = \frac{1}{u'(c_t^{\hat{e}})}$, where the equality follows from $c_t^{\hat{e}} > c_{min}$ $\left(\text{or } \phi_{1,t} = 0\right)$. If $c_{t+1}^{\hat{e}\hat{e}} > c_{\min}$, then $\phi_{1,t+1} = 0$, which implies that $c_{t+1}^{\hat{e}\hat{e}} < c_t^{\hat{e}}$. If $c_{t+1}^{\hat{e}\hat{e}} = c_{\min}$, since $c_t^{\hat{e}} > c_{min}$, it follows that $c_{t+1}^{\hat{e}\hat{e}} < c_t^{\hat{e}}$. By the budget constraint (15) and the assumption of the constant wage offer, the increasing profile of the UI fees $\tau_t^{\hat{e}}$ follows immediately. Follow the same logic as above, we can conclude that if $c_t^e > c_{min}$, then one has $c_t^e < c_{t+1}^{ee}$ and τ_t^e is decreasing over the spell of employment.

For the gap in the UI fee, by Lemma 7.5, one has $W'(V_{t+1}^{e\hat{e}}) > \hat{W}'(V_{t+1}^{\hat{e}\hat{e}})$. Then by the first order conditions (9) and (20), one has $\frac{1}{u'\left(c_{t+1}^{e\hat{e}}\right)} > \frac{1-\mu_{1,t+1}}{u'\left(c_{t+1}^{e\hat{e}}\right)} > \frac{1-\phi_{1,t+1}}{u'\left(c_{t+1}^{\hat{e}\hat{e}}\right)}$, which leads to $c_{t+1}^{e\hat{e}} > c_{t+1}^{\hat{e}\hat{e}}$. The constant wage rate received by all workers suggests $\tau_{t+1}^{e\hat{e}} < \tau_{t+1}^{\hat{e}\hat{e}}$. □

Proposition 5.2 and Corollary 5.3 show that the optimal UI contract that values the UI eligibility and rules out the moral hazard behavior delivers *differentiated* UI fees, or more generally, differentiated after-tax wages (wages net of UI fees), between UI-eligible and UI-ineligible employed workers over the spell of employment. Given the common wage rate, the UI fees paid by these two types of workers differ in both level and the profile over the job tenure. Our model predicts that the fees paid by the UI-ineligibles are typically higher in magnitude than those paid by the UI-eligibles. The key insight lies in the entitlement effect: the high fees paid by the ineligible workers essentially serve as a fair price to buy the valuable UI eligibility to be obtained in the future. Put it differently, the ineligible workers are willing to pay high fees in hope of gaining the UI eligibility and paying low fees in the future. In addition, the UI fees display different evolutions during the employment: increasing for the UI-ineligibles, while decreasing for the UI-eligibles.[17] The increasing shape is driven by the binding (Valuable-UI) constraint. In our model, the UI agency promotes the valuability of the UI, and therefore, encourages workers to accept

[17] Admittedly, if we assume a fixed duration of employment for the UI eligibility as opposed to the jump process as modelled in this paper, a UI-ineligible worker would experience a fall in UI fees when he or she becomes entitled to UI. However, the increasing shape of the UI fee over the UI-ineligible employment might not exist.

the UI upon earning it. To this end, it keeps punishing the worker who renounces the UI eligibility by increasing the UI fee in each period. So, the punishment drives the UI fee to rise over time. The decreasing shape is driven by the binding (No-Quit) constraint. In order to prevent a job quit by the UI-eligibles, the UI agency backloads their wage in each period.

It is easy to see that if the (Valuable-UI) constraint is slack, then by Proposition 5.1, the (No-Quit) constraint is slack. Thus, Eqs.(20), (21), and (23) suggest that $\hat{W}\left(V_t^{\hat{e}}\right) = \hat{W}\left(V_{t+1}^{\hat{e}\hat{e}}\right) = W\left(V_{t+1}^{e\hat{e}}\right)$, and the first order conditions (9) and (20) imply $c_t^{\hat{e}} = c_{t+1}^{\hat{e}\hat{e}} = c_{t+1}^{e\hat{e}}$. This result, together with the constant wage rate, gives rise to the following two predictions. Firstly, the UI fees by the UI-ineligibles are constant over the spell of employment. Secondly, the UI-ineligible worker experiences no change in the UI fee in the period when he or she gains the UI entitlement. However, if the (No-Quit) constraint binds for the worker who newly gains UI eligibility, such a worker starts to pay decreasing UI fees over the UI-eligible employment as predicted by Corollary 5.3. In this view, the gap in the UI fees between eligible and ineligible workers still exists. [18]

The differentiated UI fee scheme contrasts with the monotonicity of the UI fees argued in Hopenhayn and Nicolini (2009), but is consistent with various evidence found the empirical studies. It is worth pointing out that the after-tax wages—the difference between the wage and the UI fee— received by the employed are *essentially* the reservation wages for unemployed workers in our model. To see this, the optimal UI fees are part of the solution to the optimal contract that minimizes the cost of granting a certain level of the promised utility to a worker. Thus, the corresponding after-tax wage is indeed the minimal wage that attracts an unemployed worker to accept a job offer. Given this, the theoretical implication of the differentiated UI fee result is that the reservation wage for the UI-eligibles should be higher than that for the UI-ineligibles, which is in line with the empirical findings. The direct evidence comes from Fishe (1982). By using the Continuous Wage and Benefit History (CWBH) for the state of Florida for the years 1971 to 1974, Fishe estimates the effect on reservation wages of the duration of unemployment and finds strong evidence that the reservation wage decreases over the compensation period and drops dramatically upon exhaustion of benefits. Particularly, the reservation wage falls, on average, by 15 percent when UI benefits are exhausted. More recently, a similar conclusion is reached by DellaVigna and Paserman (2005) by using different data set and different methodologies. They compare self-reported reservation wages from the NLSY between workers who are receiving benefits and not, and find that receiving benefits raises the reservation wage by 4.7 percent. Admittedly, the results in DellaVigna and Paserman (2005) are subject to the endogeneity and sample selection bias problems as pointed out in

[18] It is easy to see that if the (No-Quit) constraint is slack for the worker with newly gained UI eligibility, then both types of employed workers would pay constant and equal UI fees over the spell of employment. However, this happens when the expected utility of $V_{t+1}^{e\hat{e}}$ is large, which is beyond the utility set within which we study the optimal UI contract as emphasized at the beginning of this section.

Shimer and Werning (2007), but it is worth understanding the quantitative implications of their estimates; there exists positive gains from gaining UI eligibility.

The introduction of the UI eligibility nicely preserves some properties of the optimal UI contract found in the literature. Particularly, the following corollaries and propositions show that in our model the replacement ratio is less than one, increases with the previous job tenure, and the UI benefits are decreasing over the spell of unemployment.

Corollary 5.4 *If the consumption of an eligible employed worker satisfies $c_t^e > c_{min}$, then the UI replacement ratio is less than one if the worker becomes eligible unemployed next period.*

Proof With the (No-Quit) constraint binds $\left(\mu_{2,t} > 0\right)$, by inequality (41) in Appendix 7.1.1, one has $C'(V_{t+1}^{ue}) < W'(V_t^e)$. By Eqs. (9) and (31), one has $\dfrac{1-\eta_{1,t+1}}{u'(c_{t+1}^{ue})} < \dfrac{1-\mu_{1,t}}{u'(c_t^e)}$. By assumption $c_t^e > c_{min}$, one has $\mu_{1,t} = 0$. Thus, it follows that $c_{t+1}^{ue} < c_t^e$. $\square$

Corollary 5.5 *When $\pi > s$, the expected utility of an eligible unemployed worker is increasing with the length of his or her previous employment.*

Proof By Lemma 7.4 in the appendix, one has $\mu_{2,t} > 0$ in all periods if the (No-Quit) constraint (5) binds in period t. By Eq. (12), one has $\mu_{5,t} > 0$ for all periods. So, $V_{t+1}^{\hat{u}e} = V_{t+2}^{\hat{u}ee} = V_{\min}$. By Proposition 5.2, one has $V_{t+1}^{ee} < V_{t+2}^{eee}$, then the binding (No-Quit) constraint (5) suggests $V_{t+1}^{ue} < V_{t+2}^{uee}$ for $\pi > s$. $\square$

Proposition 5.6 *When $d + \pi > 1$, for a UI-eligible unemployed worker with $V_t^u < \dfrac{1-d}{f(d+\pi-1)} + V_{min}$ and $b_t > c_{min}$, if the (Search-Incentive) constraint (28) binds, then the benefit decreases over the spell of UI-eligible unemployment.*

Proof By Lemmas 7.7 and 7.12 in the appendix, one has $V_{t+1}^{uu} < V_t^u$. Moreover, if $b_t > c_{\min}$, then by FOC (31), one has $\frac{1}{u'(b_t)} > \frac{1-\eta_{1,t+1}}{u'(b_{t+1})}$. It follows that $b_t > b_{t+1}$. $\square$

Consistent with the findings in the literature, the optimal UI benefits in our model are decreasing over the spell of unemployment. The reasons are twofold. Firstly, as explained in Section 2, the condition $d + \pi > 1$ implies that it is likely for a UI-eligible unemployed worker to lose UI entitlement exogenously, which motivates the worker to search for a job. However, this channel per se is not strong enough to implement positive search effort in the optimal UI contract. Secondly, as proved in the appendix 7.2.1, it is optimal for the UI agency to promote positive search effort only when the expected utility for a UI-eligible unemployed worker V_t^u is not too large.[19] Thus, the value of V_t^u is bounded from

[19] The same argument is shown in Hopenhayn and Nicolini (2009).

above. It is worth noting that the condition of $V_t^u < \frac{1-d}{f(d+\pi-1)} + V_{min}$ is the sufficient condition for the decreasing benefits. Under this condition, the (No-Rejection) constraint is slack so that UI-eligibles have no incentive to turn down the job offer.

Zhang and Faig (2012) shows that in a model with risk-neutral workers, a fully funded UI system that eliminates the moral hazard problem is also optimal. In what follows, we argue that this result might not hold with risk aversion preference. According to the their definition, a fully funded UI refers to the one that the expected present utility for a worker who newly enters the labor market, but yet not entitled to UI is zero. Given this definition, a fully funded UI system in our model requires the cost of offering $V_t^{\hat{e}}$ to an ineligible employed worker is zero, $\hat{W}\left(V_t^{\hat{e}}\right) = 0$. For such a UI system (contract) to be optimal, the UI contract has to satisfy all the FOCs for the ineligible employed worker. However, it is easy to see that the FOC (20) could be violated for some promised utility $V_t^{\hat{e}}$ with $c_t^{\hat{e}} > c_{min}$. This result leads to the following proposition.

Proposition 5.7 *When $c_t^{\hat{e}} > c_{min}$, the UI program corresponding to the optimal UI contract is not fully funded.*

6 Conclusion

This paper studies the optimal UI contract by relaxing the universal benefits as assumed in the standard literature. In our model, workers have to earn their UI entitlement and the benefit does not last forever. To be realistic, we focus on an environment where the UI agency cannot perfect monitor job quits and rejections, upon which the workers might lose their UI entitlement with a positive probability. The detailed rules of how workers earn and lose their UI entitlement nicely generate entitlement effect, which shapes the term of the optimal UI contract differently from the findings in the literature using the dynamic contracting approach. Particularly, we find that even with a large value of effort at work, the loophole emphasized in Hopenhayn and Nicolini (2009) does not necessarily exist in our model. This difference is due to the entitlement effec; in our model, the workers are concerned with losing the attractive UI eligibility, which mitigates the moral hazard quits, and thus, fixes the loophole in the optimal UI contract.

Secondly, we find that if the constraint that makes the UI eligibility valuable binds, and the constraint that rules out the moral hazard quits binds, then the optimal UI contract generates differentiated UI fee scheme between eligible and ineligible workers. Again, the gap in the UI fees is driven by the entitlement effect; the desire of gaining UI eligibility allows the UI agency to charge the UI-ineligibles higher fees. This result contrasts with the monotonicity of the UI fees in the literature, but is supported by the empirical evidence.

For the purpose of tractability, the presented model abstracts from many important factors that could also influence a worker's incentive to work. For example, the worker in our model is not allowed to save, and the firm is simply a passive agent and offers a constant wage.[20] Qualitatively, these assumptions do not affect the results highlighted in this contribution due to the presence of the entitlement effect. However, if one pursues a quantitative question, these assumptions need to be released. Thus, one possible direction for future research might be to endogenize the wage determination to explore the effect of the UI eligibility rule on the firm's job creation decisions, which, in turn, affects the worker's labor supply decisions. By doing so, one can conduct a social welfare analysis in a general equilibrium environment.

7 Appendix

7.1 Discussions of FOCs

We assume that all state variables V_t^i, except for $V_t^{\hat{u}}$, are larger than V_{min}. So, $V_t^i > V_{min}$ $\forall\ i \in \{e, \hat{e}, u\}$ unless stated clearly.[21]

7.1.1 Eligible Employed Workers

Consider the FOCs of eligible employed workers (9) - (12), and we focus on the case $\pi > s$, that is, the UI agency have a relative weak monitoring power.[22]

- If $\mu_{2,t} > 0$, i.e. $(1-s)V_{t+1}^{ee} = (\pi - s)V_{t+1}^{ue} + (1-\pi)V_{t+1}^{\hat{u}e}$, then by Eq. (12), one has $\mu_{5,t} > 0$, i.e. $V_{t+1}^{\hat{u}e} = V_{min}$.
  - If $\mu_{4,t} = 0$, i.e. $V_{t+1}^{ue} > V_{min}$, then the binding (No-Quit) constraint implies that $\mu_{3,t} = 0$, i.e. $V_{t+1}^{ee} > V_{min}$. In addition, suggested by Eqs. (9), (10) and (11), one has
  $$C'(V_{t+1}^{ue}) < W'(V_t^e) < W'(V_{t+1}^{ee}). \tag{41}$$
  - (rule out) If $\mu_{4,t} > 0$, i.e. $V_{t+1}^{ue} = V_{min}$, then by the binding (No-Quit) constraint, $V_{t+1}^{ee} = V_{min}$, i.e. $\mu_{3,t} > 0$. Suggested by Eqs. (9) and (10), one has $W'(V_{min}) = W'(V_{t+1}^{ee}) > W'(V_t^e) > W'(V_{min})$, which is a contradiction.

- If $\mu_{2,t} = 0$, i.e. $(1-s)V_{t+1}^{ee} > (\pi - s)V_{t+1}^{ue} + (1-\pi)V_{t+1}^{\hat{u}e}$, then by Eq. (12), one has $\mu_{5,t} = 0$, i.e. $V_{t+1}^{\hat{u}e} > V_{min}$. In turn, it suggests that $V_{t+1}^{ee} > V_{min}$, i.e. $\mu_{3,t} = 0$.

[20] Or equivalently, hidden saving is not allowed.

[21] Since V_{min} is the lower bound, the first order derivatives of all cost functions $W(\cdot)$, $\hat{W}(\cdot)$, $C(\cdot)$, $\hat{C}(\cdot)$ taken at V_{min} are not well defined. For the purpose of analysis of the FOCs, we extend all the cost functions linearly to the set of $(-\infty, V_{min})$, so the derivatives of the cost functions at V_{min} are equal to their right derivatives.

[22] This is a reasonable assumption given that the exogenous separation shock is typically small in reality as documented in Shimer (2005).

Therefore, by Eqs. (9) and (10), one has

$$W'(V_t^e) = W'(V_{t+1}^{ee}). \tag{42}$$

- If $\mu_{4,t} = 0$, i.e. $V_{t+1}^{ue} > V_{min}$, then by Eqs. (9) and (11), one has

$$C'(V_{t+1}^{ue}) = W'(V_t^e). \tag{43}$$

- (rule out) If $\mu_{4,t} > 0$, i.e. $V_{t+1}^{ue} = V_{min}$. By Eq. (42) and the convexity of $W(\cdot)$, one has $V_{t+1}^{ee} = V_t^e$. Then Eq. (3) can be rewritten as:

$$u(c_t^e) - m = (1-\beta(1-s))V_t^e - \beta s V_{min}.$$

  Since $V_t^e > V_{min}$, the RHS of the above equation has

$$(1-\beta(1-s))V_t^e - \beta s V_{min} > (1-\beta)\, V_{min} = u\left(c_{\min}\right).$$

  Hence, it has $u(c_t^e) - m > u\left(c_{\min}\right)$. Given that $m > 0$, and $u(\cdot)$ is monotone increasing, it has $c_t^e > c_{min}$. Therefore, $\mu_{1,t} = 0$. In addition, by strict concavity of $u(\cdot)$, there exists some $\epsilon_0 > 0$ such that $\frac{1}{u'(c_t^e)} > \frac{1}{u'(c_{min})} + \epsilon_0$. By Eq. (11), one has

$$C'(V_{t+1}^{ue}) = C'(V_{min}) = \frac{1}{u'(c_t^e)} + \frac{\mu_{4,t}}{\beta s} > \frac{1}{u'(c_{min})} + \epsilon_0 + \frac{\mu_{4,t}}{\beta s}.$$

  On the other hand, consider $V_t^u = V_{min} + \rho$, where ρ is positive but sufficiently small. By Eq. (31), one has

$$C'(V_t^u) = C'(V_{min} + \rho) = \frac{1-\eta_{1,t}}{u'(c_t^u)}.$$

  As $\rho \to 0$, by the Maximum theorem, $c_t^u \to c_{min}$. It is easy to see that there always exists some ρ_0 such that for $\rho < \rho_0$, one has $C'(V_t^u) = \frac{1-\eta_{1,t}}{u(c_t^u)} < \frac{1}{u'(c_{min})} + \epsilon_0 + \frac{\mu_{4,t}}{\beta s}$, which leads to a contradiction.

Lemma 7.1 *If the (No-Quit) constraint (5) for a UI-eligible employed worker with* $V_t^e > V_{min}$ *is absent, then* $C'(V_{t+1}^{ue}) = W'(V_t^e) = W'(V_{t+1}^{ee})$ *and* $c_t^e = c_{t+1}^{ee} = c_{t+1}^{ue}$.

Proof The result of $C'(V_{t+1}^{ue}) = W'(V_t^e) = W'(V_{t+1}^{ee})$ follows directly from the above discussions of the FOCs for the eligible employed workers. Next, we prove $c_t^e = c_{t+1}^{ee} = c_{t+1}^{ue}$.

By Eq. (9), one has $\dfrac{1-\mu_{1,t}}{u'(c_t^e)} = \dfrac{1-\mu_{1,t+1}}{u'(c_{t+1}^{ee})}$. If $\mu_{1,t} = \mu_{1,t+1} \geq 0$, one has $c_t^e = c_{t+1}^{ee}$. If $\mu_{1,t} = 0$ and $\mu_{1,t+1} > 0$, i.e. $c_t^e > c_{\min} = c_{t+1}^{ee}$, then $\dfrac{1-\mu_{1,t}}{u'(c_t^e)} = \dfrac{1-\mu_{1,t+1}}{u'(c_{t+1}^{ee})}$ suggests

$c_t^e < c_{t+1}^{ee}$, which leads to a contradiction. If $\mu_{1,t} > 0$ and $\mu_{1,t+1} = 0$, i.e. $c_t^e = c_{\min} < c_{t+1}^{ee}$, then $\dfrac{1-\mu_{1,t}}{u'(c_t^e)} = \dfrac{1-\mu_{1,t+1}}{u'(c_{t+1}^{ee})}$ suggests $c_t^e > c_{t+1}^{ee}$, which also leads to a contradiction. So, $c_t^e = c_{t+1}^{ee}$.

By using the same reasoning, one can prove $c_t^e = c_{t+1}^{ue}$. $\square$

When the (No-Quit) constraint is relaxed or is slack, the choice of $V_{t+1}^{\hat{u}e}$ is only subjects to the regularity constraint. Thus, the purpose of minimizing the overall cost of UI implies that it is optimal to set $V_{t+1}^{\hat{u}e} = V_{min}$.

Lemma 7.2 *If the (No-Quit) constraint (5) for a UI-eligible employed worker with $V_t^e > V_{min}$ is absent or is imposed but is not binding, then $V_{t+1}^{\hat{u}e} = V_{min}$.*

Lemma 7.3 *If the (No-Quit) constraint (5) for a UI-eligible employed worker with $V_t^e > V_{min}$ binds, then $V_t^e < V_{t+1}^{ee}$.*

Proof Since the (No-Quit) constraint (5) binds ($\mu_{2,t} > 0$), by Eqs. (9) and (10), $W'(V_t^e) < W'(V_{t+1}^{ee})$. The convexity of $W(\cdot)$ implies that $V_t^e < V_{t+1}^{ee}$. $\square$

Lemma 7.4 *Given $\pi > s$, if the (No-Quit) constraint (5) for a UI-eligible employed worker with $V_t^e > V_{min}$ binds in period t, then it would bind in period $t+1$.*

Proof Suppose by the way of contradiction that the (No-Quit) constraint binds in period t, but it does not in period $t+1$, i.e., $\mu_{2,t} > \mu_{2,t+1} = 0$, then one has

$$(1-s)\,V_{t+1}^{ee} + sV_{t+1}^{ue} = \pi V_{t+1}^{ue} + (1-\pi)\,V_{t+1}^{\hat{u}e}. \tag{44}$$

$$(1-s)\,V_{t+2}^{eee} + sV_{t+2}^{uee} > \pi V_{t+2}^{uee} + (1-\pi)\,V_{t+2}^{\hat{u}ee}. \tag{45}$$

By the discussion of FOCs in Appendix 7.1.1, one has

$$C'\left(V_{t+1}^{ue}\right) < W'\left(V_t^e\right) < W'\left(V_{t+1}^{ee}\right). \tag{46}$$

$$C'\left(V_{t+2}^{uee}\right) = W'\left(V_{t+1}^{ee}\right) = W'\left(V_{t+2}^{eee}\right). \tag{47}$$

By the convexity of $W(\cdot)$ and $C(\cdot)$, the inequality (46) and Eq. (47) suggest that

$$V_t^e < V_{t+1}^{ee} = V_{t+2}^{eee}, \text{ and } V_{t+1}^{ue} < V_{t+2}^{uee}.$$

Subtracting Eq. (45) from Eq. (44) and combining with $V_{t+1}^{ee} = V_{t+2}^{eee}$ gives

$$(\pi - s)\left(V_{t+1}^{ue} - V_{t+2}^{uee}\right) > (1-\pi)\left(V_{t+2}^{\hat{u}ee} - V_{t+1}^{\hat{u}e}\right).$$

Since $\mu_{2,t} > 0$, by Eq. (12), one has $\mu_{5,t} > 0$, which implies that $V_{t+1}^{\hat{u}e} = V_{min}$. Therefore, $V_{t+2}^{\hat{u}ee} - V_{t+1}^{\hat{u}e} \geq 0$. In addition, $\pi > s$ implies that $V_{t+1}^{ue} > V_{t+2}^{uee}$. Hence, a contradiction arises. $\square$

7.1.2 Ineligible Employed Workers

- When $\phi_{2,t} = 0$, i.e. $gV_{t+1}^{e\hat{e}} + (1-g)\,V_{t+1}^{\hat{e}\hat{e}} > V_{t+1}^{\hat{u}\hat{e}}$:
  - If $\phi_{3,t} > 0$, i.e.$V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}}$, then the (No-Quit) constraint (16) can be written as follows,
    $$gV_{t+1}^{e\hat{e}} + (1-g)\,V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}} > V_{t+1}^{\hat{u}\hat{e}}.$$
    The regularity constraint (19) requires that $V_{t+1}^{\hat{u}\hat{e}} \geq V_{min}$. Thus, one has $V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}} > V_{min}$, i.e. $\phi_{4,t} = \phi_{5,t} = 0$. Therefore, Eqs. (20), (21) and (23) suggest that
    $$\hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) < \hat{W}'(V_t^{\hat{e}}) < W'(V_{t+1}^{e\hat{e}}).$$
  - If $\phi_{3,t} = 0$, i.e. $V_{t+1}^{e\hat{e}} > V_{t+1}^{\hat{e}\hat{e}}$, then
    * (rule out) If $\phi_{4,t} > 0$, i.e. $V_{t+1}^{\hat{e}\hat{e}} = V_{min}$, then by Eqs. (20) and (21), one has
      $$\hat{W}'(V_{min}) = \hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) > \hat{W}'(V_t^{\hat{e}}) \geq \hat{W}'(V_{min}),$$
      which is a contradiction.
    * If $\phi_{4,t} = 0$, i.e.$V_{t+1}^{\hat{e}\hat{e}} > V_{min}$, then by Eqs. (20) (21) and (23), one has
      $$\hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) = \hat{W}'(V_t^{\hat{e}}) = W'(V_{t+1}^{e\hat{e}}).$$
- When $\phi_{2,t} > 0$, i.e. $gV_{t+1}^{e\hat{e}} + (1-g)\,V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}}$.
  - if $\phi_{3,t} > 0$, i.e. $V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}}$, then $V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}}$. Thus, one has either $\phi_{4,t}, \phi_{5,t} > 0$ or $\phi_{4,t} = \phi_{5,t} = 0$.
    * If $\phi_{4,t} = \phi_{5,t} = 0$, i.e. $V_{t+1}^{\hat{e}\hat{e}}, V_{t+1}^{\hat{u}\hat{e}} > V_{min}$, then one has $V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}} > V_{min}$.
      By Eq. (59) as derived in Appendix 7.1.4 and $V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}}$, one has
      $$\hat{C}'(V_{t+1}^{\hat{u}\hat{e}}) \geq \hat{W}'(V_{t+1}^{\hat{u}\hat{e}} + \frac{1}{\beta f}) > \hat{W}'(V_{t+1}^{\hat{u}\hat{e}}) = \hat{W}'(V_{t+1}^{\hat{e}\hat{e}}).$$
      By Eqs. (20) and (22), one has
      $$\hat{C}'(V_{t+1}^{\hat{u}\hat{e}}) < \hat{W}'(V_t^{\hat{e}}).$$
      Therefore, one has
      $$\hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) < \hat{W}'(V_t^{\hat{e}}).$$
    * If $\phi_{4,t} > 0$, $\phi_{5,t} > 0$, i.e. $V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}} = V_{min}$, then $V_{t+1}^{e\hat{e}} = V_{t+1}^{\hat{e}\hat{e}} = V_{t+1}^{\hat{u}\hat{e}} = V_{min} < V_t^{\hat{e}}$. Since $\hat{W}(\cdot)$ is monotone increasing, one has
      $$\hat{W}'(V_{t+1}^{\hat{e}\hat{e}}) < \hat{W}'(V_t^{\hat{e}}).$$

Therefore, when $\phi_{2,t} > 0$ and $\phi_{3,t} > 0$, one has

$$\hat{W}'(V^{\hat{e}\hat{e}}_{t+1}) < \hat{W}'(V^{\hat{e}}_t).$$

– (rule out) If $\phi_{3,t} = 0$, i.e. $V^{e\hat{e}}_{t+1} > V^{\hat{e}\hat{e}}_{t+1}$, then one has $\phi_{4,t} = 0$, i.e. $V^{\hat{e}\hat{e}}_{t+1} > V_{min}$.[23] Therefore, $V^{e\hat{e}}_{t+1} > V^{\hat{e}\hat{e}}_{t+1} > V_{min}$. In addition, $gV^{e\hat{e}}_{t+1} + (1-g)V^{\hat{e}\hat{e}}_{t+1} = V^{\hat{u}\hat{e}}_{t+1}$ and $V^{e\hat{e}}_{t+1} > V^{\hat{e}\hat{e}}_{t+1}$ imply $V^{\hat{u}\hat{e}}_{t+1} > V^{\hat{e}\hat{e}}_{t+1} > V_{min}$, i.e. $\phi_{5,t} = 0$.
Thus, Eqs. (20), (21) and (22) suggest that

$$\hat{W}'(V^{\hat{e}\hat{e}}_{t+1}) > \hat{W}'(V^{\hat{e}}_t) > \hat{C}'(V^{\hat{u}\hat{e}}_{t+1}).$$

However, by Eq. (59) and $V^{\hat{u}\hat{e}}_{t+1} > V^{\hat{e}\hat{e}}_{t+1}$, one has $\hat{C}'(V^{\hat{u}\hat{e}}_{t+1}) \geq \hat{W}'(V^{\hat{u}\hat{e}}_{t+1} + \frac{1}{\beta f}) > \hat{W}'(V^{\hat{u}\hat{e}}_{t+1}) > \hat{W}'(V^{\hat{e}\hat{e}}_{t+1})$, which leads to a contradiction.

Lemma 7.5 *If the (Valuable-UI) constraint (17) of a UI-ineligible employed worker with $V^{\hat{e}}_t > V_{min}$ binds, then $\hat{W}'(V^{\hat{e}\hat{e}}_{t+1}) < \hat{W}'(V^{\hat{e}}_t) < W'(V^{e\hat{e}}_{t+1})$.*

Proof The conclusion of $\hat{W}'(V^{\hat{e}\hat{e}}_{t+1}) < \hat{W}'(V^{\hat{e}}_t)$ directly follows the above discussion of FOCs for the ineligible employed workers. Since the (Valuable-UI) constraint (17) binds, one has $\phi_{3,t} > 0$. By (20) and (23), it follows that $\hat{W}'(V^{\hat{e}}_t) < W'(V^{e\hat{e}}_{t+1})$. □

Lemma 7.6 *If the (Valuable-UI) constraint (17) of a UI-ineligible employed worker with $V^{\hat{e}}_t > V_{min}$ binds in period t, then it would bind in period $t+1$.*

Proof Consider a worker who is employed and ineligible for UI in period t and remains the same employment and UI eligibility status in period $t+1$. Suppose that the (Valuable-UI) constraint (17) binds in period t, i.e. $V^{e\hat{e}}_{t+1} = V^{\hat{e}\hat{e}}_{t+1}$, $\phi_{3,t} > 0$, but it does not bind in period $t+1$, i.e. $V^{e\hat{e}\hat{e}}_{t+2} > V^{\hat{e}\hat{e}\hat{e}}_{t+2}$, $\phi_{3,t+1} = 0$.

Since $\phi_{3,t} > 0$, by Lemma 7.5, one has

$$\hat{W}'\left(V^{\hat{e}\hat{e}}_{t+1}\right) < \hat{W}'\left(V^{\hat{e}}_t\right) < W'\left(V^{e\hat{e}}_{t+1}\right). \tag{48}$$

By the discussion of the FOCs in Appendix 7.1.2, $\phi_{3,t+1} = 0$ implies that $\phi_{2,t+1} = 0$ and $\phi_{4,t+1} = 0$. So,

$$\hat{W}'\left(V^{\hat{e}\hat{e}\hat{e}}_{t+2}\right) = \hat{W}'\left(V^{\hat{e}\hat{e}}_{t+1}\right) = W'\left(V^{e\hat{e}\hat{e}}_{t+2}\right). \tag{49}$$

By inequality (48) and Eq. (49), one gets

$$W'(V^{e\hat{e}\hat{e}}_{t+2}) = \hat{W}'\left(V^{\hat{e}\hat{e}\hat{e}}_{t+2}\right) = \hat{W}'\left(V^{\hat{e}\hat{e}}_{t+1}\right) < \hat{W}'\left(V^{\hat{e}}_t\right) < W'\left(V^{e\hat{e}}_{t+1}\right). \tag{50}$$

Therefore, by the convexity of $W(\cdot)$ and $\hat{W}(\cdot)$, one has $V^{\hat{e}}_t > V^{\hat{e}\hat{e}}_{t+1} = V^{\hat{e}\hat{e}\hat{e}}_{t+2}$, and $V^{e\hat{e}}_{t+1} > V^{e\hat{e}\hat{e}}_{t+2}$. However, $\phi_{3,t} > 0$ and $\phi_{3,t+1} = 0$ imply that $V^{e\hat{e}}_{t+1} = V^{\hat{e}\hat{e}}_{t+1}$ and $V^{\hat{e}\hat{e}\hat{e}}_{t+1} < V^{e\hat{e}\hat{e}}_{t+2}$. So, one has $V^{e\hat{e}}_{t+1} = V^{\hat{e}\hat{e}}_{t+1} = V^{\hat{e}\hat{e}\hat{e}}_{t+2} < V^{e\hat{e}\hat{e}}_{t+2}$, which leads to a contradiction. □

[23] Suppose by way of contradiction $\phi_{4,t} > 0$, i.e. $V^{\hat{e}\hat{e}}_{t+1} = V_{min}$, then by Eqs. (20) and (21), one has $\hat{W}'(V_{min}) = \hat{W}'(V^{\hat{e}\hat{e}}_{t+1}) > \hat{W}'(V^{\hat{e}}_t) > \hat{W}'(V_{min})$, which is a contradiction.

7.1.3 Eligible Unemployed Workers:

- Either when $\eta_{2,t} > 0$, i.e. $V_{t+1}^{eu} = \pi V_{t+1}^{uu} + (1-\pi)V_{t+1}^{\hat{u}u}$, or when $\eta_{3,t} > 0$, i.e. $\beta f(V_{t+1}^{eu} - (1-d)V_{t+1}^{uu} - dV_{t+1}^{\hat{u}u}) = 1$, or both hold, by Eqs. (31) and (32), one has

$$C'(V_t^u) < W'(V_{t+1}^{eu}).$$

  - If $\eta_{4,t} = 0$, i.e. $V_{t+1}^{uu} > V_{min}$, by Eqs. (33) and (31), one has

$$C'(V_{t+1}^{uu}) < C'(V_t^u).$$

  - If $\eta_{4,t} > 0$, i.e. $V_{t+1}^{uu} = V_{min}$, then $V_{t+1}^{uu} = V_{min} < V_t^u$. That is,

$$C'(V_{t+1}^{uu}) < C'(V_t^u).$$

  As a summary,

$$C'(V_{t+1}^{uu}) < C'(V_t^u) < W'(V_{t+1}^{eu}). \tag{51}$$

- If $\eta_{2,t} = \eta_{3,t} = 0$, i.e. $V_{t+1}^{eu} > \pi V_{t+1}^{uu} + (1-\pi)\, V_{t+1}^{\hat{u}u}$, and $\beta f\left(V_{t+1}^{eu} - dV_{t+1}^{\hat{u}u} - (1-d)\, V_{t+1}^{uu}\right) > 1$, then by Eqs. (31) and (32), one has

$$C'(V_t^u) = W'(V_{t+1}^{eu}).$$

  - If $\eta_{4,t} = 0$, i.e. $V_{t+1}^{uu} > V_{min}$, then by Eqs. (31) and (33), one has

$$C'(V_{t+1}^{uu}) = C'(V_t^u) = W'(V_{t+1}^{eu}).$$

  - (rule out) If $\eta_{4,t} > 0$, i.e. $V_{t+1}^{uu} = V_{min}$, then by Eqs. (31) and (33), one has

$$C'(V_{min}) = C'(V_{t+1}^{uu}) > C'(V_t^u),$$

    which is a contradiction.

Lemma 7.7 *If the (Search-Incentive) constraint (28) for a UI-eligible unemployed worker with $V_t^u > V_{min}$ binds, then $C'(V_{t+1}^{uu}) < C'(V_t^u) < W'(V_{t+1}^{eu})$.*

Proof The conclusion follows directly from the discussion of FOCs for the eligible unemployed workers. □

Lemma 7.8 *For a UI-eligible unemployed worker with $V_t^u = V_{min}$, no matter whether the (No-Rejection) constraint (27) is imposed, one has $c_t^u = c_{min}$, $V_{t+1}^{\hat{u}u} = V_{t+1}^{uu} = V_{min}$, and the (Search-Incentive) constraint binds in period t.*

Proof Given that $V_t^u = V_{min}$, the imposed (Search-Incentive) constraint (28) and the promise-keeping constraint (25) imply that

$$V_{min} = V_t^u \geq u(c_t^u) + \beta(dV_{t+1}^{\hat{u}u} + (1-d)V_{t+1}^{uu}). \tag{52}$$

Note that $V_{min} = c_{min} + \beta V_{min}$. In addition, one has the following constraints hold, $c_t^u \geq c_{min}$, $V_{t+1}^{uu} \geq V_{min}$, $V_{t+1}^{\hat{u}u} \geq V_{min}$, and the (Search-Incentive) constraint (28). If one of these above holds as a strict inequality, then one has $V_{min} > V_{min}$, which is a contradiction. Therefore, $c_t^u = c_{min}$, $V_{t+1}^{\hat{u}u} = V_{t+1}^{uu} = V_{min}$, and the (Search-Incentive) constraint binds. □

Lemma 7.9 *In the absence of the (No-Rejection) constraint (27), if the (Search-Incentive) constraint (28) for a UI-eligible unemployed worker with $V_t^u > V_{min}$ binds in period t, then it would bind in period $t+1$.*

Proof Supposed by the way of contradiction that the (Search-Incentive) constraint (28) binds in period t, but does not bind in period $t+1$, i.e. $\eta_{3,t} > 0$ and $\eta_{3,t+1} = 0$. By Lemma 7.8, one has $V_{t+1}^{uu} > V_{min}$, i.e. $\eta_{4,t} = 0$. Eqs. (31)-(33) suggest that $C'(V_{t+1}^{uu}) < W'(V_{t+1}^{eu})$ and $C'(V_{t+2}^{uuu}) \geq C'(V_{t+1}^{uu}) = W'(V_{t+2}^{euu})$. Therefore, one has $V_{t+2}^{uuu} \geq V_{t+1}^{uu}$ and $V_{t+1}^{eu} > V_{t+2}^{euu}$.

Claim that $V_{t+1}^{uu} = V_{t+2}^{uuu}$. It is equivalent to show that $V_{t+2}^{uuu} > V_{min}$, i.e. $\eta_{4,t+1} = 0$. Suppose by the way of contradiction $\eta_{4,t+1} > 0$, then one has $V_{t+2}^{uuu} = V_{min}$. Since the (No-Rejection) constraint is absent, one has $\eta_{2,t+1} = 0$, by the Eqs. (31) and (33), one has $C'(V_{t+2}^{uuu}) > C'(V_{t+1}^{uu})$, which gives $V_{t+2}^{uuu} = V_{min} > V_{t+1}^{uu}$. So, a contradiction arises. Thus, one has $\eta_{4,t+1} = 0$, and $V_{t+1}^{uu} = V_{t+2}^{uuu} > V_{min}$.

Claim that $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu}$. Since the (No-Rejection) constraint (27) is absent in period t, one has $\eta_{2,t} = 0$.

If $V_{t+1}^{\hat{u}u} > V_{min}$, and $V_{t+2}^{\hat{u}uu} > V_{min}$, i.e. $\eta_{5,t} = \eta_{5,t+1} = 0$, then $\hat{C}'(V_{t+1}^{\hat{u}u}) = C'(V_{t+1}^{uu}) = C'(V_{t+2}^{uuu}) = \hat{C}'(V_{t+2}^{\hat{u}uu})$, which implies that $\hat{C}'(V_{t+1}^{\hat{u}u}) = \hat{C}'(V_{t+2}^{\hat{u}uu})$. By convexity of $\hat{C}(\cdot)$, one has $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu}$.

If $V_{t+1}^{\hat{u}u} = V_{min}$, i.e. $\eta_{5,t} > 0$, then it must have $V_{t+2}^{\hat{u}uu} = V_{min}$. To prove this, suppose by the way of contradiction that $V_{t+2}^{\hat{u}uu} > V_{min}$, i.e. $\eta_{5,t+1} = 0$, then $\hat{C}'(V_{t+1}^{\hat{u}u}) > C'(V_{t+1}^{uu}) = C'(V_{t+2}^{uuu}) = \hat{C}'(V_{t+2}^{\hat{u}uu})$, which leads to a contradiction.

If $V_{t+2}^{\hat{u}uu} = V_{min}$, i.e. $\eta_{5,t+1} > 0$, then it must have $V_{t+1}^{\hat{u}u} = V_{min}$. To see this, suppose by the way of contradiction that $V_{t+1}^{\hat{u}u} > V_{min}$, i.e. $\eta_{5,t} = 0$, then $\hat{C}'(V_{t+1}^{\hat{u}u}) = C'(V_{t+1}^{uu}) = C'(V_{t+2}^{uuu}) < \hat{C}'(V_{t+2}^{\hat{u}uu})$, which leads to a contradiction.

Plugging $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu}$, $V_{t+2}^{uuu} \geq V_{t+1}^{uu}$ and $V_{t+1}^{eu} > V_{t+2}^{euu}$ into the (Search-Incentive) constraint in period t gives

$$1 = \beta f(V_{t+1}^{eu} - (1-d)V_{t+1}^{uu} - dV_{t+1}^{\hat{u}u}) > \beta f(V_{t+2}^{euu} - (1-d)V_{t+2}^{uuu} - dV_{t+2}^{\hat{u}uu}).$$

So, a contradiction arises for the (Search-Incentive) constraint in period $t+1$. $\square$

Lemma 7.10 *In the absence of the (No-Rejection) constraint (27), if the (Search-Incentive) constraint (28) for a UI-eligible unemployed worker with $V_t^u > V_{min}$ binds, then $V_{t+2}^{\hat{u}uu} \leq V_{t+1}^{\hat{u}u}$.*

Proof If $V_{t+1}^{uu} = V_{min}$, then by Lemma 7.8, $V_{t+2}^{\hat{u}uu} = V_{min}$. Therefore, $V_{t+2}^{\hat{u}uu} = V_{min} \leq V_{t+1}^{\hat{u}u}$. Proof ends.

If $V_{t+1}^{uu} > V_{min}$, i.e. $\eta_{4,t} = 0$, by the discussion of the FOCs in Appendix 7.1.3, one has

$$C'(V_{t+2}^{uuu}) < C'(V_{t+1}^{uu}). \tag{53}$$

We consider the following two cases:

Case 1: $V_{t+1}^{\hat{u}u} > V_{min}$, i.e. $\eta_{5,t} = 0$. By Eqs. (33) and (34), one has $\hat{C}'(V_{t+1}^{\hat{u}u}) = C'(V_{t+1}^{uu})$. Claim that $\hat{C}'(V_{t+1}^{\hat{u}u}) \geq \hat{C}'(V_{t+2}^{\hat{u}uu})$. To see this, if $V_{t+2}^{\hat{u}uu} > V_{min}$, i.e. $\eta_{5,t+1} = 0$, then by Eqs. (33), (34) and (53), one has $\hat{C}'(V_{t+2}^{\hat{u}uu}) \leq C'(V_{t+2}^{uuu}) < C'(V_{t+1}^{uu}) = \hat{C}'(V_{t+1}^{\hat{u}u})$. Thus, by the convexity of $\hat{C}(\cdot)$, one has $V_{t+2}^{\hat{u}uu} < V_{t+1}^{\hat{u}u}$. If $V_{t+2}^{\hat{u}uu} = V_{min}$, then $V_{t+2}^{\hat{u}uu} < V_{t+1}^{\hat{u}u}$.

Case 2: $V_{t+1}^{\hat{u}u} = V_{min}$, i.e. $\eta_{5,t} > 0$. Claim that $V_{t+2}^{\hat{u}uu} = V_{min}$. Suppose not, then $V_{t+2}^{\hat{u}uu} > V_{t+1}^{\hat{u}u} = V_{min}$, i.e. $\eta_{5,t+1} = 0$. Thus, by Eqs. (33), (34) and (53), one has $\hat{C}'(V_{t+1}^{\hat{u}u}) > C'(V_{t+1}^{uu}) > C'(V_{t+2}^{uuu}) \geq \hat{C}'(V_{t+2}^{\hat{u}uu})$, which is a contradiction. Thus, $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu} = V_{min}$. $\square$

Lemma 7.11 *When $d+\pi > 1$, for a UI-eligible unemployed worker with $V_t^u < \dfrac{1-d}{f(d+\pi-1)} + V_{min}$, the (No-Rejection) constraint (27) is slack.*

Proof Suppose by the way of contradiction that the (No-Rejection) constraint binds in period t. Since the (Search-Incentive) constraint (28) is imposed in all periods, then in period t, one has

$$V_{t+1}^{eu} - dV_{t+1}^{\hat{u}u} - (1-d)V_{t+1}^{uu} \geq \frac{1}{\beta f}. \tag{54}$$

$$V_{t+1}^{eu} - (1-\pi)V_{t+1}^{\hat{u}u} - \pi V_{t+1}^{uu} = 0. \tag{55}$$

Subtracting Eq. (55) from Eq. (54) and combining with $d+\pi > 1$ give

$$V_{t+1}^{uu} - V_{t+1}^{\hat{u}u} \geq \frac{1}{\beta f(d+\pi-1)}.$$

By Eq. (25), one has

$$\begin{aligned} V_t^u &\geq u(c_t^u) + \beta(dV_{t+1}^{\hat{u}u} + (1-d)V_{t+1}^{uu}). \\ &= u(c_t^u) + \beta V_{t+1}^{\hat{u}u} + \beta(1-d)(V_{t+1}^{uu} - V_{t+1}^{\hat{u}u}). \\ &= u(c_t^u) + \beta V_{t+1}^{\hat{u}u} + \frac{(1-d)}{f(d+\pi-1)}. \\ &\geq \frac{(1-d)}{f(d+\pi-1)} + V_{min}, \end{aligned}$$

where the last inequality comes from the budget constraint $c_t^u \geq c_{min}$, the regularity constraint $V_{t+1}^{\hat{u}u} \geq V_{t+1}^{\hat{u}u}$ and the equation $V_{min} = u(c_{min}) + \beta V_{min}$. Since $V_t^u < \dfrac{1-d}{f(d+\pi-1)} + V_{min}$, a contradiction arises. □

Lemma 7.12 *When* $d+\pi > 1$, *for a UI-eligible unemployed worker with* $V_t^u < \dfrac{1-d}{f(d+\pi-1)} + V_{min}$, *if the (Search-Incentive) constraint (28) binds in period* t, *then it would bind in period* $t+1$.

Proof Since the (Search-Incentive) constraint binds in period t, by the discussion of FOCs in Appendix 7.1.3, one has $C'(V_{t+1}^{uu}) < C'(V_t^u) < W'(V_{t+1}^{eu})$. Suppose by the way of contradiction that the (Search-Incentive) constraint does not bind in period $t+1$, i.e. $\eta_{3,t+1} = 0$. By Lemma 7.8, one has $V_{t+1}^{uu} > V_{min}$, i.e. $\eta_{4,t} = 0$. By Lemma 7.11, with the (Search-Incentive) constraint (28) imposed, the (No-Rejection) constraint is slack for all t. So, $\eta_{2,t+1} = 0$.

Claim that $V_{t+2}^{uuu} > V_{min}$, i.e. $\eta_{4,t+1} = 0$. Suppose not, that is $V_{t+2}^{uuu} = V_{min}$, i.e. $\eta_{4,t+1} > 0$, then by Eqs. (31)–(33), one has $C'(V_{t+2}^{uuu}) > C'(V_{t+1}^{uu})$, which implies that $V_{min} = V_{t+2}^{uuu} > V_{t+1}^{uu}$. So, a contradiction arises.

Thus, by Eqs. (31)–(33), one has $C'(V_{t+2}^{uuu}) = C'(V_{t+1}^{uu}) = W'(V_{t+2}^{euu})$. Therefore, one has $V_{t+2}^{euu} < V_{t+1}^{eu}$ and $V_{t+1}^{uu} = V_{t+2}^{uuu}$.

Claim that $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu}$.

If $\eta_{5,t+1} > 0$, i.e. $V_{t+2}^{\hat{u}uu} = V_{min}$. It must be true that $V_{t+1}^{\hat{u}u} = V_{min}$, i.e. $\eta_{5,t} > 0$. Suppose by the way of contradiction $V_{t+1}^{\hat{u}u} > V_{min}$, i.e. $\eta_{5,t} = 0$, by Eqs. (33) and (34), one has $\hat{C}'(V_{t+1}^{\hat{u}u}) = C'(V_{t+1}^{uu})$. Therefore, $\hat{C}'(V_{t+2}^{\hat{u}uu}) > C'(V_{t+2}^{uuu}) = C'(V_{t+1}^{uu}) = \hat{C}'(V_{t+1}^{\hat{u}u})$. The concavity of $\hat{C}(\cdot)$ implies that $V_{min} = V_{t+2}^{\hat{u}uu} > V_{t+1}^{\hat{u}u}$, which is a contradiction. Thus, $\eta_{5,t} > 0$ and $V_{t+1}^{\hat{u}u} = V_{t+2}^{\hat{u}uu} = V_{min}$.

If $\eta_{5,t+1} = 0$, i.e. $V_{t+2}^{\hat{u}uu} > V_{min}$. By Eqs. (33) and (34), one has $\hat{C}'(V_{t+2}^{\hat{u}uu}) = C'(V_{t+2}^{uuu}) = C'(V_{t+1}^{uu}) \leq \hat{C}'(V_{t+1}^{\hat{u}u})$, which implies that $V_{t+2}^{\hat{u}uu} < V_{t+1}^{\hat{u}u}$ holds if and only if $\eta_{5,t} > 0$, i.e. $V_{t+1}^{\hat{u}u} = V_{min}$. It is easy to rule out this case given $V_{t+2}^{\hat{u}uu} > V_{min}$. Thus, one has $V_{t+2}^{\hat{u}uu} = V_{t+1}^{\hat{u}u} > V_{min}$.

Plugging $V^{eu}_{t+1} > V^{euu}_{t+2}$, $V^{uu}_{t+1} = V^{uuu}_{t+2}$ and $V^{\hat{u}u}_{t+1} = V^{\hat{u}uu}_{t+2}$ into the (Search-Incentive) constraint in period t gives

$$1 = \beta f(V^{eu}_{t+1} - (1-d)V^{uu}_{t+1} - dV^{\hat{u}u}_{t+1}) > \beta f(V^{euu}_{t+2} - (1-d)V^{uuu}_{t+2} - dV^{\hat{u}uu}_{t+2}),$$

which violates the (Search-Incentive) constraint (28) in period $t+1$. A contradiction arises and this completes the proof. □

7.1.4 Ineligible Unemployed Workers:

Lemma 7.13 *For a UI-ineligible unemployed worker, the (Search-Incentive) constraint (37) always binds over the spell of UI-ineligible unemployment if and only if the expected utility* $V^{\hat{u}}_t = V_{min}$.

Proof Sufficiency: given that $V^{\hat{u}}_t = V_{min}$, one needs to show that the (Search-Incentive) constraint always binds.

By the definition of $V_{min} \equiv \frac{u(c_{min})}{1-\beta}$, one has $u(c_{min}) + \beta V_{min} = V_{min}$. Combining Eq. (36) with the inequality (37) yields

$$V^{\hat{u}}_t \geq u(c_{min}) + \beta V^{\hat{u}\hat{u}}_{t+1}. \tag{56}$$

Given that $V^{\hat{u}}_t = V_{min}$, one has $u(c_{min}) + \beta V_{min} \geq u(c_{min}) + \beta V^{\hat{u}\hat{u}}_{t+1}$. Thus, $V^{\hat{u}\hat{u}}_{t+1} = V_{min}$, and the inequality (56) holds as an equality, which implies that the (Search-Incentive) constraint (28) binds.

With the same reasoning, one can prove that given $V^{\hat{u}\hat{u}}_{t+1} = V_{min}$, the (Search-Incentive) constraint binds in next period. By iteration, it is straightforward to conclude that the (Search-Incentive) constraint binds in all subsequent periods.

Necessity: if the (Search-Incentive) constraint always binds in the unemployment spell, one needs to show that $V^{\hat{u}}_t = V_{min}$.

Prove by the way of contradiction. Suppose that if the (Searching-Incentive) constraint binds in each period, i.e. $\varphi_{1,t} > 0$, for all $t > 0$, and $V^{\hat{u}}_t > V_{min}$.

Combining the binding (Searching-Incentive) constraint with the promise-keeping constraint Eq. (36) gives $V^{\hat{u}}_t = u(c_{min}) + \beta V^{\hat{u}\hat{u}}_{t+1}$. Since the (Search-Incentive) constraint binds in all subsequent periods, by iteration, the value of $V^{\hat{u}}_t$ can be expressed as:

$$V^{\hat{u}}_t = u(c_{min}) + \beta u(c_{min}) + \beta^2 u(c_{min}) + \cdots = \frac{u(c_{min})}{1-\beta} = V_{min},$$

which leads to a contradiction. □

Lemma 7.14 *The expected utility of a UI-ineligible unemployed worker is a constant over the spell of the UI-ineligible unemployment, i.e.* $V_t^{\hat{u}} = V_{t+1}^{\hat{u}\hat{u}}$.

Proof If $V_t^{\hat{u}} = V_{min}$, by Lemma 7.13, it is easy to see that $V_t^{\hat{u}} = V_{t+1}^{\hat{u}\hat{u}} = V_{min}$.
If $V_t^{\hat{u}} > V_{min}$, then it can be shown that $\varphi_{1,t} = 0$ and $\varphi_{2,t} = 0$. Suppose by the way of contradiction that $\varphi_{1,t} > 0$. By Eq. (40) and the convexity of $\hat{C}(\cdot)$, one has

$$\begin{aligned} V_t^{\hat{u}} &> V_{t+1}^{\hat{u}\hat{u}} > V_{min}, \text{ if } \varphi_{2,t} = 0, \text{ and} \\ V_t^{\hat{u}} &> V_{t+1}^{\hat{u}\hat{u}} = V_{min}, \text{ if } \varphi_{2,t} > 0. \end{aligned}$$

Thus, $V_t^{\hat{u}} > V_{t+1}^{\hat{u}\hat{u}}$. Combining the binding (Search-Incentive) constraint (37) with Eq. (36) gives

$$V_t^{\hat{u}} = u(c_{min}) + \beta V_{t+1}^{\hat{u}\hat{u}} < u(c_{min}) + \beta V_t^{\hat{u}}.$$

So, one has $V_t^{\hat{u}} < \frac{u(c_{min})}{1-\beta} = V_{min}$, which leads to a contradiction. So, $\varphi_{1,t} = 0$.
Next, we show that $\varphi_{2,t} = 0$. By the way of contradiction, suppose that $\varphi_{2,t} > 0$. By Eq. (40) and the convexity of $\hat{C}(\cdot)$, one has $V_t^{\hat{u}} < V_{t+1}^{\hat{u}\hat{u}} = V_{min}$, which is a contradiction. So, $\varphi_{2,t} = 0$ and $V_t^{\hat{u}} = V_{t+1}^{\hat{u}\hat{u}}$. This completes the proof. $\square$

By Lemma 7.14, the value of $V_{t+1}^{\hat{e}\hat{u}}$ in Eq. (36) can be derived as:

$$V_{t+1}^{\hat{e}\hat{u}} = \frac{1}{\beta f}\left[(1-\beta(1-f))V_t^{\hat{u}} - (u(c_{min}) - 1)\right]. \tag{57}$$

And the function $\hat{C}(\cdot)$ can be fully characterized by $\hat{W}(\cdot)$ as:

$$\hat{C}(V_t^{\hat{u}}) = \frac{1}{1-\beta(1-f)}\left[\beta f \hat{W}\left(\frac{1}{\beta f}\left[(1-\beta(1-f))V_t^{\hat{u}} - (u(c_{min}) - 1)\right]\right)\right]. \tag{58}$$

Taking the derivative of Eq. (58) with respective to $V_t^{\hat{u}}$ leads to

$$\hat{C}'(V_t^{\hat{u}}) = \hat{W}'\left(\frac{1-\beta+\beta f}{\beta f}V_t^{\hat{u}} + \frac{1}{\beta f} - \frac{u(c_{min})}{\beta f}\right).$$

Plugging $u(c_{min}) = (1-\beta)\,V_{min}$ into the above equation gives

$$\hat{C}'(V_t^{\hat{u}}) = \hat{W}'\left(V_t^{\hat{u}} + \frac{1}{\beta f} + \frac{1-\beta}{\beta f}\left(V_t^{\hat{u}} - V_{min}\right)\right).$$

Since $V_t^{\hat{u}} \geq V_{min}$, the monotone increasing property of $\hat{W}'(\cdot)$ suggests that

$$\hat{C}'(V_t^{\hat{u}}) \geq \hat{W}'\left(V_t^{\hat{u}} + \frac{1}{\beta f}\right). \tag{59}$$

7.2 Optimization of Constraints

In this part, we analytically show that it is optimal to impose the (Search-Incentive), (Valuable-UI) and (No-Quit) constraints in the UI contract when promised lifetime utilities to workers are not too high. For simplicity, we relax only one of the three constraints at a time. For example, when we prove it is optimal to impose the (Search-Incentive) constraint, we impose the (Valuable-UI) constraint on UI-ineligible employed workers, and the (No-Quit) constraints on ineligible and eligible employed workers. In each proof, as in Hopenhayn and Nicolini (2009), we consider two cases. In Case 1, the constraint under consideration is not imposed, while is imposed in Case 2. For the presentation purpose, we add "0" in the subscript in the notation of the cost functions $C(\cdot)/\hat{C}(\cdot)/W(\cdot)/\hat{W}(\cdot)$ in Case 1, and add "1" in Case 2.[24]

7.2.1 (Search-Incentive) Constraint for the Unemployed

Different from Hopenhayn and Nicolini (2009), the (Search-Incentive) constraint for unemployed workers in our model does not necessarily bind. Intuitively, when a UI-eligible unemployed worker has a high promised utility value, the worker values the UI eligibility. Therefore, the positive probability of running out of benefits is strong enough to deter such a worker from shirking in job searches; and the positive probability of gaining UI eligibility provides an ineligible worker strong incentive to search hard for jobs. However, when the worker's promised utility is low, these effects are weak, and the (Search-Incentive) constraint is required and may bind.

Lemma 7.15 *If* $\hat{W}\left(V_{min}+\frac{1}{\beta f}\right)\leq\frac{c_{\min}}{(1-\beta)}$, *then it is optimal to impose the (Search-Incentive) constraint on a UI-ineligible unemployed worker with* $V_t^{\hat{u}}=V_{min}$.

Proof When $V_t^{\hat{u}}=V_{min}$, consider the following two cases:

Case 1: a UI-ineligible unemployed worker with an expected utility $V_t^{\hat{u}}$ does not search and the (Search-Incentive) constraint is relaxed. In this case, the worker stays unemployed and ineiligle for UI forever and receives c_{min} in each period. Thus, $V_t^{\hat{u}}\equiv V_{min}$, and $\hat{C}_0(V_{min})=c_{\min}/(1-\beta)$.[25]

Case 2: such a worker searches when the (Search-Incentive) constraint is imposed binds. By Lemma 7.14, one has $V_t^{\hat{u}}=V_{t+1}^{\hat{u}\hat{u}}$. Since $V_t^{\hat{u}}=V_{min}$, by Eq. (35) and Lemma 7.13, one has $\hat{C}_1(V_{min})=\frac{c_{\min}+\beta f\hat{W}\left(V_{min}+\frac{1}{\beta f}\right)}{1-\beta(1-f)}$. So it is straightforward to conclude that $\hat{C}_1(V_{min})\leq\hat{C}_0(V_{min})$ if $\hat{W}\left(V_{min}+\frac{1}{\beta f}\right)\leq\frac{1}{(1-\beta)}c_{\min}$. $\square$

[24] The optimal contract when workers are unemployed has a discrete choice variable, which, in general, is not convex. Following the method in Appendix A.1 of Hopenhayn and Nicolini (2009), one can easily show that the constrained optimum is convex with the use of lotteries. The proof is available upon request.

[25] The costs of $\hat{C}_0(\cdot)$ and $\hat{C}_1(\cdot)$ are included in the overall costs of the UI in this analysis.

Lemma 7.16 *For a UI-eligible unemployed worker with V_t^u, there exists a value $\bar{V}^u$ such that the (Search-Incentive) constraint binds for $V_t^u \in \left(V_{min}, \bar{V}^u\right)$, and $C_1'(V_t^u) > C_0'(V_t^u)$ for $V_t^u \in \left(V_{min}, \bar{V}^u\right)$.*

Proof By Lemma 7.8, the (Search-Incentive) constraint binds for $V_t^u = V_{min}$. By the Maximum theorem, there exists some $\bar{V}^u > V_{min}$ such that the (Search-Incentive) constraint binds for all $V_t^u < \bar{V}^u$.

Next, we prove that $C_1'(V_t^u) > C_0'(V_t^u)$ holds for $V_t^u \in \left(V_{min}, \bar{V}^u\right)$.

Case 1: the (Search-Incentive) constraint is removed, and thus, the UI-eligible unemployed worker does not search for a job. Therefore, the worker stays unemployed and runs out of benefits with a probability d. So the cost minimizing problem $C_0\left(V_t^u\right)$ can be modified as

$$\begin{aligned} C_0\left(V_t^u\right) &= \min_{b_{0,t}, V_{0,t+1}^{uu}, V_{0,t+1}^{\hat{u}u}} b_{0,t} + \beta\left[(1-d)C(V_{0,t+1}^{uu}) + d\hat{C}(V_{0,t+1}^{\hat{u}u})\right]. \\ \textit{subject to} &: \; V_t^u = u\left(b_{0,t}\right) + \beta\left[(1-d)V_{0,t+1}^{uu} + dV_{0,t+1}^{\hat{u}u}\right], \\ &: \; b_{0,t} \geq c_{min}, \\ &: \; V_{0,t+1}^{uu} \geq V_{min}, \\ &: \; V_{0,t+1}^{\hat{u}u} \geq V_{min}. \end{aligned}$$

Case 2: the (Search-Incentive) constraint is imposed and binds. The cost minimizing problem $C_1\left(V_t^u\right)$ is the same as what is stated in Section 2.

Let $\eta_{1,t}^0, \eta_{2,t}^0, \eta_{3,t}^0$ be the Lagrange coefficients of the constraints in the problem of C_0. By the envelope theorem, one has

$$C_0'(V_t^u) = \frac{1-\eta_{1,t}^0}{u'(b_{0,t})} = C'(V_{0,t+1}^{uu}) - \frac{\eta_{2,t}^0}{\beta\left(1-d\right)} = \hat{C}'(V_{0,t+1}^{\hat{u}u}) - \frac{\eta_{3,t}^0}{\beta d}. \tag{60}$$

Similarly, let $\eta_{1,t}^1, \eta_{2,t}^1, \eta_{3,t}^1, \eta_{4,t}^1, \eta_{5,t}^1$ be the Lagrange coefficients for the problem of C_1. By Eqs. (31), (33) and (34), we have

$$\begin{aligned} C_1'(V_t^u) &= \frac{1-\eta_{1,t}^1}{u'(b_{1,t})} = C'(V_{1,t+1}^{uu}) + \frac{\eta_{2,t}^1\pi - \eta_{4,t}^1}{\beta\left(1-f\right)\left(1-d\right)} + \frac{\eta_{3,t}^1}{\beta\left(1-f\right)}. \\ &= \hat{C}'\left(V_{1,t+1}^{\hat{u}u}\right) + \frac{\eta_{2,t}^1\left(1-\pi\right) - \eta_{5,t}^1}{\beta\left(1-f\right)d} + \frac{\eta_{3,t}^1}{\beta\left(1-f\right)}. \end{aligned}$$

Since the (Search-Incentive) constraint binds $\left(\eta_{3,t}^1 > 0\right)$, i.e. $\beta f(V_{1,t+1}^{eu} - dV_{1,t+1}^{\hat{u}u} - (1-d)V_{1,t+1}^{uu}) = 1$, one has

$$V_t^u = u(b_{0,t}) + \beta((1-d)V_{0,t+1}^{uu} + dV_{0,t+1}^{\hat{u}u}) = u(b_{1,t}) + \beta((1-d)V_{1,t+1}^{uu} + dV_{1,t+1}^{\hat{u}u}). \tag{61}$$

To show $C_1'(V_t^u) > C_0'(V_t^u)$, consider the following two cases:

- $b_{0,t} < b_{1,t}$. In this case, $\eta^1_{1,t} = 0$, and one has $\frac{1-\eta^0_{1,t}}{u'(b_{0,t})} \leq \frac{1}{u'(b_{0,t})} < \frac{1}{u'(b_{1,t})}$. Thus, it follows immediately that $C'_1(V^u_t) > C'_0(V^u_t)$.

- $b_{0,t} \geq b_{1,t}$. In this case, by Eq. (61), one of the following two inequalities has to hold: $V^{uu}_{1,t+1} > V^{uu}_{0,t+1}$, or $V^{\hat{u}u}_{1,t+1} > V^{\hat{u}u}_{0,t+1}$.

  - If $V^{uu}_{1,t+1} > V^{uu}_{0,t+1} > V_{min}$, then $\eta^1_{4,t} = 0$, and one has $C'_1(V^u_t) > C'(V^{uu}_{1,t+1}) > C'(V^{uu}_{0,t+1}) \geq C'_0(V^u_t)$, where the last inequality follows from Eq. (60). So, $C'_1(V^u_t) > C'_0(V^u_t)$.
  - If $V^{\hat{u}u}_{1,t+1} > V^{\hat{u}u}_{0,t+1} \geq V_{min}$, then $\eta^1_{5,t} = 0$ and one has $C'_1(V^u_t) > \hat{C}'(V^{\hat{u}u}_{1,t+1}) > \hat{C}'(V^{\hat{u}u}_{0,t+1}) \geq C'_0(V^u_t)$, where the last inequality also follows from Eq. (60). So, $C'_1(V^u_t) > C'_0(V^u_t)$.

The proof ends. □

The result of $C'_1(V^u_t) > C'_0(V^u_t)$ suggests the existence of a single crossing point between the two curves $C_0(V^u_t)$ and $C_1(V^u_t)$. And it is optimal to impose the (Search-Incentive) constraint when the promised utility V^u_t is lower than the crossing point $\bar{V}^u$.

7.2.2 (Valuable-UI) Constraint for UI-Ineligible Employed Workers

Lemma 7.17 *If the (Valuable-UI) constraint binds for some $V^{\hat{e}}_t > V_{min}$, then there exists a value $\bar{V}^{\hat{e}}$ such that this constraint binds for $V^{\hat{e}}_t \in \left(V_{min}, \bar{V}^{\hat{e}}\right)$.*

Proof If the (Valuable-UI) constraint binds for some $V^{\hat{e}}_t > V_{min}$, then by Lemma 7.6 derived in Appendix 7.1.2, it always binds if the worker remains his type. In addition, by Proposition 5.2, $V^{\hat{e}}_t$ decreases along the spell of the UI-ineligible employment until $V^{\hat{e}}_t = V_{min}$. Thus, the (Valuable-UI) constraint binds for $V^{\hat{e}}_t = V_{min}$.

By the Maximum theorem, there exists some value $\bar{V}^{\hat{e}}$ such that the (Valuable-UI) constraint binds for the UI-ineligible employed workers with $V^{\hat{e}}_t \in \left(V_{min}, \bar{V}^{\hat{e}}\right)$. □

Lemma 7.18 *For a UI-ineligible employed worker with $V^{\hat{e}}_t \in \left(V_{min}, \bar{V}^{\hat{e}}\right)$, one has $\hat{W}'_1(V^{\hat{e}}_t) > \hat{W}'_0(V^{\hat{e}}_t)$.*

Proof Consider the following two cases:

Case 1: the (Valuable-UI) constraint is not imposed and UI-ineligible employed workers renounce the UI eligibility upon gaining it. Thus, they remain UI-ineligible over the spell of employment. So the cost minimizing problem $\hat{W}_0$ is modified as

$$\hat{W}_0\left(V^{\hat{e}}_t\right) = \min_{\tau^{\hat{e}}_{0,t}, V^{\hat{e}\hat{e}}_{0,t+1}, V^{\hat{u}\hat{e}}_{0,t+1}} -\tau^{\hat{e}}_{0,t} + \beta\left[(1-s)\,\hat{W}\left(V^{\hat{e}\hat{e}}_{0,t+1}\right) + s\hat{C}\left(V^{\hat{u}\hat{e}}_{0,t+1}\right)\right]. \tag{62}$$

subject to

$$
\begin{array}{rl}
 : & V_t^{\hat{e}} = u\left(c_{0,t}^{\hat{e}}\right) - m + \beta\left[(1-s)V_{0,t+1}^{\hat{e}\hat{e}} + sV_{0,t+1}^{\hat{u}\hat{e}}\right], \\
 : & c_{0,t}^{\hat{e}} = w - \tau_{0,t}^{\hat{e}} \geq c_{min}, \\
\text{No-Quit} : & V_{0,t+1}^{\hat{e}\hat{e}} \geq V_{0,t+1}^{\hat{u}\hat{e}}, \\
 : & V_{0,t+1}^{\hat{e}\hat{e}} \geq V_{min}, \\
 : & V_{0,t+1}^{\hat{u}\hat{e}} \geq V_{min}.
\end{array}
\tag{63}
$$

Case 2: the (Valuable-UI) constraint is imposed and UI-ineligible employed workers accept the UI eligibility upon gaining it. Thus, the cost minimizing problem $\hat{W}_1$ is the same as what is stated in Section 2.

Let $\phi_{1,t}^0$, $\phi_{2,t}^0$, $\phi_{3,t}^0$, $\phi_{4,t}^0$ be the Lagrange coefficients of the problem of $\hat{W}_0$. By the Envelope theorem, one has

$$
\begin{aligned}
\hat{W}_0'(V_t^{\hat{e}}) &= \frac{1-\phi_{1,t}^0}{u'(c_{0,t}^{\hat{e}})} = \hat{W}'(V_{0,t+1}^{\hat{e}\hat{e}}) - \frac{\phi_{2,t}^0}{\beta(1-s)} - \frac{\phi_{3,t}^0}{\beta(1-s)} \\
&= \hat{C}'(V_{0,t+1}^{\hat{u}\hat{e}}) + \frac{\phi_{2,t}^0}{\beta s} - \frac{\phi_{4,t}^0}{\beta s}.
\end{aligned}
\tag{64}
$$

Similarly, let $\phi_{1,t}^1$ $\phi_{2,t}^1$, $\phi_{3,t}^1$, $\phi_{4,t}^1$ be the Lagrange coefficients for the problem of $\hat{W}_1$, and one has

$$
\begin{aligned}
\hat{W}_1'(V_t^{\hat{e}}) &= \frac{1-\phi_{1,t}^1}{u'(c_{1,t}^{\hat{e}})} = \hat{W}'(V_{1,t+1}^{\hat{e}\hat{e}}) - \frac{\phi_{2,t}^1}{\beta(1-s)} + \frac{\phi_{3,t}^1 - \phi_{4,t}^1}{\beta(1-g)(1-s)} \\
&= \hat{C}'(V_{1,t+1}^{\hat{u}\hat{e}}) + \frac{\phi_{2,t}^1}{\beta s} - \frac{\phi_{5,t}^1}{\beta s}.
\end{aligned}
\tag{65}
$$

Since the (Valuable-UI) constraint binds $\left(\phi_{3,t}^1 > 0\right)$, i.e. $V_{1,t+1}^{ee} = V_{1,t+1}^{\hat{e}e}$, one has,

$$
V_t^{\hat{e}} = u(c_{0,t}^{\hat{e}}) + \beta((1-s)V_{0,t+1}^{\hat{e}\hat{e}} + sV_{0,t+1}^{\hat{u}\hat{e}}) = u(c_{1,t}^{\hat{e}}) + \beta((1-s)V_{1,t+1}^{\hat{e}\hat{e}} + sV_{1,t+1}^{\hat{u}\hat{e}}). \tag{66}
$$

To show $\hat{W}_1'(V_t^{\hat{e}}) > \hat{W}_0'(V_t^{\hat{e}})$, consider the following two cases:

- If $c_{min} \leq c_{0,t}^{\hat{e}} < c_{1,t}^{\hat{e}}$, then $\phi_{1,t}^1 = 0$, and $\frac{1-\phi_{1,t}^0}{u'(c_{0,t}^{\hat{e}})} \leq \frac{1}{u'(c_{0,t}^{\hat{e}})} < \frac{1}{u'(c_{1,t}^{\hat{e}})}$. Then, it follows immediately that $\hat{W}_1'(V_t^{\hat{e}}) > \hat{W}_0'(V_t^{\hat{e}})$.
- If $c_{min} \leq c_{1,t}^{\hat{e}} < c_{0,t}^{\hat{e}}$, Eq. (66) suggests that one of the following two inequalities has to hold: $V_{1,t+1}^{\hat{e}\hat{e}} > V_{0,t+1}^{\hat{e}\hat{e}} > V_{min}$, or $V_{1,t+1}^{\hat{u}\hat{e}} > V_{0,t+1}^{\hat{u}\hat{e}} > V_{min}$.

– If $V^{\hat{e}\hat{e}}_{1,t+1} > V^{\hat{e}\hat{e}}_{0,t+1} > V_{min}$, by the discussions of the FOCs in Appendix 7.1.2, one has
$$\hat{W}_1'(V_t^{\hat{e}}) > \hat{W}'(V^{\hat{e}\hat{e}}_{1,t+1}) > \hat{W}'(V^{\hat{e}\hat{e}}_{0,t+1}) \geq \hat{W}_0'(V_t^{\hat{e}}).$$
So, $\hat{W}_1'(V_t^{\hat{e}}) > \hat{W}_0'(V_t^{\hat{e}})$.

– If $V^{\hat{u}\hat{e}}_{1,t+1} > V^{\hat{u}\hat{e}}_{0,t+1}$, then $\phi^1_{5,t} = 0$, and
$$\hat{W}_1'(V_t^{\hat{e}}) \geq \hat{C}'(V^{\hat{u}\hat{e}}_{1,t+1}) > \hat{C}'(V^{\hat{u}\hat{e}}_{0,t+1}).$$
If $\phi^0_{2,t} = 0$, then $\hat{C}'(V^{\hat{u}\hat{e}}_{0,t+1}) \geq \hat{W}_0'(V_t^{\hat{e}})$, and the proof ends. If $\phi^0_{2,t} > 0$, i.e. $V^{\hat{e}\hat{e}}_{0,t+1} = V^{\hat{u}\hat{e}}_{0,t+1}$, then one has
$$V^{\hat{e}\hat{e}}_{0,t+1} = V^{\hat{u}\hat{e}}_{0,t+1} < V^{\hat{u}\hat{e}}_{1,t+1} \leq V^{\hat{e}\hat{e}}_{1,t+1},$$
where the last inequality comes from the binding (Valuable-UI) and the (No-Quit) constraints in Case 2. So $V^{\hat{e}\hat{e}}_{0,t+1} < V^{\hat{e}\hat{e}}_{1,t+1}$, and it follows immediately that
$$\hat{W}_1'(V_t^{\hat{e}}) > \hat{W}'(V^{\hat{e}\hat{e}}_{1,t+1}) > \hat{W}'(V^{\hat{e}\hat{e}}_{0,t+1}) \geq \hat{W}_0'(V_t^{\hat{e}}).$$

This completes the proof. □

The result that $\hat{W}_1'(V_t^u) > \hat{W}_0'(V_t^u)$ suggests the existence of a single crossing point between the two curves $\hat{W}_0(V_t^u)$ and $\hat{W}_1(V_t^u)$. And it is optimal to impose the (Valuable-UI) constraint when the promised utility $V_t^{\hat{e}}$ is lower than the crossing point $\bar{V}^{\hat{e}}$.

7.2.3 (No-Quit) Constraint for UI-Eligible Employed Workers

Lemma 7.19 *For a UI-eligible employed workers with V_t^e, there exists a value $\bar{V}^e$ such that the (No-Quit) constraint (5) binds for $V_t^e \in \left(V_{min}, \bar{V}^e\right)$.*

Proof By the way of contradiction suppose that for any $\bar{V}^e > V_{min}$, there exists some $\epsilon > 0$ such that the (No-Quit) constraint (5) is slack for $V_t^e = V_{min} + \epsilon$. By Lemmas 7.1 and 7.2, one has $V_t^e = V^{ee}_{t+1} = V_{min} + \epsilon$, and $V^{\hat{u}e}_{t+1} = V_{min}$. Plugging these into the slack (No-Quit) constraint gives
$$V^{ue}_{t+1} < V_{min} + \frac{1-s}{\pi - s}\epsilon. \tag{67}$$
By Eq. (3), one has
$$u\left(c_t^e\right) > \left(1-\beta\right)V_{min} + m + \left(1 - \frac{\beta\pi\left(1-s\right)}{\pi - s}\right)\epsilon.$$
Given $m > 0$, one sees that there always exists an arbitrarily small value ϵ such that $c_t^e > c_{\min} + \zeta$ for some $\zeta > 0$. By Lemma 7.1, one has $c^{ue}_{t+1} = c_t^e > c_{\min} + \zeta$. In addition,

by the inequality (67), one sees that when $\epsilon > 0$ is small enough, V_{t+1}^{ue} is arbitrarily close to V_{min}. Then, by Lemma 7.8 and the Maximum theorem, it follows that c_{t+1}^{ue} should be arbitrarily close to $c_{\min}$, which leads to a contradiction. $\square$

Lemma 7.20 *For a UI-eligible employed workers with $V_t^e \in \left(V_{min}, \bar{V}^e\right)$, one has $W_1'(V_t^e) > W_0'(V_t^e)$.*

Proof Case 1: the (No-Quit) constraint is not imposed and the UI-eligible employed workers quit their jobs. Thus, workers leave their current jobs to collect UI benefits, and the cost minimizing problem W_0 is modified as follows:

$$W_0\left(V_t^e\right) = \min_{\tau_{0,t}^e, V_{0,t+1}^{ue}, V_{0,t+1}^{\hat{u}e}} -\tau_{0,t}^e + \beta\left[(1-\pi)\hat{C}\left(V_{0,t+1}^{\hat{u}e}\right) + \pi C\left(V_{0,t+1}^{ue}\right)\right]. \tag{68}$$

subject to

$$\begin{aligned}
&: \quad V_t^e = u\left(c_{0,t}^e\right) - m + \beta\left[(1-\pi)V_{0,t+1}^{\hat{u}e} + \pi V_{0,t+1}^{ue}\right],\\
&: \quad c_{0,t}^e = w - \tau_{0,t}^e \geq c_{min},\\
&: \quad V_{0,t+1}^{ue} \geq V_{min},\\
&: \quad V_{0,t+1}^{\hat{u}e} \geq V_{min}.
\end{aligned}$$

Case 2: the (No-Quit) constraint is imposed and binds, and the UI-eligible employed workers do not quit their jobs. Therefore, the cost minimizing problem W_1 is the same as the one defined in Section 2.

Let $\mu_{1,t}^0$, $\mu_{2,t}^0$, $\mu_{3,t}^0$, $\mu_{4,t}^0$ be the Lagrange coefficients of the problem of W_0. By the Envelope theorem, one has

$$W_0'(V_t^e) = \frac{1-\mu_{1,t}^0}{u'(c_{0,t}^e)} = C'(V_{0,t+1}^{ue}) - \frac{\mu_{3,t}^0}{\beta\pi}. \tag{69}$$

Let $\mu_{1,t}^1$, $\mu_{2,t}^1$ $\mu_{3,t}^1$, $\mu_{4,t}^1$, $\mu_{5,t}^1$ be the Lagrange coefficients for the problem of W_1. And one has

$$W_1'(V_t^e) = \frac{1-\mu_{1,t}^1}{u'(c_{1,t}^e)} = C'(V_{1,t+1}^{ue}) + \frac{\mu_{2,t}^1(\pi - s)}{\beta s} - \frac{\mu_{4,t}^1}{\beta s}. \tag{70}$$

Since the (No-Quit) constraint (5) binds, i.e. $\mu_{2,t}^1 > 0$, then by Eq. (12), one has $\mu_{5,t}^1 > 0$, i.e. $V_{1,t+1}^{\hat{u}e} = V_{min}$. Moreover, the binding (No-Quit) constraint, i.e. $(1-s)V_{1,t+1}^{ee} + sV_{1,t+1}^{ue} = \pi V_{1,t+1}^{ue} + (1-\pi)V_{1,t+1}^{\hat{u}e}$, implies that

$$V_t^u = u(c_{0,t}^e) - m + \beta(\pi V_{0,t+1}^{ue} + (1-\pi)V_{0,t+1}^{\hat{u}e}) = u(c_{1,t}^e) - m + \beta(\pi V_{1,t+1}^{ue} + (1-\pi)V_{1,t+1}^{\hat{u}e})$$

To show $W_1'(V_t^u) > W_0'(V_t^u)$, consider the following two cases:

- If $c_{0,t}^e < c_{1,t}^e$, then $\mu_{1,t}^1 = 0$, and $\frac{1-\mu_{1,t}^0}{u'(c_{0,t}^e)} \leq \frac{1}{u'(c_{0,t}^e)} < \frac{1}{u'(c_{1,t}^e)}$. So, $W_0'(V_t^e) < W_1'(V_t^e)$.
- If $c_{0,t}^e \geq c_{1,t}^e$, then one must have $V_{1,t+1}^{ue} > V_{0,t+1}^{ue}$.[26] Therefore, $\mu_{4,t}^1 = 0$, and by Eqs.

[26] The case of $V_{1,t+1}^{\hat{u}e} > V_{0,t+1}^{\hat{u}e}$ is ruled out because of $V_{t+1}^{\hat{u}e} = V_{min}$.

(69) and (70), one has

$$W_1'(V_t^e) \geq C'(V_{1,t+1}^{ue}) > C'(V_{0,t+1}^{ue}) \geq W_0'(V_t^e).$$

This completes the proof. $\square$

The result that $W_1'(V_t^u) > W_0'(V_t^u)$ suggests the existence of a single crossing point between the two curves $W_0(V_t^u)$ and $W_1(V_t^u)$. And it is optimal to impose the (No-Quit) constraint when the promised utility V_t^e is lower than the crossing point $\bar{V}^e$.

7.3 Proof of Proposition 3.4

Suppose by the way of contradiction, given that $m > \frac{1-\pi}{1-s}(\beta\frac{\pi-s^2}{(\pi-s)(1-\beta)}+1)(u(w)-u(c_{min}))+\frac{\pi-s}{f(1-s)}$, there exists some UI-eligible employed worker with the promised utility $V_t^e \geq V_{min}$ prefers working to quitting. Therefore, the following inequality holds.

$$sV_{t+1}^{ue} + (1-s)V_{t+1}^{ee} \geq \pi V_{t+1}^{ue} + (1-\pi)V_{t+1}^{\hat{u}e}. \tag{71}$$

Let $\tilde{V}_{t+1}^{ue}$ and $\bar{V}_{t+1}^{ue}$ be the expected utility of delaying quitting for one period and quitting immediately, respectively. Hence, by the supposition, one has $\tilde{V}_{t+1}^{ue} \geq \bar{V}_{t+1}^{ue}$.

In fact,

$$\tilde{V}_{t+1}^{ue} = sV_{t+1}^{ue} + (1-s)\left[u(c_{t+1}^{ee}) - m + \beta\left(\pi V_{t+2}^{uee} + (1-\pi)V_{t+2}^{\hat{u}ee}\right)\right], \tag{72}$$

and

$$\bar{V}_{t+1}^{ue} = \pi V_{t+1}^{ue} + (1-\pi)V_{t+1}^{\hat{u}e}. \tag{73}$$

Subtracting Eq. (73) from Eq. (72) gives:

$$\tilde{V}_{t+1}^{ue} - \bar{V}_{t+1}^{ue} = (1-s)\left[u(c_{t+1}^{ee}) - m + \beta\left(\pi V_{t+2}^{uee} + (1-\pi)V_{t+2}^{\hat{u}ee}\right)\right] - (\pi-s)V_{t+1}^{ue} - (1-\pi)V_{t+1}^{\hat{u}e}.$$

Plugging (25) into the above equation, and combining with the results in Lemmas 7.1 and 7.2 gives

$$\begin{aligned}
\tilde{V}_{t+1}^{ue} - \bar{V}_{t+1}^{ue} &= \left[(1-\pi)\left(u(c_{t+1}^{ee}) - u(c_{min})\right) - (1-s)m\right] \\
&\quad +\beta\{(1-s)[\pi V_{t+2}^{uee} + (1-\pi)V_{min}] - (\pi-s)[dV_{t+2}^{\hat{u}ue} + (1-d)V_{t+2}^{uue}] - (1-\pi)V_{min}\}, \\
&\leq \left[(1-\pi)\left(u(w) - u(c_{min})\right) - (1-s)m\right] \\
&\quad +\beta\{(1-s)\pi V_{t+2}^{uee} - (\pi-s)\left(V_{t+2}^{eue} - \frac{1}{\beta f}\right) - s(1-\pi)V_{min}\},
\end{aligned}$$

where the last equation comes from the budget constraint $c_{t+1}^{ee} \leq w$ and the binding (Search-Incentive) constraint for the eligible unemployed worker. For the sake of composition, define

$$D = (1-s)\pi V_{t+2}^{uee} - (\pi - s)\left(V_{t+2}^{eue} - \frac{1}{\beta f}\right) - s(1-\pi)V_{min}.$$

Next, we show that $V_{t+2}^{eee} \leq V_{t+2}^{eue}$.

By Lemma 7.1, one has $W'(V_t^e) = W'(V_{t+1}^{ee}) = W'(V_{t+2}^{eee})$. In addition, by Eq. (11) and the absence of the (No-Quit) constraint, one has $W'(V_t^e) \leq C'(V_{t+1}^{ue})$. Since the (Search-Incentive) constraint binds for eligible unemployed workers, by Lemma 7.9 and Eqs. (31) and (32), one has $C'(V_{t+1}^{ue}) \leq W'(V_{t+2}^{eue})$. Thus, $V_{t+2}^{eee} \leq V_{t+2}^{eue}$ due to the convexity of $W(\cdot)$.

Recall inequality (71), given that $\pi > s$, it is equivalent to the follows,

$$(1-s)\pi V_{t+2}^{uee} \leq \frac{(1-s)^2\pi}{\pi - s} V_{t+2}^{eee} - \frac{(1-s)(1-\pi)\pi}{\pi - s} V_{min}.$$

Plugging this result and $V_{t+2}^{eee} \leq V_{t+2}^{eue}$ into D gives:

$$\begin{aligned} D &\leq \frac{(\pi - s^2)(1-\pi)}{\pi - s}(V_{t+2}^{eee} - V_{min}) + \frac{\pi - s}{\beta f}, \\ &\leq \frac{(\pi - s^2)(1-\pi)}{(\pi - s)(1-\beta)}(u(w) - u(c_{min})) + \frac{\pi - s}{\beta f}, \end{aligned}$$

where the last inequality follows from $V_{t+2}^{eee} \leq u(w)/(1-\beta)$.

Therefore, if $m > \frac{1-\pi}{1-s}(\beta \frac{\pi - s^2}{(\pi - s)(1-\beta)} + 1)(u(w) - u(c_{min})) + \frac{\pi - s}{f(1-s)}$, one has $\tilde{V}_{t+1}^{ue} < \bar{V}_{t+1}^{ue}$, which is a contradiction. $\square$

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