Grease or Sand: a Unified Explanation of Local Bureaucrats' Incentives in an Authoritarian System

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Abstract

This paper provides a novel micro foundation for the frequently observed slowing down problem in fast-growing economies under authoritarian systems. We establish a two-stage model to characterize the dynamic relationship between corruption and growth focusing on the formation and maintenance of political connections of local bureaucrats and firms. When local bureaucrats have concerns over local citizens' well-being (for the consideration of promotion) in addition to monetary payoffs, and when political connections have to be built and maintained through continuous illegal transfers, we produce three major findings. First, only highly competitive local firms can be protected under the threat of the entry of an outsider, which appears as patronage on the equilibrium path. Second, as long as the bureaucrat does not fully represent the central government or local citizens, patronage always occurs with positive probability. Finally, with this two-stage model, there are cases where corruption increases social welfare, but this conclusion does not hold when more stages are added.

Keywords: Corruption, Political connection, Procurement auction, Patronage, Dynamic game, Incomplete information

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1 Introduction

The role authoritarianism plays in economic growth remains ambiguous in the academic world, especially when one considers typical observations from almost all the authoritarian systems in countries and regions of Latin America, East Asia and Southeast Asia. These systems generate a high GDP growth rate at first but then fall into a low- or no-growth trap.[1] While this phenomenon has garnered much attention, theories thus far fail to give a unified explanation as to why persistent economic growth cannot be achieved under authoritarianism. For example, advocates of the developmental state hypothesis believe that a stable authoritarian system can easily push forward market intervention policies like government investment to actively promote economic growth; however, this hypothesis cannot explain why the same force stops working after the initial stage of fast growth. Alternatively, some scholars focus on the intriguing role of corruption[2] in authoritarian systems, which is believed to easily fertilize the soil for rent-seeking. Efficient corruption may be a possible explanation for the initial growth, but the reported corruption levels are always too high to be regarded as efficient in the low- and no-growth stages. In this paper, we attempt to tackle this puzzle using a game theoretic approach. In particular, we aim to explain why within the same model, the driving force (*grease*) behind high-speed growth becomes an obstacle (*sand*) to further development.

We believe that the key lies in the incentives of the local bureaucrats (or mid-level bureaucrats). However, unlike previous literature that mainly pays attention to the upstream side of the story by considering the delegation problem and information asymmetry[3] between the leader (or the central government) and local bureaucrats, we focus on the downstream side by studying the incentives of a local bureaucrat and a (domestic or local) firm. This specification means that our two-stage model incorporates the leader's incentives as external factors. In the first stage, there is no clear strategy or plan to follow for developing the local economy; thus, the bureaucrat is delegated the power of allocating public resources to any local firm (LF). He then makes a secret deal with an interested LF

[1] For example, Eichengreen et al. (2012) summarize global evidence on the slowing down of fast-growing economies. Zilibotti (2017) provides a detailed review of the pattern for China.

[2] Aidt (2003) provides an economic analysis of corruption based on different sources.

[3] For example, Tirole (1986) utilizes the principle&agent methodology to study collusion and the formation of coalition in hierarchies. Banerjee (1997) uses a theoretical model that also incorporates agency problems to explain the existence of red tape and corruption

in exchange for his executive support. In the second stage, an international firm (or still domestic but non-local) is attracted to the domestic or local market as a result of economic development. To create a better business environment and to encourage fair competition,[4] the authoritarian leader establishes a standard protocol as the competition mechanism that the bureaucrat is required to follow. However, the bureaucrat is still able to create some barriers to entry for the international firm as long as the incumbent (LF) agrees to another secret deal. This entry barrier can only be subdued when the international firm is sufficiently competitive. Interestingly, the final outcome of the dynamic game determines the evolutionary direction in this region: the market may become monopolized by the incumbent or may become more competitive following the successful entry of the outsider. If the latter pattern emerges, there appears to be a possibility of persistent development under authoritarianism.

In the aforementioned dynamic game, there are two key assumptions. The first assumption relates to the bureaucrat's incentives: we assume that in addition to monetary transfers, he also has concerns over measures of economic development and local citizens' well-being. These concerns are mainly derived from the authoritarian leader's decision on promotion.[5] As Olson (1998) argues, a non-myopic leader in a stable authoritarian system embraces economic growth and improvements in welfare; thus, the evaluation of local bureaucrats' performance should contain some relevant measure. The second relates to the formation and sustainability of political protection, which is not taken for granted. Political connections have to be built in the first stage and can only be maintained by continuous bribes: if the incumbent is not able to afford a second secret deal, the outsider enters immediately. Other than these two key assumptions, we also impose two modelling assumptions. First, we assume that the protocol requires any possible competition to be realized by a procurement auction. Second, regarding the bureaucrat and the LF's bargaining power, we assume that the bureaucrat is in the dominant position in making secret deals. This assumption fits the empirical observations because

[4] In addition to the desire for economic development and improvements in welfare, the leader always has concern for the maintenance of political stability. Therefore, he has an incentive to restrict local officials' power whenever possible by building mature mechanisms. As Silvok (2012) indicates, the leader has incentive to avoid two types of threats from sharing power with local officials, namely, abuses of power that may lead to a local riot and ambitious officials with enough power to turn the region into their own kingdoms.

[5] Other than this reason, the bureaucrat could have moral concerns for local citizens' well-being, especially when he is from the same region.

the bureaucrat serves as the monopoly in trading his power. If we assume that the bureaucrat and LF bargain over the transfer amount that satisfies both parties, we will encounter a major technical difficulty even for a one-shot bargain game with this incomplete information setting[6].

Under these assumptions, we show how the seemingly grease becomes the sand towards economic development through political connections enhanced by continuous illegal transfers. We establish a monotonic perfect Bayesian equilibrium, characterize the bureaucrat's sequentially optimal secret deals, and investigate the welfare implications. The first major finding is that although the LF's private cost to procure the project follows a continuous distribution, in equilibrium, three types of behaviour emerge intuitively, denoted by H (high), M (moderate) and L (low). The H firm cannot even build the connection in the first stage. The M firm can afford the first deal but not the second. Only the L firm is capable of building and maintaining the ally. This pattern confirms the intuition that political protection is awarded to firms with good potential, and provides a possible explanation as to why the economy can develop before foreign investment is attracted. The second major finding is intriguing: as long as the bureaucrat does not care about local citizens' well-being as much as the citizens or the benevolent central government care, corruption always occurs with a positive probability in the second stage regardless of the outsider's ex-ante cost advantage. Since this sufficient condition generally holds in reality, we know patronage is inevitable when LFs are competitive enough, even though the bureaucrat is not purely money oriented. The third result is that there are cases where the overall social welfare increases as a result of patronage compared to the benchmark case where secret deals are never possible. These cases normally require the outsider to have a fairly small cost advantage, which is quite intuitive since the welfare gain in the first stage from "efficient" corruption can then make up for the welfare loss in the second stage.

In addition to these findings, our model also has rich implications on the evolutionary path of a stable authoritarian system. We show that when the leader or the central government pursues economic development and welfare improvements, a typical two-stage pattern is observed. This pattern comprises an economically efficient but politically corrupt first stage followed by an economically inefficient and more politically corrupt second stage. The situation can be predicted to grow worse as time elapses, and the cumulative domino effect could lead to a final collapse. However, if a sufficiently competitive outsider overcomes the entry barrier in the second stage, the LF would be more

[6] See Ausubel et al. (2002) for a review of related work.

reluctant to pay for secret deals in the future, while the bureaucrat's personal cost of taking bribes also increases because he has to sacrifice GF's positive effect on his promotion. Following this direction, we can see that a highly competitive international firm could induce the formation of a benign market structure as a result of the incentives of the LF and the local bureaucrat. The economy would thus gradually get on the right track towards a more competitive environment, which further provides solid ground for the transition to a more democratic political regime. This possibility has been noted by Acemoglu and Robinson (chapter 10, 2005), who provide a theoretical framework to analyse the impact globalization has on the democratization of less developed countries. The work of these authors emphasizes the power of integration in the trade sector and the financial system, while ours focuses on the importance of foreign investment. The evolutionary trend also validates the plausibility of a common promise, that entry barriers will be removed or at least gradually reduced, made by authoritarian countries before joining trade treaties and organizations.

This paper contributes to four strands of the literature. The first strand includes a large volume of literature on the identification of political connections between politicians and firms and the estimation of the value added. Ades and Di Tella (1999) find that corruption is higher in countries where domestic firms are sheltered from foreign competition by natural and policy-based barriers. Our theoretical results support this empirical evidence. Faccio et al. (2006) show that politically connected firms are significantly more likely to be bailed out than non-connected peers, especially when their home government experiences economic distress. Many other studies use country-level data to confirm the value added for local or domestic firms with political connections, including Fisman (2001) on Indonesia, Khwaja and Mian (2005) on Pakistan, Jayachandran (2006) and Cooper et al. (2010) on US, Ferguson and Voth (2008) on Nazi Germany, Coulomb and Sangnier (2014) on France and Mironov and Zhuravskaya (2016) on Russia. Our paper reveals the micro and economic foundations of patronage as the direct result of continuous illegal transfers from firms to local bureaucrats in equilibrium. Moreover, our work sheds light on possible changes in the political regimes of stable authoritarian systems if a highly competitive international firm successfully breaks the barrier.

The second strand of the literature is the second-generation theory of fiscal federalism. Jin et al. (2005) provide empirical evidence for the helping hand (grease) role played by Chinese local governments and identify fiscal decentralization as the main incentive: when local bureaucrats seek to maximize government revenue, decentralization disciplines their behaviour and induces them to enact better policies to attract more capital.[7] Cai and Treisman (2005) raise a point that interjurisdictional competition for mobile capital only disciplines local governments when units begin with roughly equal endowments. Our model adds to this literature by analysing individual incentives: when local bureaucrats maximize their personal utility functions instead of the government revenue, whether they would appear to discipline themselves depends on the equilibrium outcome. Namely, when the international firm happens to be highly competitive, the corruption level seems to be reduced because the bureaucrat is more reluctant to make secret deals; otherwise, the corruption level may exhibit the opposite effect. From this perspective, our model unifies the results of these two papers. Intuitively, regions with better endowments have advantages in attracting more foreign investment and from more competitive firms; thus, barriers are more costly to create and are more easily broken down. Therefore, the seemingly different behaviours of bureaucrats in, for example, coastal regions and the central and western regions of China are both justifiable in equilibrium.

The third strand of the literature considers economic growth under different political regimes. There are two representative explanations of economic growth induced by stable authoritarian systems. The developmental state theory emphasizes the importance of aligning bureaucrats' interests at different hierarchical levels to mitigate information asymmetry. For example, the Chinese government uses economic growth as an important measure of bureaucrats' performance (Knight (2014)), including for promotion and demotion decisions. The efficient corruption hypothesis posits that corruption can help entrepreneurs avoid executive obstacles (or red tape) when they start a business. For example, studies on China's transition from a central planning economy to a market-oriented economy have long found that local governments in China play the role of a helping hand in local business development by supporting productive enterprises and disciplining unproductive ones. These two hypotheses cannot consistently explain both the initial growth and the later slowdown observed in reality. We make two contributions to this literature. First, we show that only local and domestic firms with high potential can procure the project in the first stage. This result implies that economic development in the first stage comes from the efficient corruption of capable firms. Second, overall, we show that the intriguing dual role of corruption as both the grease for and the sand to economic growth is caused by different stages on the equilibrium path.

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[7] Fiscal decentralization also plays a generally positive role in reducing corruption according to studies by Fisman and Gatti (2002) and Fan et al. (2009)), although government tiers and structures also matter.

The fourth strand of the literature considers empirical and theoretical studies on corruption in auctions. Corrupt behaviours in government auctions have been detected by several papers, even by those using data from developed countries. For example, Goldman et al. (2005) find that the political connections of the board of directors of publicly traded companies in the US can significantly affect the allocation of government procurement contracts. Straub (2014) and Koren et al. (2015) use data from South America and Hungary, respectively, and arrive at a similar conclusion. Schoenherr (2015) estimates the economic cost of misallocating procurement contracts as a result of political networks after the Korean Presidential election in 2007. Our model provides a theoretical characterization of corrupt behaviours in the public sector when the bureaucrat can manipulate entry. Previous theoretical papers on auctions with corruption normally focus on the private sector where the auctioneer or auction holder cares only about monetary payoffs. Papers in this vein includes Burguet and Che (2004), Menezes and Monteiro (2006) and Burguet (2017).

The rest of the paper is organized as follows: section 2 lays out the model. Section 3 analyses the equilibrium and its welfare implications and includes a subsection with numerical examples. Section 4 uses China, the largest authoritarian country, to show the model's explanatory power and reveal possible problems in the further development thereof. Section 5 discusses several modelling assumptions and possible extensions, and section 6 concludes.

2 The Model

We consider a representative two-stage model with three players and a discount rate $\delta \in (0, 1)$. The players are the bureaucrat, the local (or domestic) firm (LF), and the international (or domestic but non-local) firm (GF for short because IF is confusing). There is an indivisible periodic project of social value v that can improve local citizens' well-being. In the first stage, the LF is not able to procure this project without the bureaucrat's executive power; thus, the LF has an incentive to bribe the bureaucrat in exchange for protection. In the second stage, in order to promote effective and fair competition, the central government requires the project to be procured through a public auction. To qualify for participation in the auction, the interested firms (LF and GF) must acquire some certificate. The bureaucrat is awarded the power to supervise the certificate application so he can exert his influence on the process by accelerating or delaying it. For the model to be analytically tractable, we assume that this obstacle is not continuously measurable: the cost that an applicant incurs is either 0 or some $c > 0$. c is the social loss that does not go into the bureaucrat's pocket, which can be interpreted as the extra waiting time or the costs of preparing unnecessary documents. We further assume that the value of c is exogenously given.[8]

The costs for the LF and GF to procure the project are their private information. The LF's cost x follows a distribution function $F(\cdot)$ over $[a, b]$, where $a \geq 0$; the GF's cost y is independent from x and follows $G(\cdot)$ over $[0, b]$.[9] Both firms are risk-neutral; thus, we can use their monetary payoffs to represent the utility functions. We further assume b is the budget limit of this project and that $v = b$.[10] The auction is assumed to take the second-price sealed-bid format, which is strategically equivalent to the English auction in this model. We assume there is no reserve price, which means that there is no upper bound on the bid that any firm can submit.[11] Therefore, the auction works as follows: if there is only one participant, that firm directly wins and receives a payment of b from the local government; otherwise, the firm with a lower bid wins and receives the bid submitted by the losing firm as the payment.

The bureaucrat's utility function is quasi-linear in monetary transfers: he receives $u(b-P)+t$ in each stage, where P is the payment to the winning contractor in that stage and t is the amount of bribes he may get. We normalize $u(0)$ to 0 and impose no other restrictions on $u(\cdot)$. $b - P$ is called the budget residual throughout the paper, and this term also represents local citizens' well-being or surplus. The maximum willingness to pay is v, which is equal to b, and P is the actual payment.

The timeline is as follows. In the first stage, the bureaucrat offers a take-it-or-leave-it deal d_1 to the LF. Once d_1 is accepted, the LF becomes the contractor and receives b because the first-stage mechanism can be considered as an auction with only one participant and no reserve price. d_1 also

[8] This is to say that there is an upper bound of trouble that the official can create according to the central government's guidance on the procedure; thus, c is exogenous to the official.

[9] None of the results in this paper need the first-order stochastic dominance condition of $G(\cdot)$ over $F(\cdot)$.

[10] The assumption that the budget limit is equal to the upper bound of two firms' cost distributions is made to guarantee that there is no hard constraint on the payment made to any possible contractor. Moreover, $v \geq b$ is a standard assumption which guarantees that the implementation of this project is always efficient. $v = b$ simplifies the notation without changing any result qualitatively. In addition, if the budget limits in the two stages are different, the results will not be affected.

[11] Results do not change qualitatively if an arbitrary reserve price is imposed.

serves as a necessary primary deal in receiving any possible future protection; thus, the hope of maintaining the relationship in the second stage is alive. If d_1 is rejected, the project cannot be implemented in the first stage, and the LF loses the chance to bribe in the second stage. In the second stage, the layout of the game depends on the outcome of the first stage. When d_1 is accepted, the bureaucrat offers a second secret deal d_2 to the LF in which d_2 is *sequentially optimal* given the bureaucrat's updated belief about x. If d_2 is also accepted, the application obstacle of c would be implemented unilaterally for the GF. However, if d_2 is rejected after the acceptance of d_1, or if d_1 was rejected in the first place, there is no protection in the second stage. In these two scenarios, it is not wise for the bureaucrat to implement obstacles for any firm (because these obstacles only reduce the budget residual), so both the LF and GF acquire the certificate without incurring any application costs.

There are a number of remarks to be made about the above. The first is about the competition mechanism. Although we focus on auctions, our results are not restrictive for two reasons. First, if the project is modelled as being divisible, bidder i can submit a bid in the form of (q_i, p_i), where q_i is the share of the project (or item) he needs and p_i is the price. In this way, the model can smoothly transition to a market competition game. Second, the key issue is how the bureaucrat extracts rent from the illegal connection, while an auction is just one mechanism for generating the surplus term. The second remark is about the two firms' cost distributions. Although the LF's lowest possible cost is $a > 0$, this setting does not necessarily suggest that $G(\cdot)$ first-order stochastically dominates $F(\cdot)$: concerning generality, developing countries may not lack much in labour-intensive industries. The third remark is about the obstacle. In reality, in addition to hostile entry barriers for outsiders, the bureaucrat can also implement local policies to reimburse the incumbent. This possibility would not change the results qualitatively. The fourth remark is about the implicit assumption that both the bureaucrat and LF can foresee the policy change and the threat from GF in the second stage. The interpretation is they are aware that the leader has a contingent plan to open the domestic or local market by, for example, joining some organization like the WTO or agreeing on some regional trade treaty after some fixed date.

3 Equilibrium Analysis

The equilibrium concept we adopt is the perfect Bayesian Nash equilibrium (PBE). In particular, we focus on the following strategy with two cutoffs for the LF: there exist two cutoffs x_1 and x_2 where $x_2 \leq x_1$ such that the LF accepts both d_1 and d_2 when $x \leq x_2$; the LF accepts d_1 but rejects d_2 when $x \in (x_2, x_1]$; and the LF rejects d_1 when $x > x_1$. We label these three regions L, M and H. The bureaucrat and GF's equilibrium beliefs are constructed consistently. With this construction, patronage occurs when the LF is of type "L": the H-type LF cannot receive protection even in the first stage. The M-type LF can afford only a one-time deal d_1, and only the L-type LF is able to create real difficulty for the outsider, which is regarded as "patronage" empirically.

Note that with the above assumption of the equilibrium structure, an LF with lower cost tends to accept secret deals more. This dynamic may seem to violate a common assumption that weak firms (with a high x) have stronger incentives to seek protection. However, our equilibrium construction is based on the fact that the LF's budget for bribes comes from its profit; thus, a weak LF has a lower budget for bribery even though its incentive is stronger.

In the following, we apply backward induction and analyse the second stage first.

3.1 The Second Stage

In the second stage, the LF always participates in the auction because in the event where both d_1 and d_2 are accepted, the LF clearly participates. In the event that d_2 is rejected or d_1 was rejected in the first stage, there is no gain for the bureaucrat to create trouble for any of the two firms. If LF is the only participant, it directly becomes the winner and receives b; otherwise, bidding truthfully is a weakly dominant strategy for both firms.

Using backward induction, we then consider the GF's decision on participating. Define the expected payoff of GF conditional on entry as $\Pi_e(y)$. Clearly, the GF enters when it does not observe an obstacle[12] or when $\Pi_e(y)$ is no less than c. According to the equilibrium structure, in calculating $\Pi_e(y)$, GF holds the consistent belief that x follows the truncated distribution of $F(\cdot)$ over $[a, x_2]$. Therefore, when $y \geq x_2$, $\Pi_e(y) = 0$ and the GF abandons the application; when $y < x_2$,

[12] We assume that an obstacle can be detected before any costly effort is made in the application, meaning that it is costless to determine whether the cost is 0 or c. In reality, the obstacle may be detected after a firm puts in some effort, but this assumption does not affect our results qualitatively.

the GF enters if the following inequality holds[13]:

$$\Pi_e(y) = \int_{\max\{y,a\}}^{x_2} (x-y)\frac{f(x)}{F(x_2)}dx \geq c \tag{1}$$

$\Pi_e(y)$ clearly decreases in y, meaning that the more competitive GF enters with a higher probability. The next lemma characterizes the GF's entry decision.

Lemma 1 *If $\Pi_e(0) \geq c$, there exists a unique $\bar{y}(x_2, c) \in [0, x_2)$ such that the GF participates only when $y \leq \bar{y}(x_2, c)$. If $\Pi_e(0) < c$, the GF never participates.*

Proof. It is clear that $\Pi_e(y)$ strictly decreases in y. Since $\Pi_e(x_2) = 0$, by the monotonicity and continuity of $\Pi_e(y)$, if $\Pi_e(0) \geq c$, there exists a unique $\bar{y}(x_2, c) \in [0, x_2)$ such that $\Pi_e(y) \geq c$ when $y \leq \bar{y}(x_2, c)$. If $\Pi_e(0) < c$, the GF is not able to afford c. ■

Clearly, $\bar{y}(x_2, c)$ is continuous in x_2. For ease of notation, we write $\bar{y}(x_2, c)$ as $\bar{y}$ unless otherwise specified. As c increases, the possibility that the GF successfully enters decreases. Moreover, even if the GF overcomes the obstacle, the cost c becomes a deadweight loss.

We next analyse the LF's decision on accepting d_2. Define $\pi_u(x, \bar{y})$ to be the expected payoff of the LF conditional on accepting d_2, and define $\pi_f(x, \bar{y})$ to be its expected payoff if it rejects d_2. The sub-indexes u and f stands for "unfair" and "fair", respectively. Then, we have

$$\pi_u(x, \bar{y}) = \begin{cases} [1 - G(\bar{y})](b - x) + \int_x^{\bar{y}} (y - x)g(y)dy, \text{ if } \bar{y} \geq a \text{ and } x \leq \bar{y} \\ [1 - G(\bar{y})](b - x), \text{ otherwise} \end{cases} \tag{2}$$

and

$$\pi_f(x, \bar{y}) = \int_x^b (y - x)g(y)dy \tag{3}$$

Clearly, LF accepts d_2 only if $\pi_u(x, \bar{y}) - d_2 \geq \pi_f(x, \bar{y})$. By the equilibrium construction, the marginal type of LF (with $x = x_2$) should be indifferent, suggesting that

$$d_2 = \pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y}) \tag{4}$$

[13] When the GF is indifferent to entering, we follow the convention that it chooses an action to increase other parties' payoff. If it enters, the official and LF's payoffs as a whole are increased.

In equilibrium, the bureaucrat determines d_2 first, and the LF follows the corresponding cutoff x_2. Equation (4) helps to pin down the correspondence between d_2 and x_2, which is one-to-one according to the following lemma.

Lemma 2 *The term* $\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})$ *strictly decreases in* x_2*; thus, in equilibrium,* d_2 *and* x_2 *are one-to-one linked.*

Proof. See Appendix. ■

Lemma 2 shows that in equilibrium, it is equivalent for the bureaucrat to choose x_2 instead of d_2. Define $Rev_2(x_1, x_2, \bar{y})$ to be the expected payoff of the bureaucrat in the second stage. The bureaucrat's second stage problem is to find the optimal x_2 to maximize $Rev_2(x_1, x_2, \bar{y})$. However, before doing so, he has to verify the so-called *incentive compatibility* condition in the mechanism design; namely, given any d_2, the LF has no incentive to deviate from the proposed cutoff strategy with the corresponding x_2 of d_2 determined according to Equation (4). The following proposition confirms this condition.

Proposition 1 *The LF has no incentive to deviate from the proposed cutoff strategy when* x_2 *is the corresponding cutoff of* d_2*.*

Proof. See Appendix. ■

With Lemma 2 and Proposition 1, the bureaucrat can choose the optimal $x_2 \in [0, x_1]$ knowing that LF and GF will follow the proposed equilibrium strategies contingently. Finally, we consider the bureaucrat's optimization problem in the second stage. Define $w_u(x_1, x_2, \bar{y})$ and $w_f(x_1, x_2)$ to be his payoff conditional on the acceptance and rejection of d_2, respectively. The exact expressions of $w_u(x_1, x_2, \bar{y})$ and $w_f(x_1, x_2)$ are too tedious to be included in the paper. We have the following:

$$Rev_2(x_1, x_2, \bar{y}) = prob(x \leq x_2 | x \leq x_1) \cdot w_u(x_1, x_2, \bar{y}) + prob(x > x_2 | x \leq x_1) \cdot w_f(x_1, x_2) \qquad \text{(rev(2))}$$

The bureaucrat maximizes $Rev_2(x_1, x_2, \bar{y})$ by choosing x_2 over the interval $[a, x_1]$. Clearly, because $Rev_2(x_1, x_2, \bar{y}) = Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is continuous in x_2 for any x_1, and because $[a, x_1]$ is compact, the maximum always exists. $x_2^*(x_1, c)$ is the optimal x_2 given x_1 and c. For ease of notation, we use x_2^* unless otherwise specified. Although the existence of x_2^* has been guaranteed, it is generally impossible to get a closed-form solution. Instead, we briefly describe the steps to find x_2^*. In equilibrium, $\bar{y}(x_2, c)$ increases with x_2, so there exists a lower bound of x_2 for $\bar{y}(x_2, c)$ to be positive. Define this lower bound to be $x_2(0, c)$ with $\bar{y}(x_2(0, c), c) = 0$; similarly, we can define $x_2(a, c)$. The function forms of $Rev_2(x_1, x_2, \bar{y})$ are different over $[a, x_2(0, c)]$, $(x_2(0, c), x_2(a, c)]$ and $(x_2(a, c), x_1]$, so the bureaucrat needs to first locate the optimal x_2 over each of the three regions. x_2^* can be found by comparing the maximum $Rev_2(x_1, x_2, \bar{y})$ that these three maximizers generate.

3.2 The Foundation of Patronage

We know from previous analyses that L-type LFs can get continuous protection, which is regarded as "patronage" by outside observers. However, the existence of L-type LFs is endogenously defined by the bureaucrat's optimization problem. In this subsection, we discuss the conditions under which there can always be L-type LFs. Mathematically, this result requires $x_2^*(x_1, c)$ to be strictly larger than a for any values of x_1 and c. Intuitively, when the $u(\cdot)$ part of the bureaucrat's utility function is very important compared to the monetary transfer, or when the ex-ante advantage (if any) of the GF is sufficiently high (partly measured by the value of a), the bureaucrat has no incentive to offer any d_2. From the perspective of rent-seeking, the minimum willingness of the bureaucrat to offer any deal should be no more than a marginal LF's maximum willingness to pay by the equilibrium construction.

Although the necessary and sufficient condition can be derived, it is too tedious to offer any immediate interpretation. Fortunately, we are able to identify a concise and intuitive sufficient condition.

Proposition 2 *If $u(z) < z$ for any $z > 0$, we must have $x_2^* > a$. Therefore, corruption occurs in the second stage with a positive probability.*

Proof. See Appendix. The necessary and sufficient version is also included in the proof. ■

From this proposition, we can see that as long as the bureaucrat's concern for local citizens' well-being is not as much as the concern that citizens have for themselves, patronage occurs with a positive probability. Moreover, patronage occurs more when the $u(\cdot)$ part is less important or when the ex-ante advantage of the GF is smaller. This relation suggests that the foundation of patronage comes from the transfer of the welfare concern from local citizens to the bureaucrat.

In reality, $u(z) < z$ is not restrictive. We next discuss two interpretations of the $u(\cdot)$ part.

1. $u(\cdot)$ as the expected utility from possible promotion

When $u(\cdot)$ is interpreted as the bureaucrat's concern for promotion as a reflection of the central government's benevolent incentive, the budget residual $z = b - P$ serves as a natural measure representing the bureaucrat's performance. Ideally, a bureaucrat's true ability should be used as the measure, but this ability is generally unobservable, so this tangible variable that is closely related can be examined instead. In this way, $u(z)$ can be regarded as the expected payoff from promotion. If the explicit and implicit materialistic rewards associated with the higher position are objective, $u(z) < z$ is natural because the central government faces a budget constraint: its expenditure on rewarding this bureaucrat's contribution cannot exceed the contribution itself. This explanation can be better understood if we think of the whole government as a firm: the term $u(\cdot)$ can be viewed as a transfer from the higher hierarchy to the bureaucrat like a commission, and $u(z) < z$ means that this transfer is affordable.

2. $u(\cdot)$ as the moral concern

When $u(\cdot)$ represents the bureaucrat's intention to improve local citizens' well-being, $u(z) < z$ is also natural since the bureaucrat's personal happiness from local citizens' surplus normally cannot be compared to the surplus itself. Moreover, since the bureaucrat can also enjoy the benefits of this periodic project, his individual well-being improvement cannot exceed that of all local citizens.

We next dig deeper into the first interpretation. We normalize the payoff from no promotion to be 0 and express $u(z)$ as the expected payoff from promotion with the following form.

$$u(z) = \Pr(promotion|z) \cdot E(payoff\ from\ promotion|z) \tag{5}$$

$\Pr(promotion|z)$ is chosen by the central government. How should this decision depend on the performance measure $z = b - P$? An educated guess is that $\Pr(promotion|z)$ strictly increases with z and is strictly convex because as z increases, it becomes more difficult to marginally further increase z. Therefore, the ability to increase the performance measure z when the bureaucrat is already doing a good job is a strong indication of his talent. Intuitively, the bureaucrat does not prove his talent through routine work but rather through difficult tasks.

We pick a specific function form of $u(\cdot)$ where $\Pr(promotion|z)$ is strictly convex and $u(z) < z$. Assume $\Pr(promotion|z) = z^2/(b-a)^2$ and $E(payoff\ from\ promotion|z) = k(b-a)$, where k is a parameter, and that $k \leq 1$[14]. Clearly, because the budget residual $z \leq b-a, \Pr(promotion|z) \in [0,1]$ and is strictly convex. The importance of strict concavity can also be seen by assigning values to the parameters: when $k = 1, b - a = 1$, and z increases from 0.7 to 0.8, the chance of promotion is increased from 0.49 to 0.64. This result suggests that when it is marginally more difficult to outperform peers, the reward should pay off. Finally, $u(z) < z$ also holds: since $k \leq 1$ and $z^2/(b-a)^2 < z/(b-a)$ ($z = (b-a)$ with 0 probability in this model), $u(z) = k(b-a)z^2/(b-a)^2 < z$.

3.3 The First Stage

In the first stage, the LF is the last mover and decides whether to accept d_1: accepting would build a channel between the firm and the bureaucrat, enabling a second secret deal d_2; rejecting leads to a fair competition against the GF in the second stage. The LF compares its contingent payoffs and makes the optimal decision. In equilibrium, the LF should have no incentive to deviate from the proposed strategy when x_1 is the corresponding cutoff induced by d_1. The next lemma presents this correspondence.

Lemma 3 *In the first stage, if $x \leq x_1$, the LF accepts d_1 where $x_1 = b - d_1$.*

Proof. With backward induction, we know that the LF with $x \in [a, x_1]$ has no incentive to deviate in the second stage: if $x \in [a, x_2)$, the contingent equilibrium payoff from accepting d_2 is strictly higher than that from fair competition; if $x \in [x_2, x_1]$, the LF has no incentive to accept d_2. Given the second-stage payoff, the LF has no incentive to reject d_1: if it does, the first-stage payoff decreases from a strictly positive number to 0 (except for $x = x_1$, who is indifferent), while the second-stage payoff is weakly worse off because the bureaucrat misjudges its type. Therefore, such a firm has no incentive to deviate.

Next, we consider the LF with $x \in (x_1, b]$: if this LF deviates to accepting d_1, its first-stage payoff decreases from 0 to a strictly negative number, while its second-stage payoff does no change (and is equal to the payoff from fair competition).

[14] $k \leq 1$ is not restrictive because $b - a$ is normally very large for government procurement contracts. In addition, the level of the promotion normally does not depend on z because bureaucrats get promoted to the next level in the hierarchy.

This completes the proof. ■

By backward induction, Proposition 1 and Lemma 3 together imply that given d_1, no player has an incentive to deviate in the rest of the game. Therefore, fixing d_1, the strategies and consistent beliefs constitute a PBE.

Proposition 3 *For any $x_1 \in [a, b]$, the following strategies and consistent beliefs constructed from the strategies constitute a perfect Bayes equilibrium in this dynamic game. The bureaucrat offers $d_1 = b - x_1$ and $d_2 = \pi_u(x_2^*, \bar{y}(x_2^*, c)) - \pi_f(x_2^*, \bar{y}(x_2^*, c))$ where x_2^* is the maximizer of $Rev_2(x_1, x_2, \bar{y}(x_2, c))$. The bureaucrat constructs no obstacle when d_1 is rejected or when d_1 is accepted but d_2 is rejected. The LF accepts both deals when $x \in [a, x_2^*]$; accepts d_1 but rejects d_2 when $x \in (x_2^*, x_1]$; and rejects d_1 when $x \in (x_1, b]$. When there is an obstacle, the GF applies for the certificate when $y \leq \bar{y}(x_2^*, c)$. When there is no obstacle, both the LF and GF apply for the certificate. In the auction, bidders adopt the weakly dominant strategy of sincere bidding.*

Define $Rev(x_1)$ as the bureaucrat's overall expected payoff and $Rev_1(x_1)$ as his first-stage payoff. The next task is to find the optimal d_1 to maximize $Rev(x_1)$. Since the values of x_1 and d_1 are one-to-one linked, this task is equivalent to finding the optimal x_1. The maximization problem of the bureaucrat thus becomes:

$$\max_{x_1} Rev(x_1) = Rev_1(x_1) + \delta F(x_1) Rev_2(x_1, x_2, \bar{y}(x_2, c)) \quad (6)$$
$$+\delta[1 - F(x_1)] \cdot E(second\ stage\ payoff\ from\ a\ fair\ competition | x > x_1)$$
$$s.t. x_2 \text{ is sequentially optimal given } x_1 \text{ (equivalently, } x_2 \text{ is } x_2^*(x_1, c))$$

Define the optimal x_1 to be x_1^*, the existence of which is guaranteed by the continuity of $Rev(x_1)$ and the compactness of $[a, b]$. Because of the complexity of $Rev(x_1)'s$ function form, it is very difficult to characterize the analytical properties of x_1^*, but we are still able to derive the following result by assuming strict concavity (in x_2) of the bureaucrat's second-stage revenue.

Proposition 4 *If $Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is strictly concave in x_2 for any x_1, $Rev(x_1)$ should also be strictly concave in x_1. Define r^* as the unique solution to $b - \frac{F(r)}{f(r)} = r$[15] and $\hat{x}_2$ $ASx_2^*(1, c)$, which is*

[15] The solution exists because when $r = a$, the left-hand side is equal to $b > a$, which is the right-hand side; when

the unique maximizer of $Rev_2(1, x_2, \bar{y}(x_2, c))$*; then,* x_1^* *can be characterized as follows:*

$$\begin{cases} x_1^* = r^*, \text{ if } r^* \geq \hat{x}_2 \\ x_1^* \in (r^*, \hat{x}_2) \text{ if } r^* < \hat{x}_2, \text{ and } x_1^* \text{ is the unique solution to } \frac{dRev(x_1)}{dx_1} = 0 \end{cases}$$

Proof. See appendix. ■

Strict concavity is imposed only for illustration purposes. This property helps pin down a unique x_1^* and makes it easier to analytically characterize x_1^*.

Interestingly, if there is no bureaucrat and the local citizens can directly represent themselves, surplus maximization in the two stages should be regarded as separate problems. In the first stage, because there is only one firm, a reserve price should be imposed to guarantee an ex-ante positive consumer surplus. The optimal reserve price (and the corresponding x_1^*) is then r^*. From Proposition 4, we know that with corruption, x_1^* picked by the bureaucrat will always be no less than r^*. Therefore, the bureaucrat does allow in more LFs, which is socially more efficient in the first stage than the decision made by local citizens themselves.

3.4 Social Welfare

In this subsection, we investigate the impact of patronage on social welfare. In calculating social welfare, we take the GF into account and define social welfare as the expected net gain of the project; namely, when the project is implemented, the welfare is the difference between its social value and the winning contractor's true cost. This definition means that we do not count the $u(\cdot)$ part of the bureaucrat's utility for two reasons. First, when $u(\cdot)$ is interpreted as the expected utility from a possible promotion, there is normally only one position but several candidates; therefore, all candidates' expected payoffs from promotion should add up to the value of the position, which does not vary with the social welfare. Second, when $u(\cdot)$ is interpreted as the bureaucrat's personal joy from the implementation of the project, it is a psychological measure of satisfaction that cannot be counted as social welfare. Moreover, since the bureaucrat himself is also one of the local citizens, his welfare improvement has already been counted.

$r = b$, the left-hand side is equal to $b - 1/f(b) < b$, which is the right-hand side. The solution is unique because $F(\cdot)$ satisfies the monotonic likelihood ratio property.

We first consider the benchmark case in which secret deals are never possible. The project could not be implemented in the first stage, but the competition in the second stage is always fair. The social welfare can be written as

$$\begin{aligned} SW_{no\ corruption} &= \delta[\int_0^a \int_a^b (b-y)f(x)g(y)dxdy \\ &\quad + \int_a^b \int_a^y (b-x)f(x)g(y)dxdy + \int_a^b \int_a^x (b-y)f(x)g(y)dydx] \end{aligned}$$

When secret deals can be made for political protection, they play a positive role in the first stage. Nonetheless, these deals fertilize the soil for corruption and enable follow-up deals in the second stage, which hurts the society. Welfare loss comes from two sources: first, when the GF is not able to afford c, its cost may still be lower than that of an incumbent LF; second, when the GF can afford c to get the certificate, c does not go into any player's pocket and is thus wasted. Compared to the benchmark case, the welfare gain is $\int_a^{x_1^*}(b-x)f(x)dx$, while the welfare loss can be written as

$$\text{welfare loss} = \delta \begin{cases} \int_0^{\bar{y}} \int_a^{x_2} cf(x)g(y)dxdy + \int_{\bar{y}}^a \int_a^{x_2} (x-y)f(x)g(y)dxdy \\ \qquad + \int_a^{x_2} \int_a^x (x-y)f(x)g(y)dydx, \text{ when } \bar{y} < a \\ \int_0^{\bar{y}} \int_a^{x_2} cf(x)g(y)dxdy + \int_{\bar{y}}^{x_2} \int_{\bar{y}}^x (x-y)f(x)g(y)dydx, \text{ when } \bar{y} \geq a \end{cases},$$

where x_2 is the sequentially optimal x_2^* given x_1^*.

It is possible that the existence of secret deals increases the overall welfare. Intuitively, when δ is relatively small, or when the outsider does not have much cost advantage ex-ante (e.g., a is sufficiently small), the first-stage gain dominates the second-stage loss. The following lemma characterizes a case that fits this intuition.

Lemma 4 *When $a = 0$, $G(\cdot) = F(\cdot)$, the existence of secret deals increases social welfare for any $u(\cdot), c$ and δ.*

Proof. See appendix. ■

In this lemma, the GF does not have any ex-ante cost advantage. However, by continuity, when a is sufficiently small, or when $G(\cdot)$ and $F(\cdot)$ are sufficiently close pointwise, the result should still hold. This feature may fit stories in China during its initial development period: labour costs were small back then, so non-state-owned firms normally prospered in labour-intensive industries. These firms do not have much disadvantage compared to international competitors that use capital-intensive technologies.

Although we model only two stages in this model, there are more stages in reality, suggesting that the welfare loss will hurt the society in the end, even with the case in Lemma 4. Ironically, this is especially true for countries with a discount rate close to 1: these governments, including those under stable authoritarian systems, are patient enough because they have the ambition to make long and lasting plans for development. However, the existence of corruption also causes high losses in social welfare to these governments.

4 The Case of the Uniform Distribution

In this subsection, we assume that $F(\cdot)$ is the truncated uniform distribution over $[a, 1]$ and that $G(\cdot)$ is the uniform distribution over $[0, 1]$. In this case, $\bar{y}(x_2, c)$ can be solved in the closed form. From Equation (1), $\bar{y}$ is a piecewise function

$$\bar{y}(x_2, c) = \begin{cases} 0, \text{ if } a \leq c \text{ and } x_2 \in [a, \min\{2c-a, x_1\}] \\ \frac{x_2+a-2c}{2}, \text{ when } x_2 \in (\max\{a, \min\{2c-a, x_1\}\}, \min\{x_1, 2c+a\}) \\ x_2 - \sqrt{2c(x_2-a)}, \text{ when } x_2 \in [\min\{x_1, 2c+a\}, x_1] \end{cases} \tag{7}$$

The three regions admit values of $\bar{y}$ as $\bar{y}(x_2, c) = 0, \bar{y}(x_2, c) \in (0, a)$ and $\bar{y}(x_2, c) \in [a, x_2)$, respectively, where $\bar{y}(x_2, c) = 0$ is only possible when $a \leq c$. Conditional on the LF being of the L type, these regions have different economic interpretations of patronage.

i) $\bar{y}(x_2, c) = 0$

The GF's entry is completely prevented; thus, the incumbent wins the contract without any competition. Intuitively, this case occurs when c is large compared to a. From Equation (7), we know there is also an extreme case where $\bar{y}(x_2, c)$ is always equal to 0: $2c - a \geq 1$.

ii) $\bar{y}(x_2, c) \in (0, a)$

A highly competitive GF can still enter; if such a firm enters, the LF loses immediately because $x \geq a > \bar{y}$. However, the GF with $y \in [\bar{y}, a)$ is hurt the most, although it has a certain cost advantage over any LF: this GF is certain to win under the fair competition but is blocked from entry under patronage.

iii) $\bar{y}(x_2, c) \in [a, x_2)$

This case can only exist when $2c + a < 1$. Now, the GF with a certain cost advantage ($y < a$) can always overcome the barrier c and wins the auction because $y < a \leq x$. Meanwhile, a highly competitive LF still stands a chance after the GF's entry since it is possible that $y > a$.

We next look at a representative form of $u(z)$: $u(z) = kz, k < 1, a > 0$. We present four sets of values of a and c, fixing $k = 0.1$ and $\delta = 0.9$. The first, third and fourth examples correspond to cases above, while the second example is the cutoff case for Case i) and Case ii). It can be verified that $Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is strictly concave in x_2 for any x_1, which fits Proposition 4. We calculate $x_1^*, x_2^*(x_1^*, c), \bar{y}(x_2^*, c)$, the ex-ante probability of no entry for the GF, the expected net gains for the LF and the bureaucrat (B) under patronage, and the percentage change in social welfare under patronage. The probability of no entry is the probability of the LF being type L times the probability that the GF cannot afford c. The net gain is defined as a party's (discounted) sum of the expected payoffs in the two stages minus its payoff in the benchmark case where corruption cannot occur. For the LF, we calculate the average expected net gain for $x \in [a, x_2]$. The total rent is the sum of the net gains for both parties. The percentage change in social welfare is defined as the net change in social welfare compared to the benchmark case.

$u(z) = kz, \quad k = 0.1, \quad \delta = 0.9$

(a, c)	(x_1^*, x_2^*)	$\bar{y}$	Pr(*no entry*)	LF's gain	B's gain	Total rent	ΔSW
$(0.1, 0.4)$	$(0.568, 0.568)$	0	0.52	0.321	0.363	0.684	(+)55.3%
$(0.2, 0.4)$	$(0.6, 0.6)$	0	0.5	0.284	0.325	0.609	(+)47.8%
$(0.3, 0.4)$	$(0.65, 0.638)$	0.069	0.45	0.228	0.289	0.517	(+)40.8%
$(0.2, 0.1)$	$(0.6, 0.571)$	0.298	0.325	0.125	0.331	0.456	(+)52%

We first look at the first three examples where only the values of a are different. The values of x_1^* and x_2^* cannot be directly compared because the distributions are different. The probability of the formation of an alliance in the first stage is 52%, 50% and 50%, while the probability of patronage conditional on x_1^* is 100%, 100% and 98.2%. As a increases, $\bar{y}(x_2^*, c)$ increases while Pr(*no entry*) decreases. These facts are intuitive because a measures the ex-ante disadvantage of the LF: as a increases, the budget for the LF to bribe decreases, and it becomes more costly for the bureaucrat to make deals since he sacrifices more of his payoff from the loss in the $u(\cdot)$ part. Both parties' gains, the ally's total rent and the percentage change in social welfare decrease mainly because of the first stage: when the disadvantage is smaller, the gain from "efficient corruption" is larger. Moreover, the relative loss in the second stage under patronage is also smaller when the incumbent is more competitive.

We next compare the second and the fourth example, where the values of a are the same. The probability of the formation of an alliance is the same, while the probability of patronage decreases from 100% to 95.2%. As c deceases, Pr(*no entry*) decreases while $\bar{y}(x_2^*, c)$ increase, which is also intuitive. Interestingly, the LF's gain decreases significantly, while the bureaucrat's gain slightly increases. This is a good example that illustrates the bureaucrat's conflicting incentives: although the decrease in c means that the bureaucrat cannot ask for too much from the LF, he also benefits from the entry of a competitive GF. In this example, the gain outweighs the loss. The percentage change in social welfare also increases as c decreases. This pattern is because the welfare loss in the second stage is smaller: although there is no loss caused by the waste of c when $\bar{y} = 0$, the loss caused by the wrong assignment of the project is more than compensated by the savings in c.

Overall, there are some interesting findings. First, the sustainability of the ally increases with the decrease of $\bar{y}(x_2^*, c)$: in the first two examples where $\bar{y} = 0$, the values for x_1^* and x_2^* are the same, indicating that the potential entry of a GF does not yield any threat to the ally because any LF that can enter in the first stage will be able to afford d_2. As $\bar{y}(x_2^*, c)$ increases, the probability that an incumbent LF cannot afford d_2 increases. Second, in all four cases, the bureaucrat has more share of the total rent, even though the rent here also includes the bureaucrat's change in the $u(\cdot)$ part. This result is closely related to the fact that the bureaucrat has the right to make offers. However, LF also has its own advantage: its cost x is private information, so the bureaucrat has to deduct the implicit information rent when making offers. Nonetheless, this information advantage gradually vanishes as the duration of the alliance increases because the bureaucrat can constantly learn from more contact with the LF. Finally, in this example, the social welfare for all increases under patronage because the cost disadvantage of the LF comes only from a; thus, the effect of "efficient corruption" is dominant. If $G(\cdot)$ first-order dominates $F(\cdot)$, even with the same support, the result may change.

5 The Example of China

At the end of the 1970s, the Chinese government began a period of economic reform and realized enormous economic development. In this section, we use China as an example because the path of its economic growth fits that of stable authoritarian systems with the following features. First, the Communist Party of China (CPC) maintains its powerful governance with the support of a hierarchy of politicians and bureaucrats. Although the CPC experienced a crisis at the end of the 1980s, the party ensured the legitimacy of its governance through large and sustained economic growth. Second, a high growth rate is always accompanied by high levels of corruption. According to the Transparency International[16], China remains one of the countries with the highest levels of corruption. Third, Deng Xiaoping's southern tour of 1992[17] served as a milestone in China's economic development, illustrating a clear two-stage pattern in which international firms entered significantly more frequently in the second stage. Finally, at first, the Chinese economy did not seem to be affected by severe corruption because it maintained an annual growth rate of nearly 10% for thirty years after 1978.[18] However, the trend of high economic growth changed after the global financial crisis of 2008 and did not resume afterwards. The slowing down problem in China has garnered much attention because of the concern that it will decelerate the recovery of the global economy. Having exhausted the advantages of being a developing country, the government attempted to promote R&D activities to maintain efficiency, but so far, the progress is limited, as noted by Zilobotti (2017). We discuss this observation in more detail below.

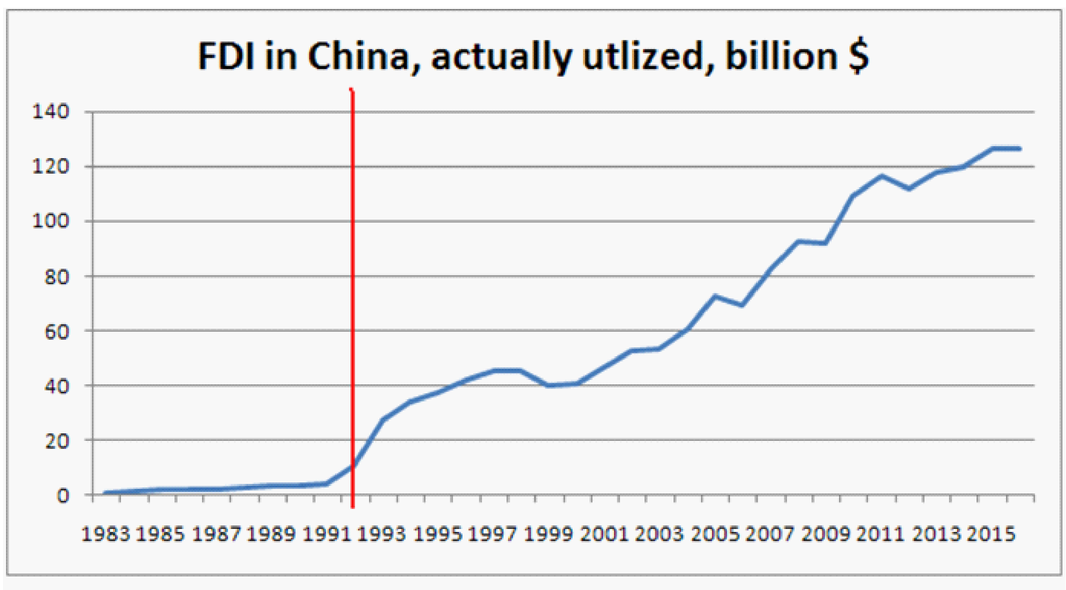
After the Cultural Revolution, Deng Xiaoping returned to the highest position of the CPC and targeted economic growth as the primary goal. Before the formal abandonment of the central planning system, Deng employed effective power sharing with local bureaucrats and enacted a series of institutional reforms to increase the flexibility of local governments. This practice was meant to create a hospitable environment for economic growth. Although Deng attempted to develop an

[16] Readers can visit www.transparency.org for more information.

[17] In the third Plenary Session of the 14th CPC Central Committee in 1993, the goal of the CPC's economic reform was stated to be to formalize market-oriented mechanisms and advance economic legislative procedures. This pronouncement is consistent with our second-stage assumption about the central government.

[18] Chen (2014) notes that the form of corruption changes in the 1990s after the fiscal recentralization, which may have caused the change in the local government's role from a helping hand to a grabbing hand.

export-led economy[19], the primary driving forces of the economic growth of the 1980s were townships and private enterprises[20], which was not predicted even by Deng himself (Jin and Qian (1998)). This pattern was mainly due to the fact that the central government passed along the arbitrary right to manage local economic affairs to local bureaucrats. With the establishment of the dual-price system, local bureaucrats had the opportunity to sell production materials to townships and private enterprises that were on the market track. The fact is modelled by our first-stage story because without the necessary materials, these enterprises cannot run their businesses. The difference between the planned price and the market price became government income, while the difference between the market price and the value of the marginal product became the rent that the ally could share. To maximize the size of the pie, the bureaucrat had an incentive to form political connections with enterprises with high potential, as our model predicts. The corruption problem became severe towards the end of the 1980s.[21]



Even in the mid of the 1980s, there was a consensus in the academic world and the central government of China that the domestic market should be unified.[22] Therefore, the ally could predict

[19] Four special economic zones and fourteen coastal open-boarder municipalities were established for this purpose.

[20] This is again consistent with our first-stage story because our model implies that the economic development was driven by the successful entry of LFs.

[21] Corruption was regarded as an important trigger for the Tiananmen Square protests of 1989.

[22] The central government drafted a plan for the simultaneous reform of the price, financial and tax system in 1986,

that the economy system would be more market oriented. After Deng's 1992 southern tour, the central government of China attempted to mitigate the corruption problem and promote economic growth by establishing a formal market system. More open policies were enacted. As a result, according to Figure 1, the actual utilized foreign investment increased from 4.37 billion (US dollars) in 1991 to 11.01 billion in 1992 and 27.52 billion in 1993. The average annual growth rate jumped from 21.56% (1983 to 1991) to 39.76% (1992 to 1998) before the Asian financial crisis. This growth indicates that the economy enters the model's "second stage", characterized by a rapid growth in FDI, after 1992. However, the central government's endeavours hit the obstacle of regional patronage. Wei (2000) notes that corruption and red tape are critical in preventing foreign investment from entering China. This fact is consistent with our second stage story. In addition, we raise a point that red tape is endogenously generated with the formation of the secret ally even in the first stage. Wang et al. (2007) calculate the marketisation index for all of China and find that the distribution is significantly biased towards the coastal regions. Our model provides a theoretical explanation for this observation: the coastal regions of China attract 85% of all FDI, so the chance for at least one GF to break through the barrier would be higher. Our results also help predict a secondary effect of FDI[23]: the improvement of regional business environments.[24] According to North (1990), the improvement of mechanisms has a profound influence on economic development.

We end this section with one caveat: although FDI can advance better policies, there is not sufficient evidence to believe that China will inevitably resume high growth. There are three concerns here. First, in most of central and western China, secret allies are stable as a result of the lack of attractiveness for potential highly competitive international firms. This pattern corresponds to the case where c is high while $G(\cdot)$ does not have much advantage, which would lead to $\bar{y} = 0$. The central government may accelerate the process by enacting region-specific policies to reduce c and improve $G(\cdot)$. Second, competitive domestic firms have already begun to improve their technologies by utilizing the advantages of backwardness. This dynamic indicates that in the long run, inter-

which was not carried out.

[23]The traditional theory on FDI indicates that FDI can increase the host country's capital stock and develop its technologies due to the spillover effect. China benefits from both effects, especially the latter (Cheung and Lin, 2004; Liu, 2008).

[24]A recent study on China by Long et al. (2015) indicates that regions with higher FDI have better business environment for investors even after controlling for endogeneity.

national firms have to be even more competitive to retain their cost advantages. Third, a portion of international firms that successfully entered are also adapting to the corrupt culture of China. There is anecdotal evidence that some multinationals deliberately hire close relatives of bureaucrats to build political connections.

6 Discussion

In this section, we discuss some of our modelling assumptions and possible extensions.

Interjurisdictional competition: If two bureaucrats in adjacent cities potentially compete for the same higher-ranking position, they would be less willing to offer the secret deal for their respective incumbents. The reason is that their regions of responsibility may both be candidates for receiving investment from a competitive international firm. Therefore, turning the GF down not only hurts a bureaucrat's promotion probability but also increases that of his direct competitor. This strategic concern would make the bureaucrat less willing to create the barrier.

Nationalism: Nationalism is a typical card played by authoritarian leaders to defend patronage. To allow this possibility in this model, we can allow the performance evaluation to be sensitive to the identity of the winner in the second stage. Specifically, for the same amount of surplus z generated, the central government strictly prefers the winning contractor to be a local or domestic firm. In this way, patronage would be more frequently observed on the equilibrium path. However, this dynamic does not suggest that the bureaucrat necessarily asks for more from the incumbent LF because now that the secret deal also benefits the bureaucrat more.

Driving force of initial economic development: In our model, the potential entry of the GF is exogenous and is independent of the outcome in the first-stage game. This assumption is not restrictive when we consider that there may be multiple LFs in the first stage. Assume there are m LFs in the first stage seeking protection. Then, the probability that one LF can successfully build a connection increases with m, which is also why empirically, better endowed regions can realize economic growth earlier and more easily (e.g., coastal regions of China) because the number of potential competitive LFs is larger. Including multiple LFs will cause an empirical concern instead of technical difficulties, and this feature is discussed next.

Number of possible contractors: We assume one local (or domestic) firm and one outsider to keep the analysis simple. If there are multiple outsiders when the market is open, as long as they face the same c as the obstacle, the analysis would not be changed qualitatively. However, if the number of LFs is greater than one, there is an issue of how corruption works in the first stage. If the bureaucrat approaches different LFs and asks them to indicate their willingness to pay, they will be reluctant to tell the truth because it will reveal information about their true cost, which will then be used against them in the second stage. This dynamic is also why the second-price sealed-bid auction cannot be applied. Although English auctions can identify the most competitive firm as the winner and mitigate the winner's concern for information revelation, it is difficult to imagine that this mechanism works in reality because it requires the bureaucrat to interact very actively with all LFs, which is too risky. However, except for this concern, having multiple LFs does not change the result qualitatively with an English auction.

Full commitment: In the current model, the bureaucrat is sequentially rational, corresponding to the non-commitment case in the literature of mechanism design: d_2 is offered after the acceptance of d_1, meaning that d_2 is sequentially optimal conditional on d_1. In contrast, there is a full commitment case in which the bureaucrat can extract the largest payoff ex-ante by announcing d_1 and d_2 together in the first stage. The LF then chooses whether to accept only d_1 or both. In equilibrium, there would still be two cutoff values, x_1 and x_2, but the optimal x_2 no longer depends on x_1. The full commitment case is easier to analyse but is not realistic because normally, the bureaucrat does not have the commitment power to stick to the d_2 he promised when the game reaches the second stage.

Anti-corruption actions: We do not assume that there is an explicit probability of being caught when secret deals are made. If the probability and the expected loss conditional on being caught do not depend on the size of secret deals, all analytical results should go through. Otherwise, suppose that the probability of being caught in stage i is a strictly increasing function $p(d_i)$ and that the expected payoff of being caught is a negative constant $-q$. The expected gross monetary payoff conditional on acceptance of the i^{th} deal is then $[1-p(d_i)]d_i-p(d_i)q$ instead of d_i. Since the function $p(\cdot)$ is not determined by the bureaucrat, we believe the results should still hold qualitatively if $[1-p(d_i)]d_i-p(d_i)q$ increases in d_i. However, if the probability of being caught in the second stage depends on d_1 and d_2 together, which means that the bureaucrat always bears the risk of previous corrupt behaviours, our results will fail to hold and the analysis in the first stage also becomes very complicated, even intractable.

Auction format: We assume a second-price sealed-bid auction, which is strategically equivalent to the English auction in our model. In reality, procurement auctions held by the government normally take the form of first-price sealed-bid auctions for two reasons: second-price sealed-bid auctions are vulnerable to tacit collusion and may raise only a low profit in some cases. However, an ascending English auction receives little criticism for the first aspect while maximizing the government's profit (which can also be viewed as the society's welfare excluding the contractor), which is not always the goal of the bureaucrat in charge. More importantly, if a first-price sealed-bid auction is applied in the second stage, the model will immediately lose tractability: the second-stage competition will always be asymmetric, even if $F(\cdot)$ and $G(\cdot)$ are the same, and asymmetric first-price auctions can seldom be solved analytically.

The procedural obstacle c: We assume that the cost to overcome the obstacle is c, while in reality, this obstacle can take multiple values and can be viewed as a continuous variable. Theoretically, as long as c takes discrete values, the equilibrium is not changed qualitatively: the bureaucrat can offer a menu of d_2 conditional on the value of c and induce different cutoff values of x accordingly. However, when c is continuous, there is the problem of information revelation: if there exists a strictly monotonic equilibrium such that LFs with different costs choose different c values, the GF can detect the type of LF immediately upon seeing c. This revelation would greatly affect GF's incentive to enter and could ruin the existence of the PBE.

7 Concluding Remarks

In this paper, we provide a novel explanation for the frequently observed growth dilemma in authoritarian systems using a game theoretic approach. This explanation assumes that local bureaucrats care about citizens' well-being as well as their own monetary payoffs and that political connections have to be built and maintained. In our model, patronage thus endogenously appears on the equilibrium path when the LF's cost is low. The political connection between a bureaucrat and such LFs explains why the initial grease becomes the sand to the wheels of economic growth within the same model.

Our results have several policy implications. First, firms that can acquire and maintain protection are highly competitive among their local and domestic peers. If R&D costs are positively associated with production costs (which is an educated guess), the central government should enact policies to encourage firms to invest in R&D activities instead of buying protection.[25] Second, the foundation of corruption is embedded deeply in the bureaucrats' incentives. To reduce corruption and to control for the costs of supervising[26], the central government can take three types of indirect measures. The first measure is to make the performance evaluation more dependent on local citizens' well-being, thus making it more costly to make secret deals.[27] Fisman and Wang (2015) raise a similar point on the importance of the alignment of incentives, although their study is about the impact of political connections on workplace fatalities. The second measure is to create a more hospitable business environment to better attract international firms. When more competitive GFs invest, both the bureaucrat and the incumbent would be less willing to make secret deals. The third measure is to lower the upper bound of the executive obstacles that local bureaucrats can impose. This action may even benefit the bureaucrat according to our example in section 4. The central government can formalize the central-local relationship and clarify the respective rights and responsibilities. In 2016, the Prime Minister of China, Li Keqiang, officially stated that the central government will clarify overlapping responsibilities with regional governments to reduce unnecessary executive costs incurred during business operations and government interventions.

There are several interesting extensions left for future research in addition to the interjurisdictional competition and nationalism lines of research that we mentioned. For example, R&D can be introduced as an option for the incumbent LF. Note that once the GF enters successfully and wins the auction, the LF cannot possibly win in the future in any fair competition. If the LF anticipates that the central government would further encourage fair competition, it can only reduce its cost through R&D. Another extension may be to model bureaucrats' personalities: a measure of integrity, which affects the incentive to take bribes, may be introduced.

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[25] R&D is important for resuming economic growth in China because according to Brandt and Zhu (2010), the fundamental reason for the decreased GDP growth rate is the deceleration of improvement in TFP. Bai et al. (2016) indicate that this problem deteriorates after the massive economic stimulus program of 2008.

[26] Acemoglu and Verdier (2000) develop a model incorporating supervision costs and conclude that the second-best intervention may involve a certain fraction of officials accepting bribes.

[27] For example, the Chinese government is believed to pursue further improvements in citizens' well-being because President Xi Jinping emphasized this as one of the CPC's primary goals in the 19th CPC national congress in 2017.

References

[1] Acemoglu, D., and Robinson, J. (2005). Economic Origins of Dictatorship and Democracy. Cambridge University Press.

[2] Acemoglu, D., and Verdier, T. (2000). The Choice between Market Failures and Corruption. *The American Economic Review*, 90(1), 194-211.

[3] Ades, A., and Di Tella, R. (1999). Rents, Competition, and Corruption. *The American Economic Review*, 89(4), 982-993.

[4] Aidt, T. (2003). Economic Analysis of Corruption: a Survey. *The Economic Journal*, 113 (491), 632-652.

[5] Ausubel, L., Deneckere, R., and Cramton, P. (2002). Bargain with Incomplete Information. Handbook of Game Theory, Vol 3., Chapter 50.

[6] Bai, C., Hsieh, C., and Song, Z. (2016). The Long Shadow of China's Fiscal Expansion. *Brookings Papers on Economic Activity*, 2016 Fall, 129-165.

[7] Birney, M., (2014). Decentralization and Veiled Corruption under China's "Rule of Mandates". *World Development*, 53, 55-67.

[8] Banerjee, A., (1997) A Theory of Misgovernance. *The Quaterly Journal of Economics*, 112(4), 1289-1332.

[9] Brandt, L. and Zhu., X. (2010). Accounting for China's Growth. Univeristy of Toronto working paper.

[10] Burguet, R., and Che, Y.K. (2004). Competitive Procurement with Corruption. *The RAND Journal of Economics*, 35(1), 50-68.

[11] Burguet, R. (2017). Procurement Design with Corruption. *American Economic Journal: Microeconomics*, 9(2), 315-341.

[12] Cai, H., and Treisman, D. (2005). Does Competition for Capital Discipline Governments? Decentralization, Globalization, and Public Policy. *The American Economic Review*, 95(3), 817-30.

[13] Chen, K. (2004). Fiscal Centralization and the Form of Corruption in China. *European Journal of Political Economy*, 20, 1001-1009.

[14] Cheung, K. and Lin, P. (2004). Spillover effects of FDI on innovation in China: Evidence from the provincial data. *China Economic Review*, 15, 25-44.

[15] Choi, J.P., and Thum, M. (2003). the Dynamics of Corruption with the Ratchet Effect. *Journal of Public Economics*, 87, 427-443.

[16] Cooper, M.J., Gulen, H., and Ovtchinnikov, A.V. (2010). Corporate Political Contributions and Stock Returns. *The Journal of Finance*, 65(2), 687-724.

[17] Coulomb, R., and Sangnier, M. (2014). The Impact of Political Majorities on Firm Value: Do Electoral Promises or Friendship Connections Matter?. *Journal of Public Economics*, 115, 158-170.

[18] Faccio, M. (2006). Politically Connected Firms. *The American Economic Review*, 96 (1), 369-386.

[19] Faccio, M., Masulis, R.W., and McConnell, J.J. (2006). Political Connections and Corporate Bailouts. *The Journal of Finance*, 61(6), 2597-2635.

[20] Fan, C.S., Lin, C., and Treisman, D. (2009). Political Decentralization and Corruption: Evidence from around the World. *Journal of Public Economics*, 93, 14-34.

[21] Fisman, R. (2001). Estimating the Value of Political Connections. *The American Economic Review*, 91(4), 1095-1102.

[22] Fisman, R., and Gatti, R. (2002). Decentralization and Corruption: Evidence across Countries. *Journal of Public Economics*, 83, 325-345.

[23] Fisman, R., and Wang, Y. (2015). The Mortality Cost of Political Connections. *The Review of Economic Studies*, 82(4), 1346-1382.

[24] Ferguson, T. and Voth, H.J. (2008). Betting on Hitler-the Value of Political Connections in Nazi Germany. *The Quaterly Journal of Economics*, 123(1), 101-137.

[25] Gandhi, J. (2008). Dictatorial Institutions and their Impact on Economic Growth. *European Journal of Sociology,* 49 (1), 3-30.

[26] Goldman, E, Rocholl, J., and Drope, J.M. (2005). Politically Connected Boards of Directors and the Allocation of Procurement Contracts. *Review of Finance*, 17, 1617-1648.

[27] Jayachandran, S. (2006). The Jeffords Effect. *Journal of Law and Economics*, 49(2), 397-425.

[28] Jin, H., Qian, Y., and Weingast, B. (2005). Regional Decentralization and Fiscal Incentives: Federalism, Chinese Style. *Journal of Public Economics*, 89, 1719-1742.

[29] Jin, H. and Qian, Y. (1998). Public versus Private Ownership of Firms: Evidence from Rural China. *The Quarterly Journal of Economics*, 113(3), 773-808.

[30] Khwaja, A.I., and Mian, A. (2005) Do Lenders Favor Politically Connected Firms? Rent Provision in an Emerging Financial Market. *The Quarterly Journal of Economics*, 120(4), 1371-1411.

[31] Knight, J. (2014). China as a Developmental State. *the World Economy*, 1335-1347.

[32] Koren, M., Szeidl, A., and Vedres, B. (2015). Political Favor Exchange in a Democreacy. Working paper.

[33] Laffont, J. (2001). Incentives and Political Economy. Oxford University Press.

[34] Lin, C., Lin, P., and Song, F. (2010). Property Rights Protection and Corporate R&D: Evidence from China. *Journal of Development Economics*, 93, 49-62.

[35] Long, C., Yang, J., and Zhang, J. (2015). Institutional Impact of Foreign Direct Investment in China. World Development, 66, 31-48.

[36] Mauro, P. (1995). Corruption and Growth. *The Quaterly Journal of Economics*, 110(3), 681-712.

[37] Menezes, F., and Monteiro, P. (2006). Corruption and Auctions. *Jourmal of Mathematical Economics*, 42(1), 97-108.

[38] Mironov, M., and Zhuravskaya, E (2016). Corruption in Procurement: Evidence from Financial Transactions Data. *American Economic Journal: Economic Policy*, 8(2), 287-321.

[39] North, D. (1990). Institutions, Institutional Change and Economic Performance. Cambridge University Press.

[40] Schoenherr, D., (2015). Political Connections and Allocative Distortions. Working paper.

[41] Straub, S. (2014). Political Firms, Public Procurement, and the Democratization Process. IDEI working paper.

[42] Tirole, J. (1986). Hierarchies and Bureaucracies: on the Role of Collusion in Organizations. *Journal of Law, Economics, & Organization.* 2(2), 181-214.

[43] Wang, X., Fan, G., and Zhu, H. (2007). Marketisation in China: Progress and Contribution to Growth. China Linking Markets for Growth. ANU E Press and Asia Pacific Press, Canberra.

[44] Wei, S. (2000). Local Corruption and Global Capital Flows. *Brooings Papers on Economic Activity*, 31(2), 303-354.

[45] Xu, C. (2011). The Fundamental Institutions of China's Reforms and Development. *Journal of Economic Literature*, 49(4), 1076-1151.

[46] Zilibotti, F. (2017). Growing and Slowing Down Like China. *Journal of the European Economic Association*, 15(5), 943-988.

8 Appendix

Proof of Lemma 2: Note that we only have to show the term $\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})$ is strictly monotonic in x_2, where $\bar{y} = \bar{y}(x_2, c)$.

On equilibrium path, $\bar{y} < x_2$. Therefore, $\pi_u(x_2, \bar{y}) = [1 - G(\bar{y})](b - x)$. Since $\pi_f(x_2, \bar{y}) = \int_{x_2}^{b} (y - x) g(y) dy$, if we take derivative with respect to x_2 in $\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})$, we have

$$\frac{d[\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})]}{dx_2} = \frac{\partial[\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})]}{\partial x_2} + \frac{\partial[\pi_u(x_2, \bar{y}) - \pi_f(x_2, \bar{y})]}{\partial \bar{y}} \cdot \frac{\partial \bar{y}(x_2, c)}{\partial x_2}$$

where by the implicit function theorem,

$$\frac{\partial \bar{y}(x_2,c)}{\partial x_2}=\begin{cases}\frac{f(x_2)(x_2-\bar{y}-c)}{F(x_2)-F(\bar{y})},\ \bar{y}\in[a,x_2]\\ \frac{f(x_2)(x_2-\bar{y}-c)}{F(x_2)},\ \text{otherwise}\end{cases}$$

Note that when $\bar{y}\in[a,x_2]$, the term $x_2-\bar{y}-c=x_2-\bar{y}-\int_{\bar{y}}^{x_2}(x-\bar{y})\frac{f(x)}{F(x_2)}dx\geq x_2-\bar{y}-(x_2-\bar{y})\int_{\bar{y}}^{x_2}\frac{f(x)}{F(x_2)}dx>0$; when $\bar{y}\in[0,a)$, $x_2-\bar{y}-c=x_2-\bar{y}-\int_{a}^{x_2}(x-\bar{y})\frac{f(x)}{F(x_2)}dx\geq x_2-\bar{y}-(x_2-\bar{y})\int_{a}^{x_2}\frac{f(x)}{F(x_2)}dx>0$. Therefore, $\frac{\partial \bar{y}(x_2,c)}{\partial x_2}>0$ always holds.

After some calculations, we have

$$\frac{d[\pi_u(x_2,\bar{y})-\pi_f(x_2,\bar{y})]}{dx_2}=G(\bar{y})-G(x_2)-g(\bar{y})(b-x_2)\cdot\frac{\partial \bar{y}(x_2,c)}{\partial x_2}$$

Since $\bar{y}<x_2$, $b\geq x_2$ and $\frac{\partial \bar{y}(x_2,c)}{\partial x_2}>0$, $\frac{d[\pi_u(x_2,\bar{y})-\pi_f(x_2,\bar{y})]}{dx_2}<0$ always holds. This completes the proof.■

Proof of Proposition 1: As long as the term $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})$ weakly decreases in x, LF has no incentive to deviate because $\pi_u(x_2,\bar{y})-\pi_f(x_2,\bar{y})=d_2$. Next we check the monotonicity of $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})$.

$$\frac{d[\pi_u(x,\bar{y})-\pi_f(x,\bar{y})]}{dx}=\begin{cases}0,\text{ if }\bar{y}\geq a\text{ and }x\leq\bar{y}\\ G(\bar{y})-G(x),\text{ if }\bar{y}<a\text{ or }\bar{y}\geq a\text{ but }x>\bar{y}\end{cases}$$

Clearly, when $\bar{y}<a$, $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})$ strictly decreases in x; when $\bar{y}\geq a$, $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})$ is a constant for $x\leq\bar{y}$ but strictly decrease in x for $x>\bar{y}$. Since $\bar{y}<x_2$ always holds, we have $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})>d_2$ for $x<x_2$ and $\pi_u(x,\bar{y})-\pi_f(x,\bar{y})<d_2$ for $x>x_2$. This completes the proof.■

Proof of Proposition 2: Note that a marginal LF (with $x=x_2$) is indifferent between accepting d_2 or rejecting it in equilibrium. Meanwhile, the official never asks for any deal that would make him worse off. This means the potential size of the pie should be positive: the minimum value of d_2 the official asks for cannot exceed the marginal LF's maximum willingness to pay in order for him to set up c.

We first consider the marginal LF's problem. Such a LF expects to receive $(b-x_2)\cdot[1-G(\bar{y})]$ if c is set up while it receives $E[y-x_2|y>x_2]\cdot[1-G(x_2)]$ otherwise. Therefore, the maximum willingness to pay should be $(b - x_2) \cdot [G(x_2) - G(\bar{y})] + E[b - y|y > x_2] \cdot [1 - G(x_2)]$, which is the difference between the above two terms.

Next we check the official's incentive to offer d_2. There are several events depending on values of x and y: when $x > x_2$, regardless of the value of y, his contingent payoff is not affected because LF cannot accept d_2; when $x \leq x_2$ and $y \leq \bar{y}$, his payoff is again not affected because GF is competitive enough to afford c; when $x \leq x_2$ and $y \in (\bar{y}, x_2]$, his payoff decreases from $E[u(b - \max\{x, y\})|y \in (\bar{y}, x_2]]$ to 0; when $x \leq x_2$ and $y \in (x_2, 1]$, his payoff decreases from $E[u(b - y)|y \in (x_2, 1]]$ to 0. After multiplying probabilities in each of these events accordingly, we have that d_2 has to be at least $E[u(b - \max\{x, y\})|y \in (\bar{y}, x_2]] \cdot [G(x_2) - G(\bar{y})] + E[u(b - y)|y \in (x_2, 1]] \cdot [1 - G(x_2)]$ for the official to break even.

The necessary and sufficient condition for a secret deal d_2 to take place is that there exists at least one $x_2 \in (a, x_1]$ for the potential size of the pie to be positive. When $u(z) < z$ for any z, it can be immediately detected that the pie size is strictly positive. Therefore, it is a sufficient condition.■

Proof of Proposition 4: When $Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is strictly concave in x_2 for any x_1, $F(x_1)Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is also strictly concave in x_2. Moreover, it is obvious that the maximizer of $Rev_2(x_1, x_2, \bar{y}(x_2, c))$ is also the maximizer of $F(x_1)Rev_2(x_1, x_2, \bar{y}(x_2, c))$. Denote this maximizer to be $x_2^*(x_1)$.

From derivation of the second-stage problem, we know $F(x_1)\frac{dRev_2(x_1,x_2,\bar{y}(x_2,c))}{dx_2}$ does not contain x_1, implying that $F(x_1)Rev_2(x_1, x_2, \bar{y}(x_2, c))$ has the same first-order properties as $Rev_2(1, x_2, \bar{y}(x_2, c))$ fixing x_2. Since $Rev_2(1, x_2, \bar{y}(x_2, c))$ has the unique maximizer at $\hat{x}_2$, we can characterize $x_2^*(x_1)$ as the following:

$$x_2^*(x_1) = \begin{cases} x_1, \text{ when } x_1 \leq \hat{x}_2 \\ \hat{x}_2, \text{ otherwise} \end{cases}$$

Hence,

$$Rev_2(x_1, x_2^*(x_1), \bar{y}(x_2^*(x_1), c)) = \begin{cases} Rev_2(x_1, x_1, \bar{y}(x_1, c)), \text{ when } x_1 \leq \hat{x}_2 \\ Rev_2(x_1, \hat{x}_2, \bar{y}(\hat{x}_2, c)), \text{ otherwise} \end{cases}$$

Next we check $\frac{dRev(x_1)}{dx_1}$. When $x_1 \leq \hat{x}_2$, $\frac{dRev(x_1)}{dx_1} = \frac{dF(x_1)(v-x_1)}{dx_1}$

$+\delta\frac{d\{F(x_1)Rev_2(x_1,x_1,\bar{y}(x_1,c))+[1-F(x_1)]E(second\ stage\ payoff\ from\ fair\ competition|x>x_1)\}}{dx_1}$, where the second term (inside the bracket) that is taken derivative of represents the official's second-stage payoff when

x_2 is restricted to be x_1. Interestingly, it thus also becomes the target function of a new problem: picking an optimal cutoff to maximize the second-stage payoff, where the cutoff has to be less than or equal to 1. There is a mathematical trick in this interpretation: now the official calls the cutoff x_1 instead of x_2. However, the name of the cutoff does not change the nature of this problem. Therefore, this problem is essentially to maximize $Rev_2(1, x_2, \bar{y}(x_2, c))$ by picking $x_2 \leq 1$. The term $F(x_1)Rev_2(x_1, x_1, \bar{y}(x_1, c)) + [1 - F(x_1)]E(second\ stage\ payoff\ from\ fair\ competition|x > x_1)$ thus has the same mathematical properties as $Rev_2(1, x_2, \bar{y}(x_2, c))$: strictly concave with a unique maximizer $\hat{x}_2$.

The above argument implies that when $x_1 \leq \hat{x}_2$, $Rev(x_1)$ is the summation of two strictly concave functions: $F(x_1)(v - x_1)$ has the maximum at r^* while the other has the maximum at $\hat{x}_2$.

When $x_1 > \hat{x}_2$, after some derivations, we have $\frac{dRev(x_1)}{dx_1} = \frac{dF(x_1)(v-x_1)}{dx_1} = f(x_1)(b - x_1) - F(x_1)$.

To find out the optimal x_1, we check the properties of $Rev(x_1)$ over the two intervals $[a, \hat{x}_2]$ and $(\hat{x}_2, 1]$. When $x_1 \in [a, \hat{x}_2]$, if $r^* > \hat{x}_2$, $\frac{dRev(x_1)}{dx_1}$ is strictly positive over this region; if $r^* \leq \hat{x}_2$, $\frac{dRev(x_1)}{dx_1}$ achieves 0 at some point inbetween r^* and $\hat{x}_2$. When $x_1 \in (\hat{x}_2, 1]$, $\frac{dRev(x_1)}{dx_1} = 0$ at r^* when $r^* > \hat{x}_2$ and $\frac{dRev(x_1)}{dx_1}$ is strictly negative when $r^* \leq \hat{x}_2$. Therefore, x_1^* is r^* when $r^* > \hat{x}_2$ and is inbetween r^* and $\hat{x}_2$ when $r^* \leq \hat{x}_2$.

This completes the proof.■

Proof of Lemma 4: In this case, the welfare loss$\leq \int_0^{\bar{y}} \int_0^{x_2} cf(x)f(y)dxdy + \int_{\bar{y}}^{x_2} \int_{\bar{y}}^{x} (x_2 - y)f(x)f(y)dydx$ since $\delta \leq 1$. Moreover, because $c < x_2 - \bar{y}$, we have

$$
\begin{aligned}
\text{the loss} &< \int_0^{\bar{y}} \int_0^{x_2} (x_2 - \bar{y})f(x)f(y)dxdy + \int_{\bar{y}}^{x_2} \int_{\bar{y}}^{x} (x_2 - y)f(x)f(y)dydx \\
&< \int_0^{x_2} \int_0^{x_2} (x_2 - y)f(x)f(y)dxdy \\
&= \int_0^{x_2} \int_0^{x_2} (x_2 - x)f(y)f(x)dydx = F(x_2) \int_0^{x_2} (x_2 - x)f(x)dx < \int_0^{x_1^*} (b - x)f(x)dx
\end{aligned}
$$

where $\int_0^{x_1^*} (b - x)f(x)dx$ is the welfare gain in the first stage. Therefore, the gain outweighs the loss.■